GALILEUS DECEPTUS, NON MINIME DECEPIT: A RE-APPRAISAL OF A COUNTER-ARGUMENT IN *DIALOGO* TO THE EXTRUSION EFFECT OF A ROTATING EARTH

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1. *Introduction*

In a justly proud reminder to himself of his achievements on centrifugal force, Christiaan Huygens noted: "Galileus deceptus.… Neutonus applicuit feliciter ad motus ellipticos Planetarum. [H]inc quanti sit haec vis centrifugae cognitio apparet."[1] Though we may doubt whether Newton would have acknowledged his debt to Huygens, and wonder what Galileo might have replied, Huygens's comment on Galileo's deluding himself on centrifugal force seems, with few exceptions, to have found favour with twentieth-century historians and philosophers of science. It is unclear exactly to which passage of Galileo's *Dialogue concerning the two chief world systems* Huygens referred, so perhaps we have to take his comment as applicable to the whole argumentative strategy propounded by Galileo in the relevant sections of the Second Day of the *Dialogue*.[2] As we shall see, Galileo basically wishes to prove that no matter how fast the Earth rotates daily on its polar axis, objects on its surface would never be extruded, i.e., they would never fly off toward the sky. That this should be the case was a rather common objection raised by anti-Copernicans at that time. Thus, in the late 1930s, Alexandre Koyré pointed out that "Galileo's argument … is extremely subtle and seductive. Unfortunately it is incorrect; and what is worse, it is manifestly incorrect".[3] Others followed Huygens and Koyré in their negative assessment of Galileo's argument.[4]

About twenty years ago, however, David K. Hill went so far as to claim that Galileo crossed the line between honest argument and conscious deception, and that he knew full well that his counter-argument to the anti-Copernicans was seriously flawed.[5] Eventually, dissent in the debate was expressed by Stillman Drake in a rejoinder note to Hill.[6]

In my view, the merit of Drake's short rejoinder consists in having exposed the bundle of sometimes confused assumptions about the behaviour of bodies on a rotating Earth, on which the contemporary chorus of negative opinion was based. Drake argued that bodies on a Earth rotating faster and faster eventually reach the condition of weightlessness, after which they continue to orbit the Earth, remaining at rest with respect to a terrestrial observer. In other words, weightless bodies behave like geostationary satellites situated in close proximity of the surface of the Earth. Thus Drake thought he could rescue Galileo's argument, on purely physical grounds, and salvage Galileo's moral reputation. However, Drake somehow missed the point of Hill's criticism of Galileo. For, as we shall see, Hill did not analyse Galileo's counter-

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argument on the basis of classical (i.e., Newtonian) physics, as had been done by other scholars in the twentieth century. On the contrary, he pointed out a flaw in Galileo's reasoning in the light of Galileo's own physics of projectile motions, an internal and destructive objection that, according to Hill, Galileo himself could not have failed to raise. Hence Hill's claim about Galileo's morally deplorable presentation of a fundamentally flawed argument, a conscious act of deception.[7] More recently, Maurice Finocchiaro analysed in great detail the logical structure of Galileo's argument.[8] Finocchiaro shifted the focus of the controversy, coming to the conclusion that "Galileo's reflections on the nature of physical mathematical reasoning, when proper contextualized…, do not conflict with the definition [of Galileo's mathematical reasoning] I extracted from his extrusion argument".[9] Finocchiaro's definition of Galileo's mathematical reasoning is as follows: "Physical-mathematical reasoning is reasoning about physical processes and phenomena such that various aspects of them are represented by mathematical entities, various mathematical conclusions are reached about these mathematical entities, and then these mathematical conclusions are applied to the physical situation."[10] In the light of his analysis, Finocchiaro was able to dissolve some of the tensions in Galileo's extrusion argument (although Finocchiaro stopped short of commenting on Hill's conclusions).

Finocchiaro's study suggested to me a possible new standpoint from which to tackle, once again, the issues raised by Galileo's extrusion argument, namely, contextualization. The relevant context in which I will place Galileo's argument is that of late sixteenth- and early seventeenth-century mathematical reasoning.

In this paper, I will re-examine Galileo's counter-argument to the extrusion effect in the context of his understanding of the "angle of contingence", a dimension of Galileo's reasoning that has so far been neglected in the debate. I will argue that it is precisely this dimension that further illuminates the counter-argument, thus resolving the apparently internal conflict in Galileo's physics, and that Hill's claim — when viewed from the standpoint of Galileo's understanding of the "angle of contingence" — becomes untenable. Galileo went wrong (by the lights of subsequent developments in mathematical physics), but he did not consciously deceive.

Section 2 will present a brief sketch of the history of interpretations of the extrusion effect. Section 3 will reconstruct Galileo's views on the angle of contingence and similar parabolic trajectories. It will focus on a letter by Galileo on the angle of contingence and on related preparatory material for *Two new sciences* — two key documents that have so far been virtually ignored. Both these sections will emphasize the need for placing Galileo's take on the anti-Copernican argument from extrusion in the context of a culture at the intersection of orality and writing. Section 4 will discuss in some detail Hill's fascinating claim and Galileo's counter-argument to the extrusion effect, on the basis of the results of the two preceding sections. I will finally draw some conclusions, and point to directions for future research in Section 5.

2. *An Historical Sketch of the Extrusion Effect*

To prepare the reader, here I give a brief sketch of the history of interpretations of the extrusion effect, only underlining certain aspects that seem more relevant for the limited scope of my paper. A broader discussion of the history of the extrusion effect and of its role in the emergence of centrifugal force can be found in a recent study by Harald Siebert.[11] In what follows, I will restrict my analysis mostly to textual aspects that I found problematic and especially significant. The extrusion effect of the diurnal rotation of the Earth is presented by Galileo in the *Dialogue* as follows.

> Now there remains the objection based upon the experience of seeing that the speed of a whirling has a property of extruding and discarding material adhering to the revolving frame. For that reason it has appeared to many, including Ptolemy, that if the Earth turned upon itself with great speed, rocks and animals would necessarily be thrown toward the stars, and buildings could not be attached to their foundations with cement so strong that they too would not suffer similar ruin.[12]

The question immediately arises of Galileo's attribution to Ptolemy of a similar argument. The implicit reference seems to be to *Almagest* Book 1, Chapter 7. Here is G. J. Toomer's translation of the relevant passage from the original Greek (on the basis of Heiberg's text).

> If the Earth had a single motion in common with other heavy objects, it is obvious that it would be carried down faster than all of them because of its much greater size: living things and individual heavy objects would be left behind, riding on the air, and the Earth itself would very soon have fallen completely out of the heavens. But such things are utterly ridiculous merely to think of.[13]

A more literal reading of the passage has been suggested to me by James G. Lennox, as follows:[14]

> But if there were some motion of the Earth that was one and the same and shared with the other heavy bodies, it is clear that it would overtake everything in descent on account of its much greater magnitude, and the animals and individual heavy bodies floating on the air would be left behind, and the Earth would very quickly fall from the very heaven itself. But even contemplating such things would appear the most laughable thing of all.

This text from the *Almagest* is highly problematic. It is not obvious, at least to my mind, what the meaning conveyed by the image of an Earth's falling from the heaven exactly is. The beginning of the passage highlights a common motion. The phrasing is consistent with both a rectilinear and a circular motion. Presumably, however, given the general context of the initial discussion in Chapter 7, a rectilinear motion is intended by Ptolemy. The subsequent portion of Chapter 7 focuses on circular motion explicitly and eventually goes on to dismiss the possibility of a diurnal rotation of the Earth around its polar axis. The challenge posed by the passage is reflected in the difficulties probably encountered by the translators of the versions circulating in the Renaissance. In the Latin edition from Greek by George of Trebizond (1395–1484) we read that the Earth "velocissime extra coelum quoque ipsum excideret".[15] Giovan Battista della Porta (1535–1615) published a partial edition from Greek of the *Almagest* limited to Book 1, in 1605, where he rendered the passage similarly, "ipsa et celerrime postremo cecidisset et ab ipso coelo".[16] The fact is that there is a potential ambiguity with the rendering of the verb ἐκπίπτω in this context. It basically means "to fall from", but it also means "to go forth, to issue forth". The Latin cognate, "excido", chosen by George of Trebizond, has two distinct semantic values, namely, "to fall from" and "to raze, to demolish". Whether these values are in fact to be found in the original ἐκπίπτω is highly debatable. Della Porta has avoided ambiguity choosing "cado". On the other hand, as we shall see in a moment, Copernicus seems to have interpreted "excido" precisely in the sense that the Earth would demolish the heavens.

Again, whether the second value of "excido", i.e., "to demolish", conveys a possible value of ἐκπίπτω, or whether it is too strong, is debatable. It also true, however, that, for those who took the heavens to be solid crystalline orbs, the Earth's falling from the heavens would have to cause some damage to the crystalline orbs enveloping the Earth.[17] It is clear that both George of Trebizond and Della Porta intended the passage in the sense of "falling". But they constructed their phrasings with different prepositions, "extra" and "ab", to reinforce the idea of motion *beyond* a place, and motion *from* a place (where "place" must not be construed as a technical term in cosmology, but simply as a placeholder for the prepositional phrase).

In the version of the *Almagest* from Arabic, however, published in 1515, and apparently in Galileo's personal library, the problematic passage is resolved somewhat more openly, in a bifurcating rendition with two verbs, "et terra velociter omnino caderet: et pertransiret celum solum".[18] Here the translator opted for a solution that emphasized both the "falling [*cado*]" (without specifying the place from which the Earth was supposed to fall, though) and the "going through [*pertranseo*]" the heavens. Moreover, Chapter 7 in the version from Arabic is headed "De eo quod indicat quod terra motum localem non habeat", whereas in the version from Greek by George of Trebizond Chapter 7 is headed "Quod terra nullo motu progressivo movetur".[19] The two texts signal slightly different interpretations, the translator from Arabic more broadly emphasizing the Earth's being deprived of local motion, while George of Trebizond spotlights the Earth's not moving by progressive motion. Finally, George of Trebizond's translation has a rather awkward "universandum deferetur". The gerundive "universandum" is problematic, in my view. I am at a loss as to how to translate it. Della Porta, who in Chapter 7 is otherwise in general agreement with George of Trebizond, gets rid of it. The 1515 edition of the *Almagest*, from Arabic, has simply "inferius iret". In sum, there is little doubt that the semantic options open to a Renaissance reader of *Almagest*'s Chapter 7 were multifarious, and many passages badly in need of interpretive work.

It is quite possible that Copernicus's reading of the *Almagest*'s difficult passage led Galileo to interpret the *Almagest*'s passage as referring to the diurnal rotation of the Earth. In Copernicus's reading the extrusion argument is attributed to Ptolemy explicitly (with the verb "excidere" used by Copernicus, I think, in the second sense, as I already anticipated). Here I follow Siebert's intimation that we should read the passage, according to grammar, taking the verb "excido" in the second sense already mentioned.[20]

Further evidence suggests, on the other hand, that very early on in his career Galileo consciously (and perhaps independently of Copernicus) attributed the extrusion argument to Ptolemy. In Galileo's rather traditional "Treatise on the sphere" — used as a basis for lectures at the university of Padua — we find a section entitled "That the Earth is immobile", in which Galileo seems to imply that he is closely following Chapter 7 of Ptolemy's *Almagest*. However, even though the text is presented as a quasi-paraphrase of Ptolemy's own rebuttal of the Earth's diurnal rotation, the series of arguments attributed to Ptolemy does not fully match *Almagest*'s Chapter 7, and surprisingly ends in crescendo with a clear statement of the extrusion effect.[21] Moreover, and to complicate matters further, Galileo's assertion that "essendo il moto circolare e veloce accommodato non all' unione, ma più tosto alla divisione e dissipazione" is strongly reminiscent of Copernicus's assertion that "[q]uae vero repentina vertigine concitantur, videntur ad collectionem prorsum inepta, magisque unita dispergi". To cap it all, in the text of the Latin version from Arabic immediately preceding the problematic passage an image is presented of moving bodies aggregating toward the centre, and remaining fixed and compressed there because of pressure coming from all parts uniformly seeking to reunite at the centre. This obviously runs counter to Copernicus's image, according to which things rotating fast around a centre are "ad collectionem prorsum inepta".[22]

To complete this historical sketch of the argument from extrusion, another relevant item of evidence needs to be considered, namely, Cristoph Clavius's presentation of the extrusion effect in his *Commentary on the Sphere*.[23]

The *Commentary on the Sphere* might indeed have reinforced the polemical appeal of the extrusion argument, its value as a target for convinced Copernicans, so to say, given the popularity of the commentary and reputation of its author.[24] Clavius rehearses the argument as follows. If the Earth rotated around the axis of the world in twenty-four hours, "all edifices would be destroyed, and in no way could they remain firm".[25] In effect the textual context in which Clavius's vision of collapsing buildings is delineated suggests an intriguing possibility. The section "That the Earth is immobile" of Galileo's *Treatise on the Sphere*, might have been modelled, at least in part, precisely on Clavius's presentation of the argument from extrusion. I believe that this conclusion is further supported by the list of arguments not matching *Almagest*'s Chapter 7 that are summarized by Galileo in that section of the *Treatise*, and which appear in Clavius's text. In particular, the argument of an arrow thrown upwards vertically, which would not fall back in the same place, and the image of a stone falling from the mast of a moving ship, are discussed by Clavius immediately following the catastrophic picture of collapsing buildings.[26] Galileo reversed the order of presentation, reserving the extrusion effect for his short finale, but kept to the substance of Clavius's argumentative strategy.

Thus, Ptolemy's, Copernicus's, and Clavius's texts coalesced in Galileo's memory, forming a converging framework of ideas. He reorganized, so to speak, the intricate network of verbal arguments and mental images, directly or indirectly related to *Almagest*'s Chapter 7, that he found in relevant contemporary works. Eventually he attributed the argument from extrusion to Ptolemy himself. We should not forget that Galileo's culture was still influenced by a style of intellectual approach to texts typical of oral cultures. Memorizing content rather than checking for the verbatim exactness of quotations was often a scholar's more urgent mode of interaction with books. As Walter Ong has masterfully taught us, oral cultures are aggregative rather than analytic. The aggregative character of orality-based thought, Ong suggests, tends to emphasize not so much integral units as clusters of units.[27] In the present case, we see not so much an integral argument, but rather a cluster of arguments, the mode of appropriation of which is the act of memorizing the cluster around a central theme.

Thus, we should not find it exceptional that Galileo aggregated a sparse network of ideas into a memorable framework for thinking about Earth's diurnal rotation and the extrusion effect.[28] Within Galileo's mind, Ptolemy simply became the attractive pole that oriented the aggregative effect of oral modes of cognition in contact with written material.

Finally, two further developments are worth noting, which tend to corroborate the conclusion that Galileo's style of reading was still part of an orality-dominated mode of assimilation of texts. The first is a gut-feeling response by Galileo himself in the form of a marginal postil to a book presenting the extrusion argument. The second is the appearance of an historical text sanctioning the legitimacy of reading the *Almagest*'s controversial text as intimating the extrusion effect.

In 1612 the philosopher Giulio Cesare La Galla (1576–1624), a friend of Galileo's, published a long dissertation refuting the plausibility of Galileo's recent astronomical discoveries.[29] Galileo wrote numerous postils in the margin of La Galla's book. La Galla discusses the argument from extrusion at length, referring to it as "that formidable argument by Ptolemy".[30] When, further on in the text, La Galla reiterates the point that if the Earth rotated diurnally then all edifices, trees, and everything else would be destroyed in less than a day, Galileo inscribed in the margin "[m]elius dixisset Ptolemaeus…".[31] When solicited by the textual cue of the extrusion effect, Galileo's memory naturally responded activating the framework of ideas converging on Ptolemy.

Four decades later, G. B. Riccioli (1598–1671), in his massive *Almagestum novum* (1651), gave a detailed résumé of the history of the argument from extrusion up to the mid-seventeenth century. Riccioli quoted many authors but anchored the progression of readings of *Almagest*'s Chapter 7 to Copernicus.[32] He juxtaposed a verbatim quotation of the latter's comments with a quotation of the difficult passage from *Almagest*'s Chapter 7 (actually in a Latin version slightly different from all of those I have mentioned, presumably his own, or one that I have not identified). Significantly, Riccioli claims that Copernicus attributed the extrusion argument to Ptolemy. At the same time Riccioli seems implicitly to accept that the *Almagest*'s problematic passage may, at least obscurely, hint at the extrusion effect, especially since he refrains from commenting on Copernicus's attribution.[33] In this way, I would argue, Riccioli sanctioned the legitimacy of reading Ptolemy's passage as the first sediment of an accretive deposit of interpretations thrusting upward to the extrusion effect. A set of sparse references, which had originally been nurtured in an amalgam of orality-shaped interactions with books, was historicized by Riccioli into an incipient, written textual tradition.

3. *Galileo on the Angle of Contingence and Similar Parabolas*

In a letter written in 1635 to the mathematician Giovanni Camillo Gloriosi (1572–1643), who had succeeded him in the chair of mathematics at Padua in 1613, Galileo expounded his views on the angle of contingence.[34] Apart from the technical content strictly relevant to our goal in this paper, which I shall discuss presently, the letter affords us a rare glimpse of ideas that Galileo never committed to writing in full, for reasons on which unfortunately we can only speculate.

Galileo begins the letter with a typical old-person's complaint about his failing memory due to his age. Then, he opens his arguments by saying that he will relate a discourse on the angle of contingence which ran into his imagination [*fantasia*] a long time before.[35] A little further on, he remembers that some time in the past he also excogitated many "discourses" on the same question, only one of which he will expand on in the letter. Both the reference to the "imagination" and the rather ambiguous use of the term "discourse" suggest that Galileo was reconstructing mental content from his memory rather than from written material in his notebooks (although, in fairness, it must be said that in 1635 he was on the brink of blindness).

The first discourse related by Galileo in the letter to Gloriosi is intended to prove that the angle of contingence is called "angle" only equivocally, it being in fact not a true angle. Galileo makes his first move from what he takes to be the accepted definition of *angle*, i.e., the inclination of two lines touching each other at a point that are not placed straight with respect to each other.[36] He then proposes the following argument (*cf*. Figure 1).

Let us consider a regular polygon inscribed in a circle. The inclinations of the sides are as many as the sides, if the number of sides is uneven, or half the number of sides if the latter is even (since in this case two opposite sides will have the same inclination). If we now imagine that a side of the polygon is applied to any straight line whatever, no angle will be formed between the side and the straight line since

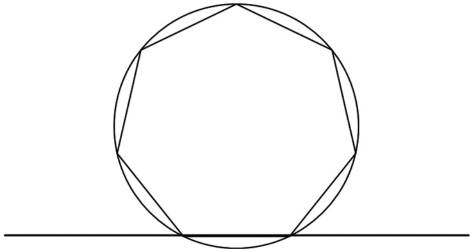

FIG. 1. This figure, I think, more accurately reflects Galileo's thinking than that printed in *Opere*, xvi, 331, which was based on the first edition of the letter given by Gloriosi. The absence of references in Galileo's text to the lettering of the diagram printed by Gloriosi might suggest that Galileo's original figure was different, or that there was no figure at all accompanying the reasoning. In the latter case my reconstruction would have only a didactic value.

they progress along the same direction. But the subsequent side will form an angle since it is inclined to the line and touches it. Given that the circle is conceived of as a polygon of infinite sides, then all directions will be found in its perimeter, that is infinite directions. There will thus be the direction of any line whatever, which can only be thought of as that of the side applied to it. Therefore the side of the circle applied to the straight line does not form an angle with the straight line, and this is the so-called point of contact. It is also inappropriate to say that although a point on the circumference does not contain an angle with the tangent at that point, the contiguous point will contain such an angle, exactly as in the polygon it is the subsequent side that forms the angle with the direction of the preceding side. The reason is that the point subsequent to the point of contact does not touch the straight line, which is touched only by one point of the circumference. Therefore since in the definition of angle both the inclination and the contact are required, the so-called angle of contingence is not a true angle and has no quantity.[37]

Galileo now goes on to propose another "discourse" in support of his view that the angle of contingence is no angle at all, which he remembers to have crafted long ago (Figure 2). Let us consider line *FG* turning on point *C*. The mixed angle *ACG* will become more and more acute until eventually it will transform into mixed angle *OCA*. This transformation cannot occur unless the angle annihilates, which, Galileo argues, can happen only when the turning line, *GF*, coincides with the horizontal line (*cf*. Figure 2). If we look at the history of the controversy on the angle of contingence we find a strikingly similar view, i.e., the angle of contingence is not a true angle and not a quantity, and a strikingly similar argument in Jacques Peletier (1517–82).[38] Galileo might have read the argument in Peletier's edition of Euclid, or in one of the publications by Peletier in which similar arguments are repeated.[39] However, I believe it is more likely that he would have seen the résumé of the discussion (with verbatim quotations) published by Peletier's opponent in the controversy, namely, Christoph Clavius.[40]

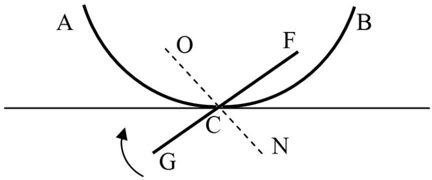

FIG. 2. A straight line forming a mixed angle passes from one side to another of a horizontal line so that the mixed angle must be annihilated. I have slightly simplified Galileo's original figure.

Peletier claims that the angle of contingence is not an angle because it forms no section with the circumference, while *angle* consists precisely in forming a section [*sectio*, or *decussatio*] not a contact [*contactus*] (Figure 3). A line, *ED*, turning on point *A,* forms angles more and more acute with the circumference because it sections it. But when the line coincides with the horizontal tangent a section will no longer occur. We might say that Peletier has a punctiform view of the angle of contingence, since for him "all angles consist in no more than one point".[41] Clavius held a conception radically different from Peletier's, according to which the angle of contingence is indeed a true angle and has quantity.[42] As we shall see in a moment, Galileo might have elaborated Peletier's punctiform view of angles, while rejecting Clavius's opinion.

In the salient part of the letter to Gloriosi, Galileo claims to refute the "discourse" [*discorso*], according to which not only is the angle of contingence a true quantity, but as such it is also infinitely divisible. Infinite divisibility, Galileo argues, is warranted

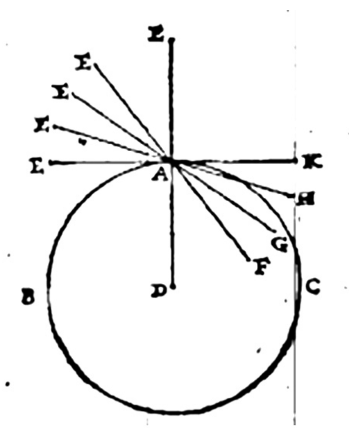

FIG. 3. The diagram accompanying Peletier's argument (*op. cit.* (ref. 39), 75). An identical diagram was published by Clavius (*op. cit.* (ref. 40), 117).

by the possibility of constructing greater and greater circles passing through the same point of contact between circumference and tangent (this example was one of Clavius's counter-arguments to Peletier). Galileo's reasoning strategy is paramount for our purposes because it involves a recourse to similar figures. We will see in the second part of this section that for Galileo parabolic trajectories are similar curves; and in the next section, that similar parabolic trajectories, in the broader context of the angle of contingence, are the hidden scaffolding of Galileo's counter-argument to the extrusion effect.

Not the *angle*, as Clavius had claimed, but the *space* between the circumference of the circle and the tangent line, Galileo argues, can actually be divided by greater and greater circumferences passing through the same point of contact between circumference and tangent.[43] This, he continues, can be shown starting with the simple example of rectilinear similar polygons (Figure 4).[44]

The perimeter of the greater hexagon divides the space between the smaller hexagon and the tangent, but angle *IBE* is not divided. In consequence, regardless of the number of sides of the similar polygons angle *IBE* will never be divided. The angle, Galileo points out, could be divided only by a dissimilar polygon, one with a greater number of sides. Hence, in Galileo's view, since *all circles are similar polygons of infinite sides*, when they are applied to the same tangent at *B*, the space between the tangent and the circumference is divided by the circumferences of the greater circles, but the angle of contingence, which is common to all, is not divided. Further, Galileo concludes, since the circles are polygons of infinite sides it cannot be said that a greater circle is a polygon of more sides and thus capable of dividing the angle, on the analogy of polygons of a finite number of sides. Interestingly, Galileo notes that, since when the number of sides of the polygons increases angle *IBE* becomes more and more acute, it looks as though the angle will be infinitely acute when the number of sides rises to infinity, in which case the angle will become "non-quantifiable, and not angle [*non quanto e non angolo*]".[45]

Something of Peletier's punctiform analysis is reflected in Galileo's line of

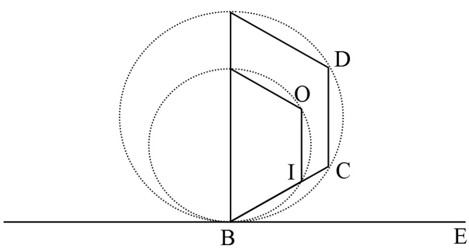

FIG. 4. Two rectilinear similar polygons (hexagons in this case, only half of which are diagrammed by Galileo) inscribed in two circles, passing through the same point of contact *B*. I have simplified the diagram by dotting the lines of the circumferences and limiting the lettering to what is needed for my discussion.

reasoning. At point *B* no true angle can be formed since, we might say, the inclinations of circumference and the tangent being the same the point alone cannot constitute a section, whereas, in Galileo's definition, it is the inclination of two lines touching each other at one point while not being placed straight with respect to each other that forms an angle.

To sum up the first part of this section, we have seen that Galileo begins his response to Gloriosi with an argument, or rather the recollection of a discourse, apparently based on a conception of the composition of lines in terms of points, since he refers to a point "subsequent" to the point of contact. He then moves on to another discourse, strongly reminiscent of one of Peletier's arguments aimed at proving that the angle of contingence is actually no angle at all. Finally, in my view, Galileo propounds his most original reflection on the angle of contingence based on similar figures, perhaps elaborating on Peletier's punctiform view of the nature of an angle.

Galileo starts by remembering one of his (presumably) first discourses about the angle of contingence. When moving to his second discourse he does not remember his past reading of Peletier's arguments, nor does he bother to clarify whether his ideas have been inspired by others. He shapes ideas on demand, so to say, solicited by Gloriosi's inquiry, through the medium of reconstructive recollection.

We have noted that for Galileo all circles are *similar* polygons of infinite sides. From a manuscript sheet, written in preparation for the calculation of the ballistic tables published in *Two new sciences*, we can gather that he held analogous views concerning parabolas (Figure 5).[46]

Parabolas can be found, Galileo says, *similar* to each other.[47] We now know that *all* parabolas are indeed similar curves.[48] Galileo is not explicit about this possible generalization, since obviously he did not have an analytic framework, that is, a Cartesian framework, for thinking about conic sections in all generality. There is, however, a tantalizing statement concerning *similar paraboloids* in Archimedes that may have been the source of Galileo's thinking about similar parabolas. Wilbur Knorr has actually claimed that Archimedes "asserts the theorem that all parabolas are similar in the Preface to *Conoids and Spheroids*".[49] I surveyed two Renaissance editions of Archimedes that Galileo would have seen, but I did not find an explicit assertion of that theorem. In the Archimedes edition annotated by Galileo, listed in his own personal library, and which we may thus assume was the one he used to consult, we find the statement that "omnia vero conoidalia rectangula [i.e., paraboloids] sunt similia". Almost the same phrasing is used in the Commandino edition.[50] I conclude, therefore, that all Galileo could have gathered from Archimedes is a pronouncement on similar paraboloids, although, admittedly, he might have extended this view to the parabolas generating paraboloids. But fortunately Galileo's views on similar parabolas emerge more clearly when we investigate in detail the text associated with the diagram presented in Figure 5, on f. 122v of *Manuscript 72*.

The text concerns the calculation of the parabolic trajectories of projectiles launched from point *D* (*cf*. lower left corner, in Figure 5, note that the parabolic trajectories are

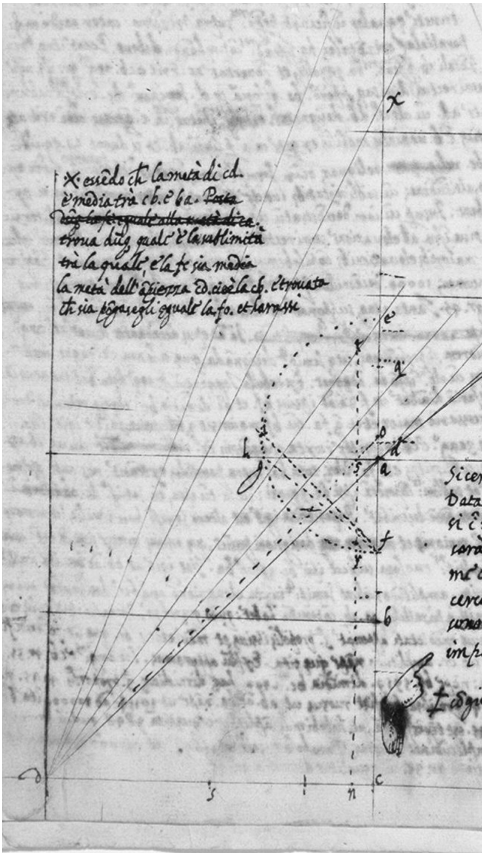

FIG. 5. The relevant portion of f. 122v, in *Manuscript 72*. The diagram printed in *Opere*, viii, 432, is incomplete.

not drawn in the diagram), at different inclinations but with the same initial energy (*impetus*, rather, in Galileo's terminology). Galileo starts from the parabolic trajectory determined by an elevation of 45°, and looks for the parabolic trajectories generated by the same impetus at point *D* with different shooting elevations, for example that with an elevation of 55°. We need not consider the technical details of the ingenious procedure excogitated by Galileo. It is basically an approximation technique based on the well-known *regula falsi*. Galileo first guesses the trajectory by assuming that its axis is the same (vertical) axis of the parabola generated by a 45° elevation, then he linearly scales the "false" trajectory to the true one by applying a simple proportionality rule. The warrant of the scaling operation is in fact the *similarity* between the two parabolas, which are (so to speak) "boxed" by, or inscribed in, *similar* triangles.

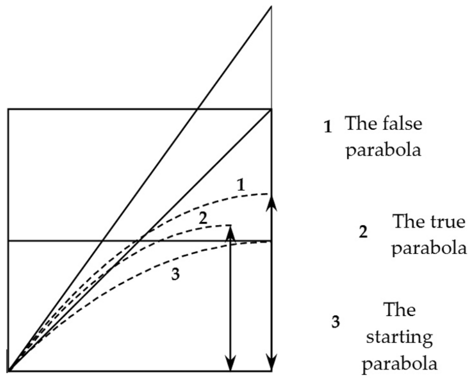

FIG. 6. A simplified version of the diagram on f. 122v, showing the similar parabolas used in the calculation but not represented by Galileo. Note that I have plotted here only semi-parabolas and their axes of symmetry.

Since Galileo did not draw the parabolas I have provided a reconstructed diagram in order to clarify the strategy of Galileo's procedure (*cf.* Figure 6).

The similarity attributed to parabolas that emerges from this procedure is reducible to Euclidean similarity between rectilinear figures, in our case simple right triangles. In Figure 7, I have drawn a diagram with the false and true semi-parabolas used in Galileo's procedure (remember that Galileo's idealized ballistic trajectories are symmetrical with respect to a vertical axis). I have "boxed" them by grey-shaded right triangles, in order to highlight their similarity. Thus, what Galileo has in mind when speaking of *similarity* between parabolas is the simple Euclidean idea that a proportionality transformation somehow connects the two similar figures. Galileo has of course no algorithm to compute a complete transformation in the case of curves such as parabolas. But all he needs in order to construct the ballistic tables are the characteristic dimensions of the parabolas, which he calls "amplitude" and "height". These are in fact characteristic dimensions of the triangle within which the semi-parabolas are inscribed. These characteristics can be proportionally transformed so as to obtain each true trajectory from the corresponding false one for any chosen elevation. The lack of a complete transformation procedure for parabolas also explains why Galileo did not draw the parabolas on his folio. He did not have a simple point-by-point drawing procedure from the linearly transformable characteristic dimensions.

To sum up, Galileo had recourse to *similar* parabolas in order to construct the ballistic tables presented in *Two new sciences*. The procedure is based on the extension of Euclidean similarity, valid for rectilinear figures, to parabolas. Similar semi-parabolas are inscribed in similar right triangles. This simply means that the semi-parabolas are tangent to the common hypotenuse of the triangles at the vertex in the lower-left corner (Figure 7), a constraint imposed by the ballistic condition of launch at the same

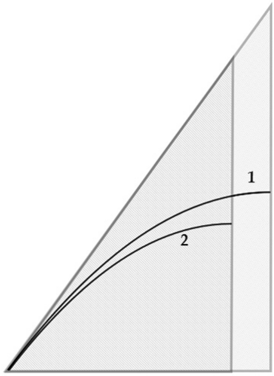

FIG. 7. The false (1) and true semi-parabolas (2) (for an elevation other than 45°), which, in Galileo's view, are similar in that they can be thought of as "boxed", or inscribed, by similar right triangles.

elevation, and their vertical axes coincide with the vertical side of the triangle.

In the next section, I shall combine Galileo's approaches to similar parabolas and to the angle of contingence, and argue that they are the two hidden structures scaffolding Galileo's counter-argument to the extrusion effect.

4. *Galileo's Counter-argument to the Extrusion Effect*

The counter-argument to the extrusion effect, presented by Galileo in the *Dialogue*, is embedded in a complex dialogical structure. The three famous interlocutors, Salviati, Sagredo, and Simplicio, a collective mouthpiece for the collage of Aristotelian positions that Galileo confronted in his career, engage in a lively discussion driven by Salviati's re-enactment of Socratic maieutics with Simplicio.[51] The questioning of Simplicio's mind, however, is coloured by pungent irony. Indeed irony, with its suspension of the literal level of meaning, is always a threatening presence in this long section of the *Dialogue*. In the process of questioning and eliciting answers, Galileo will raise objections to his own reasoning too. But since the *Dialogue*, written in Italian, was mostly aimed at neutralizing entrenched presuppositions against Copernican astronomy, which were common to a broad audience, Galileo did not cast the progression of questions and answers in a strictly technical language. He rather let ideas flow in a cyclical, wave-like movement of thinking.

The three interlocutors are agreed that any circular motion, like that of a sling, or a wheel, has a faculty of extruding objects placed on the circumference, and that the direction of the object's motion upon leaving the extruding device is the tangent to the circumference at the point of separation. Further, they agree that the motion after separation will be uniform and that if the circular motion is fast enough extrusion in slings and wheels will at some point occur. But all heavy bodies on the Earth's surface have a natural tendency downwards. The three interlocutors have no doubt on this either. Then Galileo issues his challenge to Simplicio. Galileo wishes to prove that no matter how small that downward tendency, and no matter how fast the diurnal rotation of the Earth, all heavy objects will remain firmly attached to the Earth's surface.[52]

Two objections are subsequently raised by Sagredo. I here summarize and paraphrase Galileo's text. Imagine a body a few instants along the tangent after leaving the rotating Earth. Immediately upon leaving the Earth's surface it will start descending toward the centre of the Earth in naturally accelerated fall. The first objection is as follows. The downward tendency, in terms of degrees of speed of fall, decreases *ad infinitum* as the body is thought of as approaching backwards the point of separation. Galileo has in fact already introduced in the *Dialogue* the law of falling bodies, and the idea of the uniform increase of a falling body's degree of speed with time. Second objection, how about the weight of the object? Going by Aristotelian physics the lighter the body the less fast it will fall. Thus, Sagredo concludes, by combining the two effects one has good reason to doubt that at least some objects will be able to escape the grip of the Earth and eventually be extruded. To this conclusion Galileo replies with the following counter-argument. It is this reply that has given rise to the controversy that led David Hill to his claim of dishonesty (*cf.* Figure 8).

Let us assume that at point *A* an object leaves the surface of the Earth along the tangent at *A*. Since the motion will be uniform evenly spaced points on line *AB* represent instants of equal intervals of time. The degrees of speed, and the vertical distances fallen through, are represented by segments *FG*, *HI*, *KL*.

I need to pause here. In his own explanation of this diagram, Galileo initially states that these segments represents "degrees of speed" acquired during *AF*, *AH*, *AK*. Only at some later point in the passage does Galileo equate these segments (*FG*, *HI*, *KL*) with distances fallen through as well. But this equation is questionable, not to say illegitimate, because (by Galileo's own law of fall) the distances fallen vary

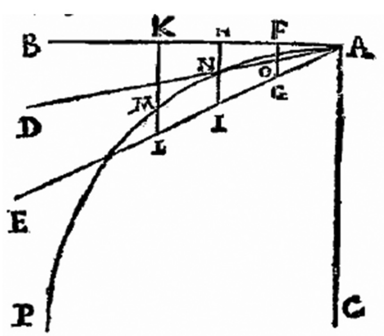

FIG. 8. The diagram supporting Galileo's counter-argument.

as the square of the times elapsed. So he may be fairly charged with committing some kind of equivocation.

Galileo can also incorporate in the argument the Aristotelian assumption that lighter bodies fall slower. He depicts this with the device of differently inclined lines, so the lighter the body the less acute the angle of inclination of lines *AE*, *AD*, i.e., the less fast the body will fall. There is a simplifying hypothesis implicitly made by Galileo. The directions of fall are vertical, not in the sense that they tend towards the centre of the Earth, but in the sense that they are parallel to the radius of the Earth at the point of separation. In Galileo's diagram the diminution *ad infinitum* of both weight and degree of speed are thus captured. Galileo continues as follows:

> The degrees of speed, infinitely diminished by the decrease of the weight of the moving body and by the approach to the first point of motion (the state of rest), are always determinate. They correspond proportionately to the parallels included between the two straight lines meeting in an angle such as the angle *BAE*, or *BAD*, or some other angle infinitely acute but still rectilinear. But the diminution of the spaces through which the moving body must go to return to the surface of the wheel is proportional to another sort of diminution included between lines which contain an angle infinitely narrower and more acute than any rectilinear angle whatever.… Now the parallels included between the straight lines, as they retreat toward the angle, always diminish in the same ratio.… But this is not thus with the line intercepted between the tangent and the circumference of the circle.[53]

Since the curvilinear angle is infinitely narrower and more acute than any rectilinear one, then the downward tendency will always be more than enough for the falling body to cover the distance between the tangent and the surface of the Earth. *But what about the actual trajectory of the projected object?* Galileo must have known that it is a parabolic arc. The problem of the actual trajectory is the hub around which the accusation of dishonesty raised by David Hill turns. Let's see how.

Hill has raised the following objection to Galileo's counter-argument to the extrusion effect. In Hill's words, it

> contains an interesting and fairly well-concealed fallacy which can be characterized either as a non-sequitur partly disguised by the vagueness of a key term or as a classical equivocation on that term. Galilei successfully argues that as we approach the point of contact, *A*, the distances which need to be covered to prevent projection necessarily vanish more quickly than the speeds of fall. But this does not imply that centripetal tendencies must overwhelm centrifugal tendencies. To prove this Galileo would have to show that the distances which *need* to be covered to prevent projection necessarily vanish more quickly than the distances a falling body would *actually cover* (as the point of contact is approached). This, however, cannot be established. These two distances vanish at the same rate, both being as the square of the speeds (and times).[54]

Further, Hill goes on to qualify Galileo's "mistaken inference as surprising and suspicious".[55] The reason for Hill's sceptical conclusion is that since, as is well known, by the time he completed the *Dialogue* Galileo had long reached his results about the parabolic trajectories of projectiles, it seems

> difficult to believe that Galileo simply never saw the relevance of the parabolic trajectory to the examination of the projection argument.… Can a projected object rise above, and remain above, the spinning Earth? Clearly, it could, *if* its speed of projection is large enough to produce a sufficiently flat parabolic arc … a parabola sharing a tangent might always *lie between* tangent and circle, in which case distances covered in fall are always less than those which must be covered to prevent projection.[56]

In Figure 9, I have visualized what Hill presumably has in mind when speaking of parabolic trajectories for projected bodies, by adding them to Galileo's original diagram. When the parabolic arcs are sufficiently flat, as Hill has suggested, they must leave the Earth's surface and thus extrusion will eventually ensue.

In the remaining part of this section, I will try to show that not only do we find in Galileo's *Dialogue* vestiges of the objection that Hill has raised (though under the guise of a language that Galileo wanted accessible to a vast audience), a point strangely missed by Hill, but that Galileo responded to that self-raised objection, in a way that needs to be unpacked and illuminated in the context of his approach to the angle of contingence and similar parabolas.

As I have already suggested, if we are to understand Galileo's counter-argument fully we need to follow the wave-like movement of his thinking carefully. The counter-argument in fact is not exhausted by the portion examined and criticized by Hill. It cannot be separated, in other words, from the self-objections subsequently raised by Galileo and the answers to those self-objections.

Sagredo, the layman not committed to any philosophical school, is dissatisfied

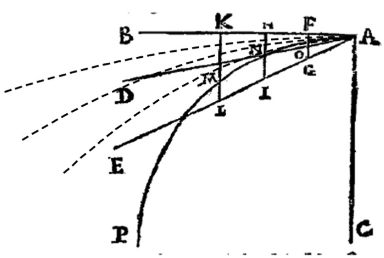

FIG. 9. Hill's objection to Galileo. The dotted lines represent parabolas tangent at *A* to the Earth (circle *AP*), i.e., the actual trajectories of projected bodies, as Galileo knew. Being open paths, when they are sufficiently flat, as Hill has suggested, they must leave the Earth's surface and thus extrusion can eventually ensue.

with Salviati's diagrammatic construction. If the speed of fall decreasing with weight (under the assumption of Aristotelian physics, for the sake of argument) followed the proportion of the line segments between tangent and circumference, or even a greater proportion, what would then happen?[57] Salviati is quick to mention that experience refutes the assumption of Aristotelian physics. The case is thus thrown out on empirical grounds. But this is besides Sagredo's point. The fact is that Salviati-Galileo is deeply intrigued by Sagredo's objection and wants to show that regardless of that proportion extrusion will never occur. We must take stock here. The language of this passage makes no sense in terms of Euclidean proportionality.[58] What does Sagredo really mean? It is in fact by making the angle of lines, such as *AD*, *AE*, with the tangent at *A* more and more acute that speed can be diminished *ad infinitum*. It is the inclination of those lines in the diagram that represents the decrease in speed of fall owing to the decrease in weight, whatever the relation between these two magnitudes might be. It makes no sense to talk of the proportion of that diminution as though "following" the proportion of the line segments between tangent and circumference approaching the point of contact! Thus, I take Sagredo's passage as intimating, though in a veiled allusion, the fact that Galileo actually imagines the parabolic path of the extruded object, exactly as Hill argues that he should have done. On the other hand, Sagredo's language makes perfect sense if we assume that he is in fact describing the trajectory of the extruded object. For, in this way it is perfectly meaningful to talk of the line segments between tangent and circumference following a certain *proportion*, or rather a certain *progression*, as they approach the point of contact. Here, then, proportionality has no Euclidean technical meaning.

Let us now turn to examining how Galileo goes about resolving this self-objection. Extrusion does not occur, even under the circumstance hinted at by Sagredo.

> What makes me believe this is that a diminution of weight made according to the ratio of the parallels between the tangent and the circumference has as its ultimate and highest term the absence of weight, *just as those parallels have for their ultimate term of reduction precisely that contact which is an indivisible point*. Now weight never does diminish to its last term, for then the moving body would be weightless; but the space of return for the projectile to the circumference does reduce to its ultimate smallness, which happens when the moving body rests upon the circumference at that very point of contact, so that no space whatever is required for its return.[59]

Here Galileo has introduced the point of contact as an *indivisible point*, the point at which no distance is required of the falling body to rejoin the surface of the Earth. The ground has been prepared for the small finale but the last movement must break through Simplicio's misconceptions about the contact between tangent and straight line. Simplicio is flabbergasted by Salviati's argument and raises the question that geometry, though it functions very well in the abstract, does not work in the real world. When it comes to matter, Simplicio claims, it makes no sense to say that *sphera tangit planum in puncto*. Thus, in Simplicio's view, the tangent at *A on the*

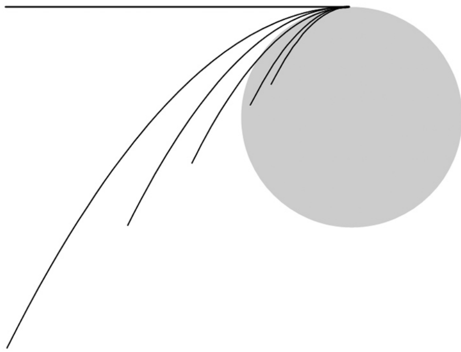

FIG. 10. A visualization of similar parabolic trajectories, according to Galileo, in the case of extrusion. The parabolic arcs are all similar to one another in that they are inscribed in similar rectangles. To avoid confusion in the diagram I have not represented the similar rectangles framing the parabolic arcs. (I have constructed these similar arcs with the help of the drawing software *Canvas 9.*)

real Earth not only touches one point, *A*, but grazes the surface for many miles. To which, Salviati replies as follows.

> But don't you see that if I grant you this, it will be so much the worse for your case? For if even assuming that the tangent lies removed except at one point, it has been proven that the projectile would not be separated, because of the extreme acuteness of the angle of contingence (*if it can indeed be called an angle*), how much less cause will it have for becoming separated if that angle is completely closed and the surface united with the tangent?[60]

Galileo's approach to the angle of contingence as no angle, no quantity, is hinted at here. It is of great importance to realize that in this final part of the argument *Galileo is exploring the limit behaviour of the falling object in proximity of the point of contact*. What happens to the falling object at the point of contact? This is the hovering question the answer to which can seal the counter-argument to the extrusion effect.

That the trajectory is parabolic in the vicinity of the point of contact Galileo has intimated already. But are the parabolic trajectories distinguishable in terms of the motion of the extruded object *in the vicinity of the point of contact*? Hill argues that this must be the case (*cf*. Figure 9). I suggest that in the framework of Galileo's mathematical physics they are not. Galileo has no calculus to fine-tune his analysis of the limit behaviour of the falling body. On Galileo's footsteps, Huygens will accomplish exactly this, a few decades later. It is only Galileo's views on the angle of contact (as being no quantity) and his views on similar parabolas that allow us to explore the limit behaviour of the falling object within the context of his mathematical physics.

Similar parabolic arcs in the case of extrusion can be represented — according to Galileo's preparatory analysis for the ballistic tables — by inscribing parabolic arcs with vertex at *A* in similar rectangles. Some arcs may intersect the surface of the Earth (thus extrusion would not follow, according to Hill's analysis), others escape from its surface (thus extrusion would follow, according to Hill's analysis). At the point of contact, however, the angle of contingence is the same for all arcs in the sense that it is no angle at all. All that can be said is that for Galileo the angle of contingence is not divided. Thus, as in the letter to Gloriosi, even in this case of extrusion, it is not the angle that can be divided. It is only the space between the circumference of the Earth and the tangent line at *A* that can actually be divided by greater and greater parabolic arcs passing through the same point of contact between the circumference of the Earth and the tangent. However, this fact that the space between the circumference of the Earth and the tangent line at *A* can actually be divided cannot serve our present purpose of exploring the limit behaviour of the falling body. Since all similar parabolic arcs may be reduced to similar polygons of infinite sides — as we may speculate in accord with Galileo's reasoning about circles being all similar polygons of infinite sides — then, when similar parabolic arcs are applied to the same tangent at *A*, we must conclude that the angle of contingence common to all parabolic arcs is not divided, even though the space between the tangent and the circumference is divided. In other words, in Galileo's physics there is available no measure whatsoever for the angles of contingence of different but similar parabolic arcs at the point of tangency.

Hence, in Galileo's physics, the limit behaviours of a falling body moving along different but similar parabolic arcs — in the vicinity of the point of contact — cannot be distinguished by discriminating among the angles of contingence at the point of contact. The angles of contingence at the point of contact are all the same. On this ground, the limit behaviour is therefore independent of the characteristics of the trajectory. It is at the point of contact, *A*, that the falling body *need fall no distance* to rejoin the Earth, regardless of the different, incipient parabolic trajectories.

What Galileo would have required to further his investigation of the limit behaviour of the extruded body, and thus come to terms with the error in his analysis, is some basic understanding of curvature of the trajectory, how to measure it, and a good helping of some form of infinitesimal calculus. It is such an understanding of curvature as a local property associated with curves (which we tend to take for granted today), that, I believe, has derailed Hill's fascinating analysis. Galileo, however, must be credited with the merit of realizing that the imagery behind the extrusion effect, the vision of buildings collapsing and animals and trees flying off toward the sky, was the fruit of deep-rooted misconceptions about centrifugal effects. Projection in rotating devices occurs *not along the radial direction* of the rotating device *but along the straight line tangent to the circumference at the point of separation*. Galileo succeeded in re-orienting discussion of the centrifugal effects of a rotating Earth, although he lacked the mathematical machinery to tame the problem.

To conclude, there is some irony in this story. Knowledge of the parabolic trajectory of projectiles, one of Galileo's lasting achievements in mathematical physics, was nowhere near enough for him to analyse the incipient behaviour of a falling body in the process of being extruded, or rather projected, by a fast rotating Earth. The analysis of the local behaviour of bodies at the point of separation from the extruding device requires a mathematical approach which goes into the infinitesimal. Galileo's understanding of the parabolic trajectory of projectiles rested on his classical approach to conic sections, as had been illustrated by Apollonius, not on the mathematics of curves analytically describable as *loci* in terms of local properties. The merit of fully understanding the properties of centrifugal effects was left for Christiaan Huygens, later in the seventeenth century, although, I am convinced, Huygens's possible dependence on at least some of Galileo's ideas deserves further scrutiny.

5. Conclusion

Galileo's attribution of the extrusion argument to Ptolemy has an intriguing history that can be illuminated in terms of the effects of orality-shaped interactions with books. The form that cognition takes in a culture at the intersection of orality and writing, such as that of the late Renaissance, is a fertile terrain for exploring a Renaissance scholar's modes of appropriations of scientific ideas. Both Galileo and the intellectuals whom we have encountered in this story still participated in that form of cognition.

In the *Almagestum novum*, as we have noted, Riccioli sketched a brief history of the arguments and counter-arguments inspired by the extrusion effect. While marking the transformation of a loose bundle of ideas floating in the elusive space between orality and writing into a written tradition he also conceded defeat. In fact he headed the main section containing his discussion of the extrusion argument as follows. "Proponuntur argumenta quinque, *sed invalida* …", against the diurnal rotation of the Earth.[61] The argument from the extrusion effect, one of the five proposed in that section, was *invalid* in Ricclioli's view. It would be fascinating to speculate what might have led Riccioli to his conclusions. Unfortunately for us he only lists some counter-arguments but does not comment on their substance. He compiles a detached reportage of the *status quaestionis* but mutes his personal voice. Thus we are left with a tantalizing question, why is the extrusion argument against the diurnal rotation of the Earth invalid for Riccioli? Together with lesser works, Riccioli mentions Galileo's *Dialogue*, Kepler's *Epitome*, and Ismael Boulliau's *Philolai, sive dissertationis de vero systemate mundi*. However, both Kepler's and Boulliau's discussions of the extrusion argument are rather obscure, and boil down to no more than a few comments in passing.[62]

Thus I believe that the only credible source of Riccioli's conviction must have been Galileo, to whom, on the other hand, vast portions of the *Almagestum novum* are devoted. This is also indirectly suggested by one comment that Riccioli makes, referring the reader who wishes to know more about centrifugal effects to what Galileo relates on this subjects in the *Dialogue*.[63] A history of the reception of Galileo's counter-argument and of the eventual demise of the argument from extrusion would be highly rewarding for historians of seventeenth-century science. It remains for future research.[64]

Finally, if my reconstruction of the context of this Galilean counterargument is correct, we must reject Hill's conclusion that Galileo engaged in a morally deplorable act of conscious deception; that, deploying mathematical "trickery" to strengthen the rhetorical force of persuasion of his reasoning, Galileo published what he knew was a fundamentally flawed argument.[65] This conclusion is untenable, the fruit of historical anachronism.

I have argued that Galileo's thinking in the *Dialogue* cannot be disembodied from its dialogical framework. It is a wave-like flow of argumentation that incorporates self-objections and answers to them. In this maieutic process Galileo quite possibly adumbrated the type of objection from the parabolic trajectory of projectiles that, in Hill's view, should have proven to him the blatant inconsistency of his counter-argument. But Galileo's views on the angle of contingence and similar parabolic arcs cast a raking light on the counter-argument. They allow us to catch a glimpse of the thought-processes that prevented him from seeing that inconsistency. To think locally about the point of separation from the extruding device Galileo should have mastered the apparatus of calculus and curvature that Hill seems to have taken for granted in his analysis. Huygens was right, Hill went wrong. *Galileus deceptus, non minime decepit*.

Acknowledgements

I am grateful to the Biblioteca Nazionale in Florence, Italy, for permission to publish the image of a portion of a folio from Galileo's *Manuscript 72*, and I wish to thank James G. Lennox for his help with the Greek texts, and Curtis Wilson and two anonymous referees for their helpful comments on previous drafts of this paper.

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2. *Cf.* Galileo Galilei, *Le opere di Galileo Galilei, edizione nazionale*, edited by Antonio Favaro (20 vols, Florence, 1890–1909; hereafter: Galileo, *Opere*), vii, 214–44, and the translation by Drake, in Galileo Galilei, *Dialogue concerning the two chief world systems*, transl. by Stillman Drake, 2nd edn (Berkeley and Los Angeles, 1967), 188–217.
3. *Cf.* A. Koyré, *Galileo studies*, transl. from the French by J. Mepham (Hassocks, Sussex, 1978), 195, and the original French, *Études galiléennes* (Paris, 1966), 268.
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Studies in history and philosophy of science, xiv (1983), 315–40, p. 321; Y. Yoder, *Unrolling time: Christiaan Huygens and the mathematization of nature* (Cambridge, 1988), 35–41; D. Hill, "The projection argument in Galileo and Copernicus: Rhetorical strategy in the defence of the new system", *Annals of science*, xli (1984), 109–33; M. Finocchiaro (ed.), *Galileo on the world systems: A new abridged translation and guide* (Berkeley and Los Angeles, 1997), 179–95; and *idem*, "Physical-mathematical reasoning: Galileo on the extruding power of terrestrial rotation", *Synthese*, cxxxiv (2003), 217–44, p. 234. Negative conclusions, on the basis of a reconstruction of Galileo's argument according to Newtonian mechanics, were also reached by P. Palmieri, in "Re-examining Galileo's theory of tides", *Archive for history of exact sciences*, liii (1998), 223–375, pp. 281–94. It is important to realize that Galileo gives several other counterarguments to the extrusion effect, which have been analysed in detail especially by Maurice Finocchiaro and by A. Chalmers and R. Nicholas (references above): a physical counterargument comparing the extrusion along the tangent with fall along the secant; another physical counterargument contrasting how extrusion depends on linear speed and how it depends on the radius; and two other mathematical counterarguments: one claiming that on a rotating Earth extrusion is mathematically impossible because the ratio of an exsecant to the corresponding tangent segment tends toward zero, the other claiming that it is impossible because the ratio of one exsecant to another (at twice its distance from the point of tangency) tends to zero.

5. Hill, "The projection argument in Galileo and Copernicus" (ref. 4), 133, for example.
6. S. Drake, "Galileo and the projection argument", *Annals of science*, xliii (1986), 77–79.
7. "Though there is presumably a rhetorical dimension to all argument meant to persuade, there must always be a basic distinction between honest argument and conscious deception. I have argued that Galileo crossed the line in the case at hand", Hill, "The projection argument in Galileo and Copernicus" (ref. 4), 133.
8. Finocchiaro, "Physical-mathematical reasoning" (ref. 4).
9. Finocchiaro, "Physical-mathematical reasoning" (ref. 4), 235.
10. Finocchiaro, "Physical-mathematical reasoning" (ref. 4), 235.
11. *Cf.* Harald Siebert, *Die Große kosmologische Kontroverse: Rekonstruktionsversuche anhand des Itinerarium exstaticum von Athanasius Kircher SJ (1602–1680)* (Stuttgart, 2006); see especially the discussion on pp. 132–54.
12. Galileo, *Dialogue concerning the two chief world systems* (ref. 2), 188.
13. *Ptolemy's Almagest*, transl. and annotated by G. J. Toomer (Princeton, NJ, 1998; first edn, London, 1984), 44. *Cf.* the Greek text in Heiberg's edition: εἰ δέ γε καὶ αὐτῆς ἦν τις φορὰ κοινὴ καὶ μία καὶ ἡ αὐτὴ τοῖς ἄλλοις βάρεσιν, ἔφθανεν ἄν πάντα δηλονότι διά τήν τοσαύτην τοῦ μεγέθους ὑπερβολήν καταφερομένη, καὶ ὑπερλείπετο μέν τά ζῷα καί τά κατά μέρος τῶν βαρῶν ὀχούμενα ἐπὶ τοῦ ἀέρος, αύτὴ δὲ τάχιστα τέλεον ἂν ἐκπεπτώκει καὶ αύτοῦ τοῦ οὐρανοῦ. ἀλλὰ τὰ τοιαῦτα μέν καὶ μόνον ἀλλὰ τὰ τοιαῦτα μέν καὶ μόνον ἐπινοηθέντα πάντων ἄν φανείη γελοιότατα. *Cf.* Ptolemy, *Syntaxis mathematica*, ed. by J. L. Heiberg, Part 1, Books 1–6 (Leipzig, 1898), 23–24. We may notice that even the great Heiberg shied away from translating the *Almagest*, commenting at the end of his preface to the 1898 edition, "interpretationem meam sive Latinam sive linguae recentioris in tanta rerum difficultate addere ausus non sum; de re videant astronomi, si interpretationem desideraverint" (*ibid.*, p. vi). It is ultimately for the astronomers to tackle the issues raised by Ptolemy's text!
14. I wish to thank James Lennox for translating the passage and sharing with me his insights into the semantic complexities of the original Greek.
15. Ptolemy, *Omnia quae extant opera* (Basel, 1541), 7. The whole passage is rendered as follows, "Quod si communis caeteris ponderibus singularisque motus ipsi quoque inesset, patet quia propeter tantum (sui magnitudine) excessum universandum deferetur, praeveniret caeterisque relictis in aerem animalibus, dico aliisque ponderibus, ipsa velocissime extra coelum quoque ipsum excideret. Verum haec ridiculosissima omnium intellectu videntur" (*ibid.*). The translation was first published in a 1528 edition. *Cf.* R. De Vivo's Introduction, in G. B. Della Porta, *Claudii Ptolemaei Magnae constructionis liber primus*, ed. by R. De Vivo (Naples, 2000; first edn,

Naples, 1605), pp. viii ff.

16. Della Porta, *Claudii Ptolemaei Magnae constructionis liber primus* (ref. 15), 84. The whole passage is rendered as follows, “Si vero et ipsius esset aliqua latio communis, et una et eadem aliis ponderibus, praeoccuparet utique omnia, videlicet ob tantum magnitudinis excessum deorsum lata, et relinquerentur quidem et animalia et vecta in aere secundum partem ponderum, ipsa et celerrime postremo cecidisset et ab ipso coelo, sed talia quidem et tantum excogitata maxime omnium ridicula viderentur” (*ibid.*).
17. On the solid nature of the celestial orbs in the period spanning the late Middle Ages to the Renaissance and the seventeenth century, see Edward Grant, “Celestial orbs in the Latin Middle Ages”, *Isis*, lxxviii (1987), 153–73; B. R. Goldstein and P. Barker, “The role of Rothmann in the dissolution of the celestial spheres”, *The British journal for the history of science*, xxviii (1995), 385–403; B. R. Goldstein and Giora Hon, “Kepler’s move from orbs to orbits: Documenting a revolutionary scientific concept”, *Perspectives on science*, xiii (2005), 74–111; and more generally M. A. Granada, *Sfere solide e cielo fluido: Momenti del dibattito cosmologico nella seconda metà del Cinquecento* (Milan, 2002).
18. Ptolemy, *Almagestum Cl. Ptolemei* (Venice, 1515), 4. The whole passage is rendered as follows: “Quo si terre et reliquorum corporum gravium que sunt preter eam esset motus unus communis: terra propter superfluitatem sue molis et gravitatis vinceret omnia gravia que sunt preter ipsam: et inferius iret. Et remanerent animalia et relique species gravium sita in aere. Et terra velociter omnino caderet: et pertransiret celum solum. Tamen imaginari hoc et eius simile est derisio et illusio imaginantis ipsum” (*ibid.*). I have preserved the Latin morphology of the 1515 printed Latin, only expanding the abbreviations. *Cf.* A. Favaro, “La libreria di Galileo Galilei”, *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, xix (1886), 219–90, for the catalogue of Galileo’s library, in which the 1515 edition of Ptolemy’s *Almagest* is listed.
19. Ptolemy, *Omnia quae extant opera* (ref. 15), 6; and Ptolemy, *Almagestum Cl. Ptolemei* (ref. 18), 4. Della Porta repeats George of Trebizond’s title almost verbatim, “Quod terra neque motum progressivum aliquem facit”. *Cf.* Della Porta, *Claudii Ptolemaei Magnae constructionis liber primus* (ref. 15), 83.
20. Copernicus’s rendition of Ptolemy is as follows: “Si igitur, inquit Ptolemaeus Alexandrinus, terra volueretur, saltem revolutione cotidiana, oporteret accidere contraria supradictis. Etenim concitatissimum esse motum oporteret, ac celeritatem eius insuperabilem, quae in xxiiii. horis totum terrae transmitteret ambitum. Quae vero repentina vertigine concitantur, videntur ad collectionem prorsum inepta, magisque unita dispergi, nisi coharentia aliqua firmitate contineantur: et iam dudum, inquit, dissipata terra caelum ipsum (quod admodum ridiculum est) excidisset, et eo magis animantia atque alia quaequnque soluta onera haud quaquam inconcussa manerent.” N. Copernicus, *De revolutionibus orbium coelestium, Libri VI* (Nurenberg, 1543), ff. 5r–v. I agree with Siebert’s suggestion (*Die Große kosmologische Kontroverse* (ref. 11), 138) that here the meaning of *excido* should be that of “demolish” since the sentence is constructed with an accusative. Siebert renders the passage as follows: “Und schon lngst, sagt er, hätte die zersprengte Erde das Himmelsgwölbe selbst (was völlig lächerlich ist) zerstört …” (p. 134, and discussion in footnote 6). As for *excido* in the sense of “fall out”, on the other hand, I found that when it is intended to mean “fall out” then it is generally constructed with a prepositional phrase and an ablative, not with a direct object expressed in the accusative form. Siebert comments: “Die ptolemischen Ausdruck ‘extra coelum excidere’ verkürzt er [i.e., Copernicus] in ein ‘coelum excidere’, wodurch ‘excidere’ nicht mehr ‘herausfallen’ (ex+cadere) bezeichnet, sondern sich in dieser transitiven Verwendung trotz gleicher Schreibung als ein anderes Verb entpuppt (ex-caedere), welches die Bedeutung hat von ‘heraushauen, aufbrechen, zerstören’” (p. 138). Hill, too, in “The projection argument in Galileo and Copernicus” (ref. 4), 112–15, discusses at length Copernicus’s reasons for interpreting Ptolemy’s passage in such a way. A referee suggested a further, broader interpretative possibility in terms of “rhetorical” strategy. I report his suggestion almost verbatim. “Copernicus makes a very personal appropriation of Ptolemy’s argument. With an intentional deformation of the *Almagest* text, he explains (in *De revolutionibus* I 8, at f. 5v), that if the supposed violent Earth’s rotation were to disperse all things not firmly bound together, and

eventually bring the terrestrial globe itself to disintegrate and fall out the heavens (a consequence not imaginated by Ptolemy), then it is even more obvious (and this is a consequence Ptolemy should fear), that, due to the ever increasing speed of their violent circular motion, the heavens would become more and more immense, if not infinite, in being driven away from the centre. This being so, Copernicus's use of Ptolemy is to be understood not as the result of a faulty reading of the *Almagest* passage (either in Greek, or in one of its Latin translations), but the product of a rhetorical strategy." I thank the anonymous referee for the suggestion.

21. "Considerando Tolomeo questa opinione, per distruggerla argomenta in questa guisa.... E finalmente, essendo il moto circolare e veloce accommodato non all' unione, ma pi tosto alla divisione e dissipazione, quando la terra così precipitosamente andasse a torno, le pietre, gli animali, e l' altre cose, che nella superficie si ritrovano, verriano da tal vertigine dissipati, sparsi e verso il cielo tirati; così le città e gli altri edificii sariano messi in ruina." Galileo, *Opere* (ref. 2), ii, 223–4.
22. "... et aggregantur mota: et stant fixa in medio ex sustentatione et coangustatione vel fulcimento et impulsione eorum ad invicem ab omnibus partibus equaliter et similter." *Cf.* Ptolemy, *Almagestum Cl. Ptolemei* (ref. 18), 4.
23. See C. Clavius, *In sphaeram Ioannis de Sacro Bosco commentarius* (Rome, 1585).
24. Various editions of the *Commentary* appeared in 1570, 1581, 1585, 1593, 1596, 1601 and 1607, and also in 1611 (as the third volume of the *Opera mathematica*).
25. "Praeterea, si terra tanta celeritate circa axem mundi volueretur, ut videlicet circuito expleret spacio 24. horarum, sicut quidam fabulantur, omnia aedificia corruerent, et nulla ratione diu consistere possent." See Clavius, *In sphaeram Ioannis de Sacro Bosco commentarius* (ref. 23), 196. Clavius does not mention Ptolemy, however. *Cf.* James Lattis's comment, "Clavius's apocalyptic vision of collapsing buildings is not a recitation of anything found in Ptolemy…" (*Between Copernicus and Galileo: Christoph Clavius and the collapse of Ptolemaic cosmology* (Chicago, 1994), 121).
26. Clavius, *In sphaeram Ioannis de Sacro Bosco commentarius* (ref. 23), 196.
27. W. Ong, *Orality and literacy: The technologizing of the word* (London, 1988), 38.
28. An analogous situation can be described in a case which has puzzled Galileo scholars for a long time, i.e., the myth concerning the formation of planets that Galileo explicitly attributes to Plato, for example, in the Fourth Day of *Two new sciences*. Fabio Acerbi, in a paper summarizing the *status quaestionis* of this little Galileo mystery, has shown that in Galileo's text there is a marked correspondence of themes and syntactic structures with *Timaeus* 38c 7–8, 38e 3–6, but it is impossible to pin down precise textual references to Renaissance editions of *Timaeus*, in both Greek and Latin, potentially available to Galileo. F. Acerbi, "Le fonti del mito Platonico di Galileo", *Physis*, xxxvii (2000), 359–92. Though Acerbi does not take this possibility into consideration, I suggest that we are here, once again, in the presence of a typical effect of orality-shaped modes of appropriation of written material; hence the loss of precise correspondences between texts.
29. The text of La Galla's dissertation (in Latin) has been re-published partially, together with all Galileo's postils, in Galileo, *Opere* (ref. 2), iii, 311–99.
30. Galileo, *Opere* (ref. 2), iii, 345.
31. Galileo, *Opere* (ref. 2), iii, 346.
32. G. B. Riccioli, *Almagestum novum* (2 vols, Bologna, 1651), ii, 432–3.
33. Riccioli, *Almagestum novum* (ref. 32), ii, 433. However, the quotation from Ptolemy is introduced by Riccioli with the possibly adversative "Ptolemaei autem verba lib. 1 cap. 7 fuerant…", which might suggest that he saw a certain discrepancy between Copernicus's reading and Ptolemy's original passage. On the other hand, it has been suggested that Riccioli, traditionally portrayed as a staunch anti-Copernican, might in fact have harboured doubts about the geocentric model of the universe. *Cf.* A. Dinis, "Was Riccioli a secret Copernican?", in M. T. Borgato (ed.), *Giambattista Riccioli e il merito scientifico dei Gesuiti nell' età barocca* (Florence, 2002), 49–77, espec. pp. 59ff. Therefore one might read his self-effacement in the historical reconstruction of the extrusion effect as part of a conscious rhetoric of ambiguity.

34. Galileo, *Opere* (ref. 2), xvi, 330–4. The letter was written in 1635 and first published by Gloriosi himself, in G. C. Gloriosi, *Exercitationum mathematicarum decas tertia* (Naples, 1639), 146–51. This fascinating document has not been studied in detail so far. On Gloriosi, see Pier Daniele Napolitani, "Galileo e due matematici napoletani: Luca Valerio e Giovanni Camillo Glorioso", in F. Lomonaco and M. Torrini (eds), *Galileo e Napoli* (Naples, 1987), 159–95. *Cf.* also C. R. Palmerino, "Una nuova scienza della materia per la Scienza nova del moto: La discussione dei paradossi dell' infinito nella Prima Giornata dei Discorsi galileiani", in E. Festa and R. Gatto (eds), *Atomismo e continuo nel XVII secolo* (Naples, 2000), 275–319, pp. 284–6, for a few comments, in passing, on the angle of contingence, in relation to her claim that there is a certain similarity between Galileo's letter and his solution to the Rota Aristotelis paradox (published later on in *Two new sciences*). Finally, see Carl B. Boyer, "Galileo's place in the history of mathematics", in E. McMullin (ed), *Galileo: Man of science* (New York, 1967), 232–55. Boyer's article contains a brief discussion of Galileo on the angle of contingence, but Boyer sees in Galileo's reasoning only an anticipation of the concept of "the order of an infinitesimal", an opinion which, in my view, is anachronistic.
35. "… certo mio discorso che gran tempo fa mi passò per la fantasia", Galileo, *Opere* (ref. 2), xvi, 331.
36. "… angolo sia l' inclinazione di due linee che si toccano in un punto e non son poste tra di loro per diritto", Galileo, *Opere* (ref. 2), xvi, 331.
37. Galileo, *Opere* (ref. 2), xvi, 331–2.
38. The controversy was prompted by Euclid's Proposition 16, in Book III of the *Elements*. "The straight line drawn at right angles to the diameter of a circle from its end will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed, further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilinear angle." *Cf.* Euclid, *The thirteen books of the Elements*, transl. and ed. by Thomas Heath, 2nd edn (3 vols, New York, 1956), ii, 37. An important Renaissance episode in the controversy was the debate between Jacques Peletier and Christoph Clavius. *Cf.* L. Maierù, "In Christophorum Clavium de contactu linearum Apologia: Considerazioni attorno alla polemica fra Peletier e Clavio circa l' angolo di contatto (1579–1589)", *Archive for history of exact sciences*, xli (1990), 115–37. For a detailed study of that episode, and more generally, on the history of the controversy, see Heath's comments, *loc. cit.*, ii, 39–43.
39. J. Peletier, *In Euclidis Elementa geometrica demonstrationum libri sex* (Lyons, 1557), 73–78; *Commentarii tres* (Basel, 1563), 28–48; *In Christophorum Clavium De contactu linearum apologia* (Paris, 1579), 3r–9v; and *De contactu linearum, commentarius* (Paris, 1581).
40. Galileo was familiar with Clavius's edition of Euclid's *Elements*, in which the résumé was republished together with Clavius's response (P. Palmieri, "The obscurity of the equimultiples: Clavius' and Galileo's foundational studies of Euclid's theory of proportions", *Archive for history of exact sciences*, lv (2001), 555–97). In fact Clavius quotes Peletier's comments in the latter's edition of Euclid verbatim. *Cf.* C. Clavius, *Commentaria in Euclidis Elementa geometrica* (Hildesheim, 1999), facsimile edition of the first volume of *Christophori Clauii Bambergensis e Societate Iesu Opera mathematica V tomis distributa* (5 vols, Mainz, 1611–12), 117.
41. "Cum igitur omnis angulus in pluribus punctis non consistat, quam uno" (Clavius, *Commentaria in Euclidis Elementa geometrica* (ref. 40), 117). An interpretive caveat is necessary, however. For the passage continues as follows: "fit ut punctum *A* tam sit ineptum angulo constituendo, quam modo erat punctum sectionis *E*, linearum rectarum." It seems difficult to reconcile the punctiform view with the claim that point *A* is "inept" to form an angle. I owe this insight to Curtis Wilson.
42. Clavius, *Commentaria in Euclidis Elementa geometrica* (ref. 40), 119.
43. Clavius asserted that "... quemvis angulum contactus, etsi ab Euclide minor ostensus est omni acuto rectilineo, dividi posse in partes infinitas". Clavius, *Commentaria in Euclidis Elementa geometrica* (ref. 40), 119.
44. Galileo, *Opere* (ref. 2), xvi, 332ff.
45. Galileo, *Opere* (ref. 2), xvi, 334.

46. See Galileo Galilei, *Two new sciences: Including centres of gravity and force of percussion*, ed. by Stillman Drake (Madison, 1974), 249– 52, for the ballistic tables. Folio 122v, *Manuscript 72*, is preserved in the National Library at Florence. The sheet was published in Galileo, *Opere* (ref. 2), viii, 432. It is now also available on-line, retrievable at: http://echo.mpiwg-berlin.mpg.de/content/scientific_revolution/galileo.
47. Galileo, *Opere* (ref. 2), viii, 432.
48. *Cf.* a classic treatise, such as, for example, W. H. Besant, *Conic sections* (London, 1895), 154, for an example and a discussion. It can be easily seen, for instance, by writing the polar equation of a parabola.
49. W. Knorr, *The ancient tradition of geometric problems* (New York, 1993), 335.
50. Archimedes, *Archimedis Syracusani philosophi ac geometrae excellentissimi Opera* (Basel, 1544), 59. In the Commandino edition, we find "omnia conoidea rectangula sunt similia". See Archimedes, *Opera Archimedis Syracusani philosophi et mathematici ingeniosissimi* (Venice, 1558), 27v.
51. Galileo, *Dialogue concerning the two chief world systems* (ref. 2), 188ff.
52. *Ibid.*, 197.
53. *Ibid.*, 200–1.
54. Hill, "The projection argument in Galileo and Copernicus" (ref. 4), 121.
55. *Ibid.*, 122.
56. *Ibid.*, 123.
57. Galileo, *Dialogue concerning the two chief world systems* (ref. 2), 201–2.
58. Galileo's proportional reasoning is a form of reasoning based on the principled manipulation of ratios and proportions, according to the rules set forth in the fifth book of Euclid's *Elements*. As for Galileo's use of Euclidean proportionality, a considerable body of literature is now available, which allows us to understand most of its technical aspects better. *Cf.* C. Armijo, "Un nuevo rol para las definiciones", in J. Montesinos and C. Solís (eds), *Largo campo di filosofare: Eurosymposium Galileo 2001* (La Orotava, Tenerife, 2001), 85–99; S. Drake, "Velocity and Eudoxan proportion theory", *Physis*, xv (1973), 49–64 (reprinted in S. Drake, *Essays on Galileo and the history and philosophy of science* (3 vols, Toronto, 1999), ii, 265–80); *idem*, "Galileo's experimental confirmation of horizontal inertia: Unpublished manuscripts", *Isis*, lxiv (1973), 291–305 (reprinted in Drake, *Essays*, ii, 147–59); *idem*, "Mathematics and discovery in Galileo's physics", *Historia mathematica*, i (1974), 129–50 (reprinted in Drake, *Essays*, ii, 292–306); *idem*, "Euclid Book V from Eudoxus to Dedekind", *Cahiers d' histoire et de philosophie des sciences*, n.s., xxi (1987), 52–64 (reprinted in Drake, *Essays*, iii, 61–75); A. Frajese, *Galileo matematico* (Rome, 1964); E. Giusti, "Aspetti matematici della cinematica Galileiana", *Bollettino di storia delle scienze matematiche*, i (1981), 3–42; *idem*, "Ricerche Galileiane: Il trattato 'De motu equabili' come modello della teoria delle proporzioni", *Bollettino di storia delle scienze matematiche*, vi (1986), 89–108; *idem*, "Galilei e le leggi del moto", in Galileo Galilei, *Dicorsi e dimostrazioni matematiche intorno a due nuove scienze attinenti alla meccanica ed i movimenti locali*, ed. by Enrico Giusti (Turin, 1990), pp. ix–lx; *idem*, "La teoria galileiana delle proporzioni", in L. Conti (ed.), *La matematizzazione dell' universo: Momenti della cultura matematica tra '500 e '600* (Perugia, 1992), 207–22; *idem*, *Euclides reformatus: La teoria delle proporzioni nella scuola galileiana* (Turin, 1993); *idem*, "Il filosofo geometra: Matematica e filosofia naturale in Galileo", *Nuncius*, ix (1994), 485–98; *idem*, "Il ruolo dela matematica nella meccanica di Galileo", in A. Tenenti *et al.*, *Galileo Galilei e la cultura veneziana* (Venice, 1995), 321–38; F. Palladino, "La teoria delle proporzioni nel Seicento", *Nuncius*, vi (1991), 33–81; and P. Palmieri, "The obscurity of the equimultiples" (ref. 38). For a general treatment of the various aspects of the Euclidean theory of proportions I have relied on: I. Grattan-Guinness, "Numbers, magnitudes, ratios, and proportions in Euclid's elements: How did he handle them?", *Historia mathematica*, xxiii (1996), 355–75; C. Sasaki, "The acceptance of the theory of proportions in the sixteenth and seventeenth centuries", *Historia scientarum*, xxix (1985), 83–116; K. Saito, "Compounded ratio in Euclid and Apollonius", *Historia scientiarum*, xxxi (1986), 25–59; and *idem*, "Duplicate ratio in Book VI of Euclid's *Elements*", *Historia scientiarum*, l (1993), 115–35. P. L. Rose, *The Italian renaissance*

of mathematics: Studies on humanists and mathematicians from Petrarch to Galileo (Geneva, 1975) is an extensive, immensely erudite survey of Renaissance mathematics in Italy from a non-technical point of view. *Cf.* also E. D. Sylla, “Compounding ratios: Bradwardine, Oresme, and the first edition of Newton’s *Principia*”, in E. Mendelsohn (ed.), *Transformation and tradition in the sciences: Essays in honor of I. Bernard Cohen* (Cambridge, MA, 1984), 11–43.

59. Galileo, *Dialogue concerning the two chief world systems* (ref. 2), 202–3. Emphasis is mine.
60. *Ibid*., 203. I have slightly altered Drake’s translation. Emphasis is mine.
61. Riccioli, *Almagestum novum* (ref. 32), 429. Emphasis is mine.
62. *Cf.* J. Kepler, *Opera omnia*, ed. by C. Frish (8 vols, Frankfurt A. M., 1858–70), vi, 183–4; and I. Boulliau, *Philolai, sive dissertationis de vero systemate mundi* (Amsterdam, 1639), 20–21.
63. Once again, however, Riccioli pairs the Galileo reference with a “balancing” reference to the *Commentary* on Aristotle’s *Meteorologica* by Niccolò Cabeo (1586–1650), who squarely opposes Galileo on centrifugal force.
64. The history might reveal interesting insights not only about seventeenth-century astronomy but also about seventeenth-century natural philosophies; for example, William Gilbert thought the extrusion argument to be “frivolous” and of “no moment”. See W. Gilbert, *De mundo nostri sublunari philosophia nova* (Amsterdam, 1651), 161–2. Antonio Rocco (1586–1652), an Aristotelian natural philosopher who attacked Galileo’s *Dialogue* in 1633, thought the argument to be of no value and “di niun momento e falso”, qualifying it with almost the same words as Gilbert’s (Galileo, *Opere* (ref. 2), vii, 682).
65. Hill, “The projection argument in Galileo and Copernicus” (ref. 4), 133.