Print your first and last name legibly above the line:

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Calculus III

Professor Piotr Hajłasz

Second Exam

November 13, 2015.

Problem	Possible points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1.

(a) Evaluate the integral

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{y}{x}\, dx\, dy.$$

(b) Evaluate the integral

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dx\, dy$$

by converting it to polar coordinates.

Problem 2. Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} e^{(x^2+y^2+z^2)^{3/2}}\, dz\, dy\, dx.$$

Problem 3. Rewrite the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\,dz\,dy\,dx \quad \text{as} \quad \int_?^? \int_?^? \int_?^? f(x,y,z)\,dx\,dz\,dy.$$

Problem 4. Find a function f such that $\mathbf{F} = \nabla f$ and use it to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \sin y\,\mathbf{i} + (x \cos y + \cos z)\,\mathbf{j} - y \sin z\,\mathbf{k}$ and C is parametrized by $\mathbf{r}(t) = \sin t\,\mathbf{i} + t\,\mathbf{j} + 2t\,\mathbf{k}$, $0 \le t \le \pi/2$.

Problem 5. Use Green's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle y - \cos y, x \sin y \rangle$ and C is the circle $(x-3)^2 + (y+4)^2 = 4$ oriented **clockwise**.