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Calculus III

Professor Piotr Hajłasz

Second Exam

March 30, 2015.

Problem	Possible points	Score
1	20	
2	10	
3	10	
4	20	
5	10	
6	20	
7	10	
8	10	
Total	110	

You need 100 to have a perfect score. 10 additional points is a bonus.

Problem 1. (20p.) Using the method of Lagrange multipliers, find the area of the largest rectangle with pairs of sides parallel to the coordinate axes that can be inscribed in the ellipse $x^2 + 4y^2 = 1$. Also give the coordinates of the corner of the rectangle in the first quadrant.

Problem 2. (10p.) Evaluate the integral $I = \int_0^2 \int_{\sqrt{y/2}}^1 y \exp(x^5)\, dx\, dy$.

Exercise 3. (10p.) Evaluate the integral

$$I = \int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\, dy\, dx$$

by converting to polar coordinates. (In order to compute the integral it might be useful to note that $(-(\cos\theta)^5/5)' = (\cos\theta)^4 \sin\theta$.)

Problem 4. (20p=10p+10p). Let E be the region in the first octant bounded by the surfaces $2y^2 + z^2 = 8$ and $x + y = 2$, and let $f(x, y, z)$ be a function whose domain contains E. Denote $I = \iiint_E f(x, y, z), dV$.

(a) Set up the iterated triple integral $\int_?^? \int_?^? \int_?^? f(x, y, z)\, dz\, dx\, dy$.

(b) (Continuation from the previous page.) Set up the iterated triple integral $\int_{?}^{?}\int_{?}^{?}\int_{?}^{?} f(x,y,z)\,dx\,dy\,dz$

Problem 5 (10p.) Find the volume of the solid bounded by the plane $z = 0$ and the surface $z = 1 - x^2 - y^2$.

Problem 6 (20p.) Evaluate the integral $\displaystyle\iiint_E x^2\, dV$ where E is bounded by the xy-plane and the hemispheres $z = \sqrt{9 - x^2 - y^2}$, $z = \sqrt{16 - x^2 - y^2}$.

Hint: You can use without any explanations the following integrals: $\displaystyle\int_0^{\pi/2} \sin^3 x\, dx = 2/3$, $\displaystyle\int_0^{2\pi} \cos^2 x\, dx = \pi$.

Problem 7 (10p=5p+5p.) Let $\mathbf{F} = \langle yz(2x+y), xz(x+2y), xy(x+y)\rangle$.
(a) Find a function $f(x,y,z)$ such that $\nabla f = \mathbf{F}$.

(b) (Continuation from the previous page.) Evaluate the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C has parametrization $\mathbf{r} = \langle 1+t+\sin(\pi t), 1+2(1+\sin(\pi t))t^2, 1+3t^2\rangle$, $0 \le t \le 1$.

Problem 8 (10p.) Use Green's theorem to find the integral $\int_C (x^2 - y)\,dx + (x + y^2)\,dy$, where C is the perimeter of the unit square with vertices $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$, with positive orientation.[1]

[1]No Green's theorem, no credit :(