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Calculus III
Professor Piotr Hajłasz
Second Second Exam
November 16, 2015.

Problem	Possible points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1. Evaluate the integral $\iiint_T x^2 dV$, where T is the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

Problem 2. Evaluate the integral

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2 + y^2 + z^2} dz dy dx.$$

Problem 3. Rewrite the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) dz dy dx \quad \text{as} \quad \int_{?}^{?} \int_{?}^{?} \int_{?}^{?} f(x, y, z) dy dz dx.$$

Problem 4. Find a function f such that $\mathbf{F} = \nabla f$ and use it to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = yze^{xz} \mathbf{i} + (e^{xz} + e^y) \mathbf{j} + (xye^{xz} + 2z) \mathbf{k}$ and C is parametrized by $\mathbf{r}(t) = (t^2 + 1) \mathbf{i} + (t^2 - 1) \mathbf{j} + (t^2 - 2t) \mathbf{k}$, $0 \leq t \leq 2$.

Problem 5. Use Green's theorem to evaluate $\int_C xy^2 dx + 2x^2y dy$, where C is the positively oriented triangle with vertices $(0,0)$, $(2,2)$ and $(2,4)$.