"On the Electrodynamics of moving Bodies"

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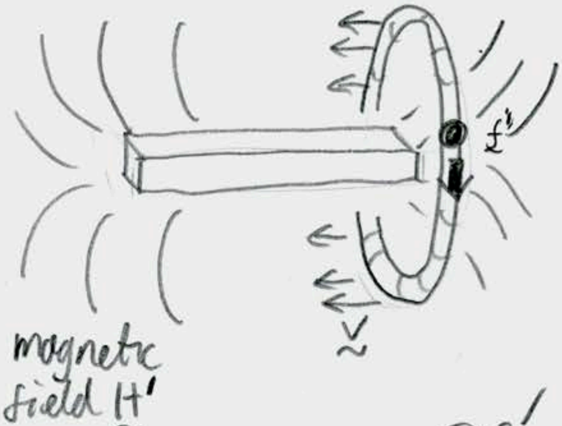
[Notes by John D. Norton]

Overview

Problem: Ether state of rest of contemporary electrodynamics is superfluous – not manifest in actual optical experiments – theory predicts it is elusive ⇩	Introduction
Solution New theory of space and time based on principle of relativity, light postulate	Part A. Kinematical Part
Eradicates state of rest in electrodynamics, while leaving theory unchanged Provides useful tool for solving problems in the electrodynamics of moving bodies	Part B. Electrodynamic Part

Introduction: magnet & conductor Thought Experiment

Rest frame of magnet

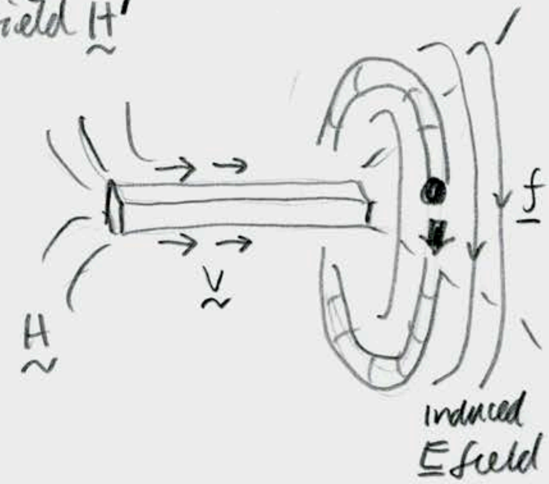


current generating force $\underset{\sim}{f}$ on charge q in conductor:

$$\boxed{\frac{\underset{\sim}{f}'}{q} = \frac{1}{c}\underline{V} \times \underline{H}'}$$

denote magnet rest frame

Rest frame of conductor



Rules for transforming for rest frame magnet to rest frame conductor

$$\underset{\sim}{H} = \underset{\sim}{H}'$$

$$t = t'$$

$$\underset{\sim}{r} = \underset{\sim}{r}' - \underset{\sim}{V}t'$$

Hence

$$\frac{\partial}{\partial t'} = \frac{\partial t}{\partial t'}\cdot\frac{\partial}{\partial t} + \frac{\partial r}{\partial t'}\cdot\underset{\sim}{\nabla} = \frac{\partial}{\partial t} - \underline{V}\cdot\underline{\nabla}$$

$\underline{H}'$ field static in magnet rest frame

$$\frac{\partial \underline{H}'}{\partial t'} = 0 \Rightarrow \frac{\partial \underline{H}}{\partial t} = (\underline{V}\cdot\underline{\nabla})\underline{H}$$

$$= -\underline{\nabla}\times(\underline{V}\times H) + \underline{V}\underbrace{(\underline{\nabla}\cdot\underline{H})}_{\text{0 by maxwell's equations } \underline{\nabla}\cdot\underline{H}=0}$$

Identity for constant $\underline{V}$

$$\underline{\nabla}\times(\underline{V}\times\underline{H}) = -(\underline{V}\cdot\underline{\nabla}) + \underline{V}(\underline{\nabla}\cdot\underline{F})$$

maxwell's equation

$$\underline{\nabla}\times E = -\frac{1}{c}\frac{\partial H}{\partial t}$$

$$\underline{\nabla}\times\underline{E} = \frac{1}{c}\,\underline{\nabla}\times(\underline{V}\times\underline{H})$$

$$\therefore\ \underset{\sim}{E} = \frac{1}{c}(\underline{V}\times\underline{H}) + \underbrace{\underline{\nabla}\varphi}$$

arbitrary φ

No net contribution to current since for current loop $\oint \underline{\nabla}\cdot\varphi\cdot d\underline{r} = 0$

$$\underbrace{\underset{\sim}{E} = \underline{f}/q}$$

Hence same force as seen in rest frame magnet

Einstein's morals

- Observables (current) depends only on relative motion
- Theoretical account depends on absolute motion

motion magnet only ⇒ NEW ENTITY induced $\underline{E}$ field "with a definite energy"

"Examples of this sort" + "unsuccessful attempts to detect motion of earth relatively to light medium" ⇒ Principle of relativity (at least to first order")

Plausible: moving magnet carries same field with it.

Problems

1. Transformation $\underline{H} = \underline{H}'$ cannot be assumed. must be deduced from Maxwell's equations.
   Turns out to hold ONLY at first order v/c

2. Other analogous thought experiments fail. e.g. Föppl, two charges

$\underline{f}'$, $\underline{f}$ differ in quantities second order.

Einstein promises to solve problem

New kinematics based on

1. Principle of relativity
2. Light postulate

A. Kinematical Part

§1 Definition of Simultaneity

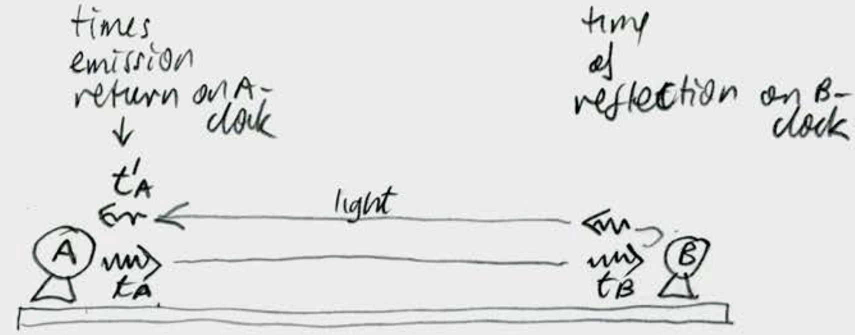
Goal | Principle of relativity
Light postulate
"apparently irreconcilable"
(introduction)

⇐ show:
Relativity of simultaneity reconciles them.

Definition of properly synchronised clocks A, B at different places

⇓

Definition needed.
Hence big literature follows in "conventionality of simultaneity"



"the two clocks are synchronous by definition if

$$t_B - t_A = t'_A - t_B \text{"}$$

[Equality of one-way transit times]

[A is synchronous with B assumed
• symmetric • transitive]

Round trip speed of light measured on one clock
∴ not conventional

⇓

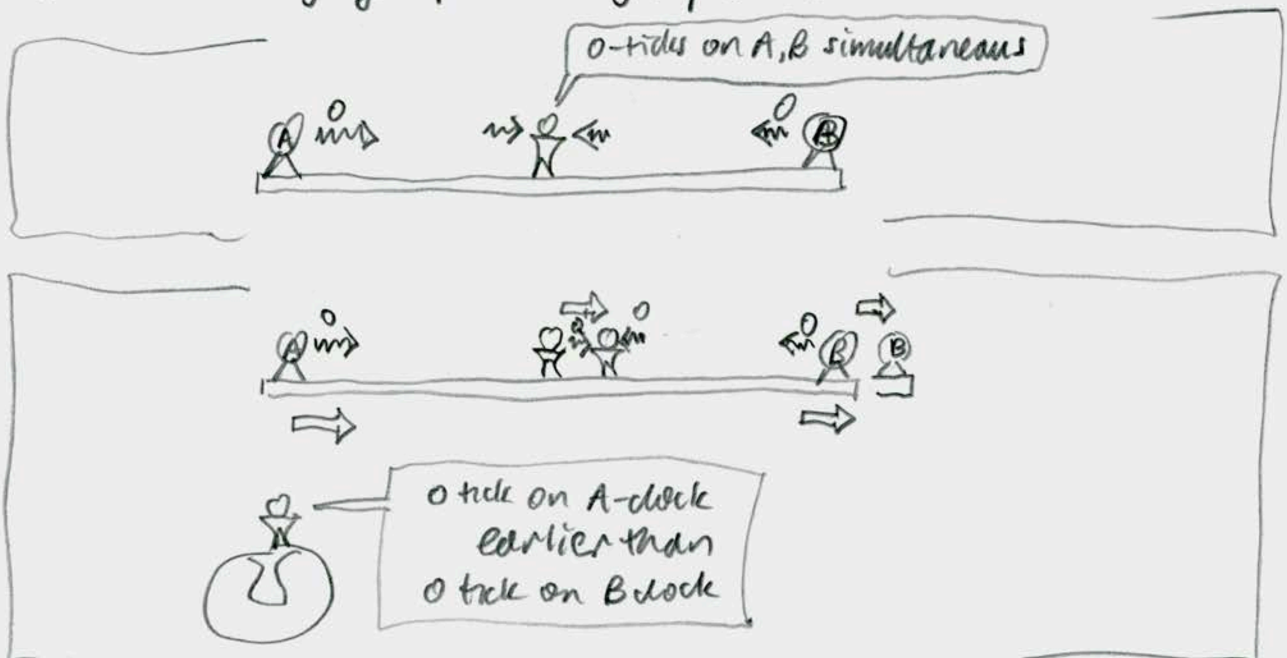
" Based on experience, we further stipulate that the quantity $\frac{2\overline{AB}}{t'_A - t_A} = c$

be a universal constant (the velocity of light in empty space) "

Factual presumption
Restates light postulate if assumed for all inertial frames.

What Einstein did not say, but was implicit (?):

- Observers in relative motion disagree on simultaneity of spatially separated events



Two views of same process

- If clocks are set by Einstein's definition then light postulate can hold for all inertially moving observers

  ... chasing light does not slow it, since clocks reset to obliterate the slowdown → comoving observer always measures equal transit times

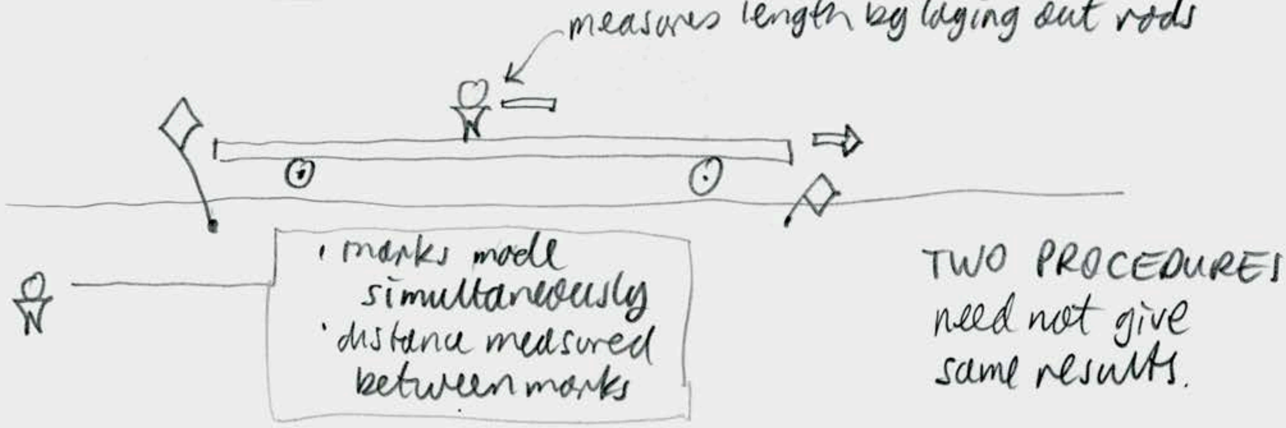
  Only possible as long as factually light postulate holds.

  ... else round trip speed would be affected by motion of observer.

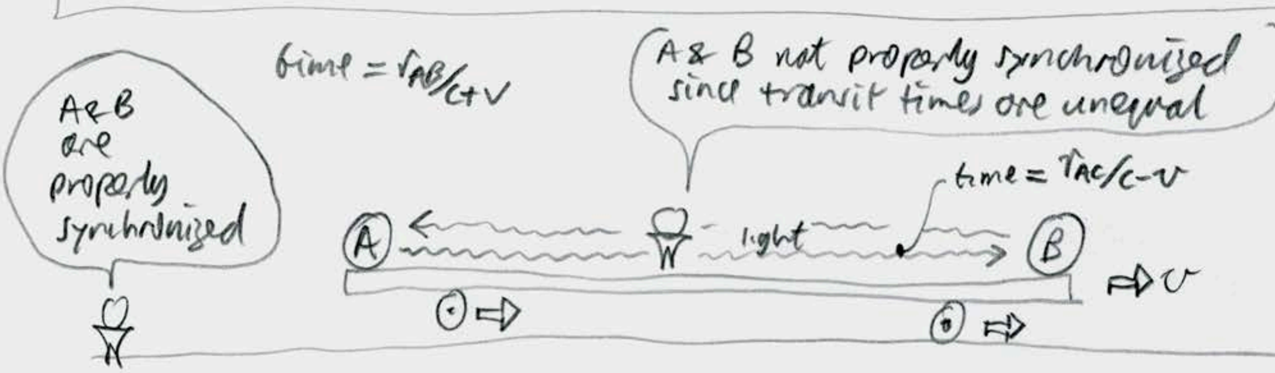
§2 On the Relativity of Lengths & Times

converted to "is" in section 3

Length of rigid rod AT REST	NEED NOT equal	Length of rigid rod IN MOTION



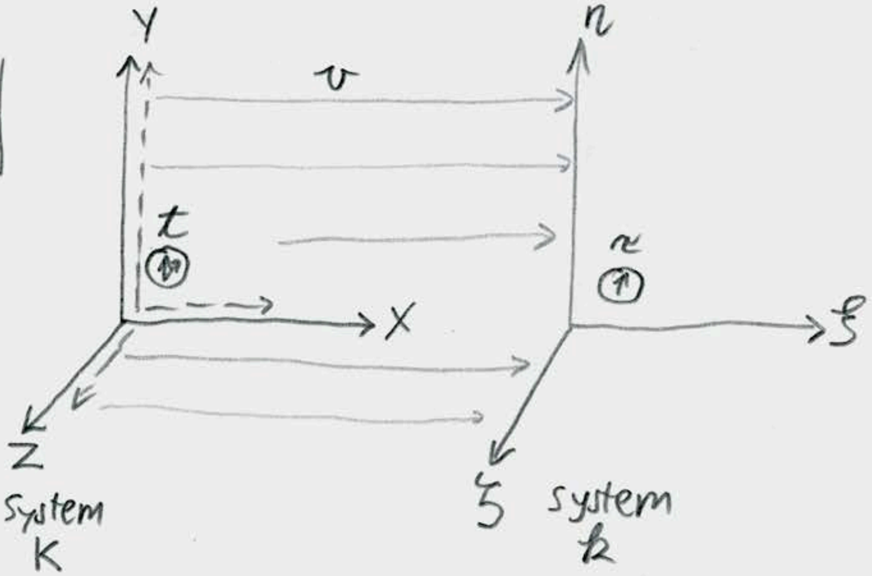
clocks well synchronized for resting observer	⟹	clocks may not be well-synchronised for moving observer



Hence relativity of simultaneity

§3 Einstein's (unbelievably cumbersome) derivation of the Lorentz transformation

Set up

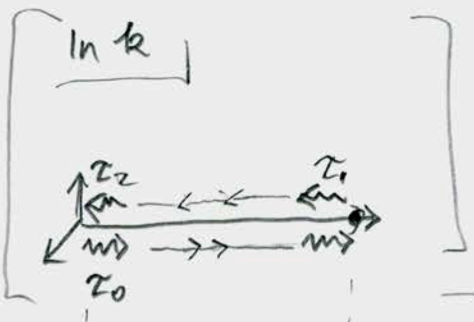


K and k coincide at $\tau = t = 0$

Find τ, ξ, η, ζ as function of t, x, y, z

Homogeneity of space & time
$\Downarrow$
Transformation must be linear

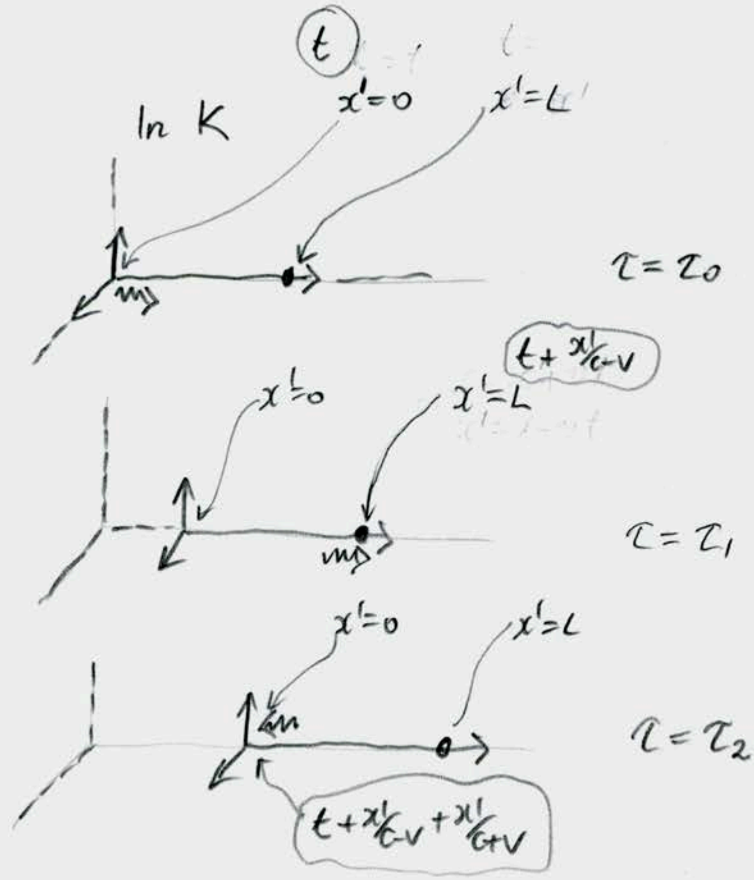
Reflected light signal



In K: $\longleftarrow L \longrightarrow$

$x = vt$ $\qquad$ $x = vt + L$

$\underbrace{x' = x - vt = 0 \qquad x' = x - vt = L}_{x' \text{ values of both points indep. of } t}$



Einstein's definition of clock synchrony for clocks in k:

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1$$

$$\tau = \tau(x', y, z, t)$$

x' used as coordinate!

K time for outward trip

K time for return

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,\left\{t + \frac{x'}{c-v} + \frac{x'}{c+v}\right\}\right)\right] = \tau\left(x',0,0,t + \frac{x'}{c-v}\right)$$

Let x' become very small

$$\tau\left(x',0,0,t+\frac{x'}{c+v}\right) \approx \tau(0,0,0,t) + x'\frac{\partial\tau}{\partial x'} + \frac{x'}{c-v}\frac{\partial\tau}{\partial t}$$

$$\tau\left(0,0,0,t+\frac{x'}{c-v}+\frac{x'}{c+v}\right) \approx \tau(0,0,0,t) + x'\left(\frac{1}{c-v}+\frac{1}{c+v}\right)\frac{\partial\tau}{\partial t}$$

$$\Downarrow$$

$$\frac{1}{2}\left(\frac{1}{c-v}+\frac{1}{c+v}\right)\frac{\partial\tau}{\partial t} = \frac{\partial\tau}{\partial x'} + \frac{1}{c-v}\cdot\frac{\partial\tau}{\partial t}$$

$$\frac{1}{c-v} - \frac{1}{2}\left(\frac{1}{c-v}+\frac{1}{c+v}\right) = \frac{c+v}{c^2-v^2} - \frac{1}{2}\,\frac{c+v-(c-v)}{c^2-v^2} = \frac{v}{c^2-v^2}$$

$$\Downarrow$$

$$\boxed{\frac{\partial\tau}{\partial x'} + \frac{v}{c^2-v^2}\cdot\frac{\partial\tau}{\partial t} = 0}$$

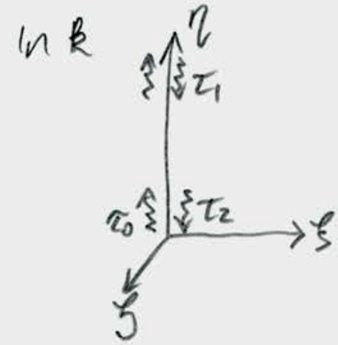
$\leftarrow$ holds at all x', y, z

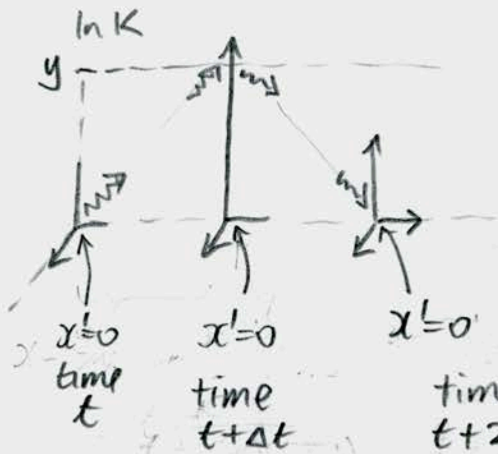
(Repeat argument with origin replaced by (x', y, z))

$$\boxed{\text{Analogous reasoning on } y, z \implies \frac{\partial \tau}{\partial z} = 0 \quad \frac{\partial \tau}{\partial y} = 0}$$

(Not given)

JDN: For y-axis





never actually need to compute it!

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1$$

$$\therefore \underbrace{\frac{1}{2}\left[\tau(0,0,0,t) + \tau(0,0,0,t+2\Delta t)\right] = \tau(0,y,0,t+\Delta t)}$$

small y

$$\tau(0,0,0,t+2\Delta t) \approx \tau(0,0,0,t) + 2\Delta t \frac{\partial \tau}{\partial t}$$

$$\tau(0,y,0,t+\Delta t) \approx \tau(0,0,0,t) + y\frac{\partial \tau}{\partial y} + \Delta t \frac{\partial \tau}{\partial t}$$

$$\Downarrow$$

$$\frac{1}{2}\left(\tau + \tau + 2\Delta t \frac{\partial \tau}{\partial t}\right) = \tau + y\frac{\partial \tau}{\partial y} + \Delta t \frac{\partial \tau}{\partial t}$$

$$0 = y\frac{\partial \tau}{\partial y}$$

$$\boxed{\frac{\partial \tau}{\partial y} = 0}$$

$$\boxed{\tau = a t + B x' + D y + E z}$$ since linear in t, x, y, z

$a = \frac{\partial \tau}{\partial t}$

$D = 0$ since $\frac{\partial \tau}{\partial y} = 0$

$E = 0$ since $\frac{\partial \tau}{\partial z} = 0$

$$B = \frac{\partial \tau}{\partial x'} = -\frac{v}{c^2 - v^2}\frac{\partial \tau}{\partial t} = -a\frac{v}{c^2-v^2}$$

$$\boxed{\tau = \underbrace{a}_{\text{"}\varphi(v)\text{"}}\left(t - \frac{v}{c^2-v^2}x'\right)}$$

Find ξ as function t, x', y, z

Light signal $\xi = c\tau$ in k $\longrightarrow$ $\overbrace{x' = (c-v)t}^{\text{same as } x - vt = (c-v)t,\ x = ct}$ in K

$$\tau = a\left(t - \frac{v}{c^2-v^2}\cdot x'\right) = a\underbrace{\left(\frac{x'}{c-v} - \frac{v}{c^2-v^2}x'\right)}$$

$$= a\frac{c^2}{c^2-v^2}x'$$ since $\frac{1}{c-v} - \frac{v}{c^2-v^2} = \frac{c+v}{c^2-v^2} - \frac{v}{c^2-v^2} = \frac{c}{c^2-v^2}$

$$\boxed{\xi = c\tau = a\frac{c^2}{c^2-v^2}x'}$$

True for all t, x, y, z since transformation is linear

Find η as function of t, x, y, z

Light signal $\eta = c\tau \longrightarrow y = c_{\text{in } y \text{ dirn}}\, t$

$\uparrow$

$c\sqrt{1-v^2/c^2}$

$x' = 0$

$$\therefore \eta = c\tau = a\left(t - \frac{v}{c^2-v^2}x'\right) = a\,\frac{y}{c\sqrt{1-v^2/c^2}} = a\,\frac{c}{\sqrt{c^2-v^2}}\cdot y$$

True for all x', y, z, t due to linearity

Analogously $\zeta = a\,\frac{c}{\sqrt{c^2-y^2}}\,z$

$$\boxed{\tau = a\left(t - \frac{v}{c^2-v^2}x'\right) \quad \xi = a\,\frac{c^2}{c^2-v^2}x' \quad \eta = a\,\frac{c}{\sqrt{c^2-v^2}}\,y \quad \zeta = a\,\frac{c}{c^2-v^2}\,z}$$

$$\tau = a\left(t - \frac{v}{c^2-v^2}(x-vt)\right) \qquad \xi = a\,\frac{c^2}{c^2-v^2}(x-vt)$$

$$= a\left(t + \frac{v^2}{c^2-v^2}t - \frac{v}{c^2-v^2}x\right)$$

$$= \underbrace{a\cdot\frac{c^2}{c^2-v^2}}\left(t - \frac{v}{c^2}x\right)$$

$$a\,\frac{c^2}{c^2-v^2} = \underbrace{\frac{a}{\sqrt{1-v^2/c^2}}}_{\text{"}\varphi(v)\text{"}} \cdot \underbrace{\frac{1}{\sqrt{1-v^2/c^2}}}_{\text{"}\beta\text{"}}$$

$$\boxed{\tau = \varphi(v)\,\beta\left(t - \frac{v}{c^2}x\right) \quad \xi = \varphi(v)\,\beta(x - vt) \quad \eta = \varphi(v)\,y \quad \zeta = \varphi(v)\,z}$$

Compatability with light postulate with $\varphi(v)$ undetermined

Light shell expanding at c in K

$x^2+y^2+z^2=c^2t^2$

$\Longrightarrow$

Light shell expanding at c in k

$\xi^2+\eta^2+\zeta^2=c^2\tau^2$

... by direct substitution

Determine $\varphi(v)=1$

since

$$K(x,y,z,t) \xrightarrow[\text{in } x\text{-direction}]{+v} k(\xi,\eta,\zeta,\tau) \xrightarrow[\text{in } \xi \text{ direction}]{-v} K'(x',y',t',z')$$

must return original K so that $t'=t$, $x'=x$, ...

By direct substitution, find $t'=\varphi(v)\varphi(-v)t$ $\quad y'=\varphi(v)\varphi(-v)y$

$x'=\varphi(v)\varphi(-v)x$ $\quad z'=\varphi(v)\varphi(-v)z$

e.g.

$$t'=\varphi(-v)\beta(-v)\left(\tau-\tfrac{v}{c^2}\xi\right)$$

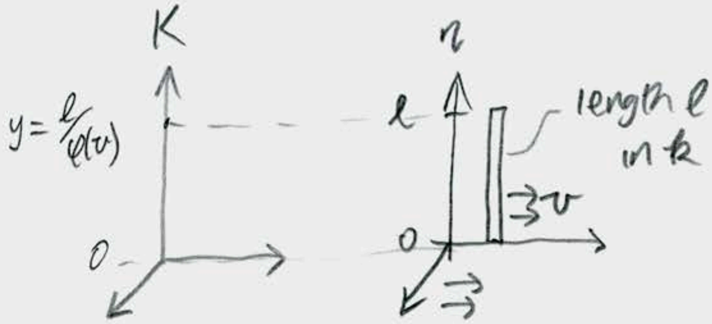
$$=\varphi(-v)\varphi(v)\underbrace{\beta(-v)\beta(v)}\left(t-\cancel{\tfrac{v}{c^2}x}-\cancel{\tfrac{v}{c^2}}(x-vt)\right)$$

$$=\varphi(v)\varphi(-v)\frac{1}{1-v^2/c^2}\left[t\left(1-v^2/c^2\right)\right]$$

$$=\varphi(v)\varphi(-v)\,t$$

$$\boxed{\varphi(v)\varphi(-v)=1}$$

and | $\varphi(v) \sim$ length change of rod perpendicular to direction of motion



motion v : $\underbrace{\ell \longrightarrow \frac{\ell}{\varphi(v)}}_{\text{same effect for } v \text{ or } -v}$

$$\boxed{\varphi(v) = \varphi(-v)}$$

so | $\varphi(v)\,\varphi(-v) = 1 \xrightarrow[\varphi(-v)=\varphi(v)]{} [\varphi(v)]^2 = 1 \quad \varphi(v) = \pm 1$

choose $\varphi(v) = +1$ since ξ, x axes point in same direction

Final Result |

$$\tau = \beta\left(t - \frac{v}{c^2}x\right) \qquad \eta = y$$

$$\xi = \beta\,(x - v t) \qquad \zeta = z$$

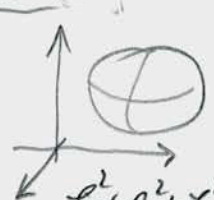
$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}}$$

§4 Physical meaning...

Rigid sphere $\xrightarrow{\text{motion}}$ Flattened to ellipsoid

N.B. Lorentz's electron!

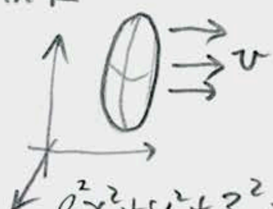
In k



$\xi^2+\eta^2+\zeta^2=R^2$

at $t=0$
$\xi=\beta x$

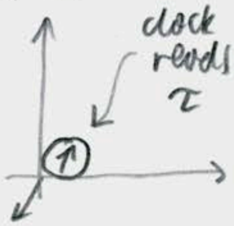
In K



$\beta^2x^2+y^2+z^2=R^2$

moving clocks run slower

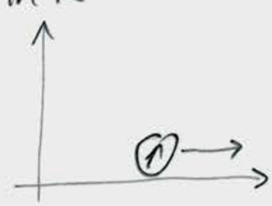
In k



$\tau=\beta\left(t-\frac{v}{c^2}x\right)$
$x=vt$

$\therefore\ \tau=\sqrt{1-v^2/c^2}\,t$

In K



$\tau=\sqrt{1-v^2/c^2}\;t\approx\left(1-\tfrac{1}{2}(v/c)^2\right)t$

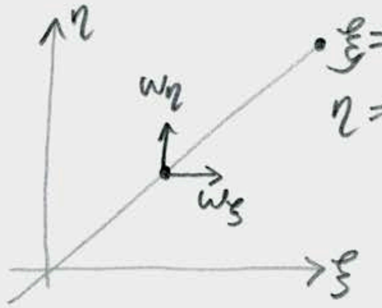
what moving clock reads

time of clock at rest.

$\therefore$ clocks at equator run slower

§5 Addition Theorem for Velocities

In k



$\xi = w_\xi \tau$

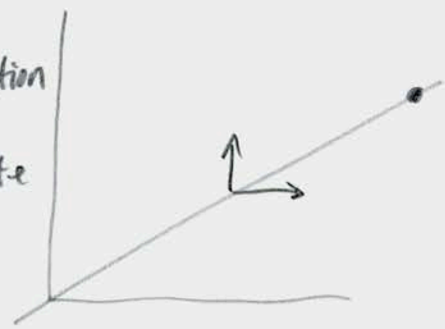
$\eta = w_\eta \tau$

loosely: Add v in ξ direction

$\Longrightarrow$

substitute directly

In K



$$x = \frac{w_\xi + v}{1 + \frac{v w_\xi}{c^2}}\, t$$

$$y = \frac{\sqrt{1-(v/c)^2}}{1 + \frac{v w_\xi}{c^2}}\, t$$

Read alternative rules of velocity composition directly from these.

Not AE's symbol!

e.g. v, w parallel $\Longrightarrow v \oplus w = U = \dfrac{v+w}{1+\frac{vw}{c^2}}$

$$\boxed{\text{subluminal velocities} \xrightarrow[\text{compounding}]{\not\Leftarrow\ \not\Rightarrow} \text{superluminal velocities}}$$

$v = c - k$

$w = c - \lambda$

need not be small

$$v \oplus w = \frac{2c-k-\lambda}{1+\frac{(c-k)(c-\lambda)}{c^2}} = \frac{2c-k-\lambda}{\underbrace{2c-k-\lambda+\frac{k\lambda}{c^2}}_{\text{always less than 1}}} \cdot c$$

c unchanged if compounded with $w < c$

$$c \oplus w = \frac{c+w}{1+\frac{cw}{c^2}} = \frac{c\,(1+w/c)}{(1+w/c)} = c$$

Lorentz transformation forms a group

$$\begin{tikzcd} K \arrow[r, "\substack{\text{velocity}\\ v}"] \arrow[rr, bend right=40, "\substack{\text{Lorentz}\\ \text{transformation}\\ \text{with velocity}\\ \frac{v+w}{1+\frac{vw}{c^2}}}"'] & k \arrow[r, "\substack{\text{velocity}\\ w}"] & k' \end{tikzcd}$$

(same direction)

B. Electrodynamical Part

§6 Transformation of Maxwell-Hertz Equations for Empty Space

CORE SECTION!

Establishes

- New kinematics ⟶ Principle of relativity holds in electrodynamics.

  Maxwell's equations (source free) preserve form under Lorentz transformation of space & time, of $\underbrace{\text{fields}}_{\text{derived here.}}$

- Lorentz transformation for field ⟹ New way to conceive electric & magnetic forces.

  Lorentz transformation for field: Relative existence of electric & magnetic field

  New way to conceive electric & magnetic forces: In rest frame of charge, all forces are electric.

Derivation of Lorentz Transformation for Field

See "Electrodynamics 001" for translation of modern statement of Maxwell's equations into Einstein's component form.

NB AE does not address $\underline{\nabla}\cdot\underline{E}=0$, $\underline{\nabla}\cdot\underline{H}=0$
Taken as definition of electric/magnetic source free case?
... but definition must also be shown to be Lorentz covariant

Preparation: Transformations of space, time derivative operators

$$\tau=\beta(t-\tfrac{v}{c^2}x) \quad \xi=\beta(x-vt) \quad \eta=y \quad \zeta=z$$

$$\left[\frac{\partial}{\partial t}=\underbrace{\frac{\partial\tau}{\partial t}}_{\beta}\cdot\frac{\partial}{\partial \tau}+\underbrace{\frac{\partial\xi}{\partial t}}_{-\beta v}\cdot\frac{\partial}{\partial\xi}+\underbrace{\frac{\partial\eta}{\partial t}}_{0}\cdot\frac{\partial}{\partial\eta}+\underbrace{\frac{\partial\zeta}{\partial t}}_{0}\cdot\frac{\partial}{\partial\zeta}=\beta\frac{\partial}{\partial\tau}-\beta v\frac{\partial}{\partial\xi}\right]$$

$$\left[\frac{\partial}{\partial x}=\underbrace{\frac{\partial\xi}{\partial x}}_{\beta}\cdot\frac{\partial}{\partial\xi}+\underbrace{\frac{\partial\tau}{\partial x}}_{-\beta v/c^2}\cdot\frac{\partial}{\partial\tau}+\underbrace{\frac{\partial\eta}{\partial x}}_{0}\cdot\frac{\partial}{\partial\eta}+\underbrace{\frac{\partial\zeta}{\partial x}}_{0}\cdot\frac{\partial}{\partial\zeta}=\beta\frac{\partial}{\partial\xi}-\frac{\beta v}{c^2}\frac{\partial}{\partial\tau}\right]$$

$$\left[\frac{\partial}{\partial y}=\frac{\partial}{\partial\eta}\right] \qquad \left[\frac{\partial}{\partial z}=\frac{\partial}{\partial\zeta}\right]$$

Inverses $\frac{\partial}{\partial\tau}=\beta\frac{\partial}{\partial t}+\beta v\frac{\partial}{\partial x}$ $\quad \frac{\partial}{\partial\xi}=\beta\frac{\partial}{\partial x}+\frac{\beta v}{c^2}\frac{\partial}{\partial\tau}$

$$\frac{1}{c}\frac{\partial \underline{E}}{\partial t} = \underline{\nabla}\times\underline{H} \qquad \underline{E}=(X,Y,Z) \quad \underline{H}=(L,M,N)$$

Maxwell's equations in $K(x,y,z,t)$ transform to $k(\xi,\eta,\zeta,\tau)$

x-component

$$\frac{1}{c}\frac{\partial X}{\partial t} = \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}$$

$\Downarrow$ Add $\frac{v}{c}\frac{\partial X}{\partial x}$ to both sides.

where $\frac{v}{c}\frac{\partial X}{\partial x} = -\frac{v}{c}\frac{\partial Y}{\partial y} - \frac{v}{c}\frac{\partial Z}{\partial z}$ since $\underline{\nabla}\cdot\underline{E}=0$, $\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}=0$

multiply by β

$$\frac{1}{c}\beta\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)X = \beta\,\frac{\partial(N-\frac{v}{c}Y)}{\partial y} - \beta\,\frac{\partial(M-\frac{v}{c}Z)}{\partial z}$$

$\Downarrow$ substitute for space, time operators

$$\frac{1}{c}\frac{\partial X}{\partial \tau} = \frac{\partial}{\partial \eta}\beta\left(N-\frac{v}{c}Y\right) - \frac{\partial}{\partial \zeta}\beta\left(M-\frac{v}{c}Z\right)$$

y-component

$$\frac{1}{c}\frac{\partial Y}{\partial t} = \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}$$

$\Downarrow$ Add $-\beta^2\frac{v}{c^2}\frac{\partial N}{\partial t} + \beta^2\frac{v}{c}\frac{\partial Y}{\partial x}$ to both sides.

Insert factors $1=\beta^2(1-v^2/c^2)$

$$\frac{1}{c}\overbrace{\beta^2\left(1-\frac{v^2}{c^2}\right)}^{1}\frac{\partial Y}{\partial t} - \beta^2\frac{v}{c^2}\frac{\partial N}{\partial t} + \beta^2\frac{v}{c}\frac{\partial Y}{\partial x} = \frac{\partial L}{\partial z} - \beta^2\frac{v}{c^2}\frac{\partial N}{\partial t} + \beta^2\frac{v}{c}\frac{\partial Y}{\partial x} - \overbrace{\beta^2\left(1-\frac{v^2}{c^2}\right)}^{1}\frac{\partial N}{\partial x}$$

$$\frac{1}{c}\beta^2\frac{\partial Y}{\partial t} - \beta^2\frac{v}{c^2}\frac{\partial N}{\partial t} + \beta^2\frac{v}{c}\frac{\partial Y}{\partial x} - \beta^2\frac{v^2}{c^2}\frac{\partial N}{\partial x} = \frac{\partial L}{\partial z} - \beta^2\frac{v}{c^2}\frac{\partial N}{\partial t} + \beta^2\frac{v^2}{c^3}\frac{\partial Y}{\partial t} - \beta^2\frac{\partial N}{\partial x} + \beta^2\frac{v}{c}\frac{\partial Y}{\partial x}$$

$$\frac{1}{c}\left(\beta\frac{\partial}{\partial t}+\beta v\frac{\partial}{\partial x}\right)\beta\left(Y-\frac{v}{c}N\right) = \frac{\partial L}{\partial z} - \beta\left(\frac{v}{c^2}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\beta\left(N-\frac{v}{c}Y\right)$$

$\Downarrow$ substitute for space, time operators

$$\frac{1}{c}\frac{\partial}{\partial \tau}\beta\left(Y-\frac{v}{c}N\right) = \frac{\partial L}{\partial \zeta} - \frac{\partial}{\partial \xi}\beta\left(N-\frac{v}{c}Y\right)$$

... etc. for remaining maxwell equations

see §a for more details

If maxwell's equations also hold in k,
$\equiv$ then fields must transform as

in k → ← in K

$$X' = \psi(v)X \qquad L' = \psi(v)L$$
$$Y' = \psi(v)\beta\left(Y - \tfrac{v}{c}N\right) \qquad M' = \psi(v)\beta\left(M + \tfrac{v}{c}Z\right)$$
$$Z' = \psi(v)\beta\left(Z + \tfrac{v}{c}M\right) \qquad N' = \psi(v)\beta\left(N - \tfrac{v}{c}Y\right)$$

Fix $\psi(v)$ = since

- Transformation forms a group.

$$X \xrightarrow{\text{add } v} X' = \psi(v)X \xrightarrow[v]{\text{subtract}} X'' = \psi(-v)X'$$

But $X'' = X$ $\therefore$ $\psi(-v)\psi(v)X = X$

$$\boxed{\psi(v)\psi(-v) = 1}$$

- By symmetry

$$\boxed{\psi(v) = \psi(-v)}$$

see footnote 4.
Brief new $\underline{E}$ induced from $\underline{H}$ by change frame of reference must flip direction if $v \to -v$.

$$\psi(v)\psi(-v) = (\psi(v))^2 = 1$$
$$\psi(v) = \pm 1$$

↑ choose plus to retain agreement in direction axes
i.e. rule out $\psi(0) = -1$

Standard worries	: Does the derivation really work?!

- If we are allowed to group terms any way we please, why can't we preserve the principle of relativity for ANY law?

    It work work for just any law.
    The resulting transformation formulae must form a group that is compatible with the space, time transformation.
    Generically this is not possible.

- The derivation generates a candidate for the transformation formulae. Is it unique?

    It is unique, but that is not shown.
    My hunch is that a proof would first note that the field transformation law must be linear in the fields.] Why? see over
    It should then be easy to prove that any linear terms added to the standard equations must vanish if the transformations are still to work

- Why are the transformed fields REAL on a par with the originals? Why take $\beta(Y - \frac{v}{c}N)$ seriously? Why aren't the fields of one frame (ether frame?) THE TRUE fields and all the rest fake?

That the transformed fields are equally real physically is a physical assumption you cannot be compelled to take. (Lorentz didn't)

But it is hard to escape. The calculations show that the composite quantities $\beta(Y - \frac{v}{c}N)$ etc. in k satisfy every property required by fields in Maxwell's theory.

If you want to insist that some extra property distinguishes the fields of K as real, then that extra property is not expressed in Maxwell's theory.

Why must transformations be linear in the fields?

Assume – addition of fields has coordinate independent meaning.
– multiplication of fields by scalar

In K: $(\underline{E}_1, \underline{H}_1)$, $(\underline{E}_2, \underline{H}_2)$ satisfy maxwell's equations $\Longrightarrow$ $(\alpha \underline{E}_1 + \beta \underline{E}_2, \alpha \underline{H}_1 + \beta H_2)$ satisfy maxwell's equations

True since maxwell's theory is linear

$$\Big\Downarrow T$$

In $\bar{K}$: $(T(E_1), T(H_1))$, $(T(E_2), T(H_2))$ $\Longrightarrow$ $\underbrace{(\alpha T(E_1) + \beta T(E_2), \alpha T(H_1) + \beta T(H_2))}$

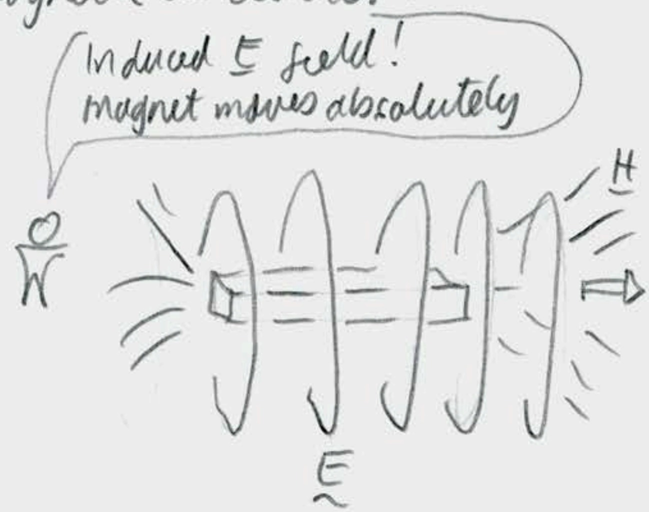
must be same as outcome of first adding & then transforming.

$$= T(\alpha E_1 + \beta E_2, \alpha H_1 + \beta H_2)$$

$\boxed{\text{... this is linearity of } T\text{!}}$

"... On the Nature of the Electromotive Forces Arising Due to motion in a magnetic Field

Resolution of problem of magnet & conductor:



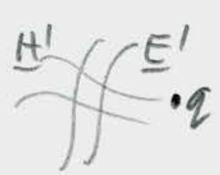
... No Induced $\underline{E}$ field due solely to relative motion of observer & magnet.
Observer on magnet sees no $\underline{E}$ field.

Old mode expression

$\underline{f} = \underline{E} + \frac{\underline{v}}{c} \times \underline{H}$ (first order)

New mode expression

(1) Transform to rest frame charge

(2)


$\underline{f}' = \underline{E}'$

Also: Unipolar Induction

§9 Transformation of the Maxwell-Hertz Equations when $\underbrace{\text{convection currents}}_{=\text{charge currents}}$ are taken into account

Reverse direction of AE' calculation: Transform from $k \to K$

operator identities

$t = \beta(\tau + \frac{v}{c^2}\xi)$

$x = \beta(\xi + v\tau)$

$y = \eta \quad z = \zeta$

$$\frac{\partial}{\partial\tau} = \frac{\partial t}{\partial\tau}\frac{\partial}{\partial t} + \frac{\partial x}{\partial\tau}\frac{\partial}{\partial x} = \beta\frac{\partial}{\partial t} + \beta v\frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial\xi} = \frac{\partial t}{\partial\xi}\cdot\frac{\partial}{\partial t} + \frac{\partial x}{\partial\xi}\frac{\partial}{\partial x} = \beta\frac{v}{c^2}\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial x}$$

Solve for three representative cases. Note manipulation is invertible ∴ invert to recover the calculation AE describes.

$$\boxed{\frac{1}{c}\left\{\frac{\partial X'}{\partial\tau} + u_\xi\rho'\right\} = \frac{\partial N'}{\partial\eta} - \frac{\partial M'}{\partial\zeta}}$$

$X' = X$

$u_\xi = \dfrac{u_x - v}{1 - u_x v/c^2}$

$\rho' = \beta(1 - u_x v/c^2)\rho$

$N' = \beta(N - \frac{v}{c}Y)$

$M' = \beta(M + \frac{v}{c}Z)$

$$\frac{1}{c}\beta\frac{\partial X}{\partial t} + \beta\frac{v}{c}\frac{\partial X}{\partial x} + \underbrace{\frac{1}{c}\left(\frac{u_x - v}{\cancel{1 - u_x v/c^2}}\right)\beta\left(\cancel{1 - \frac{u_x v}{c^2}}\right)\rho}_{\frac{1}{c}\beta u_x\rho - \beta\frac{v}{c}\rho} = \beta\frac{\partial N}{\partial y} - \beta\frac{v}{c}\frac{\partial Y}{\partial y} - \beta\frac{\partial M}{\partial z} + \beta\frac{v}{c}\frac{\partial Z}{\partial z}$$

Maxwell:

$$\rho = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z}$$

∴ These terms cancel

divide by β

divide by β

$$\boxed{\frac{1}{c}\left\{\frac{\partial X}{\partial t} + u_x\rho\right\} = \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}}$$

$$\boxed{\underbrace{\frac{1}{c}\left\{\frac{\partial Y'}{\partial \tau} + u_\eta \rho'\right\}}\ = \ \frac{\partial L'}{\partial \zeta} - \frac{\partial N'}{\partial \xi}}$$

$Y' = \beta(Y - \frac{v}{c}N)$

$\frac{\partial}{\partial \tau} = \beta\frac{\partial}{\partial t} + \beta v\frac{\partial}{\partial x}$

$u_\eta = u_y \big/ \beta(1 - u_x v/c^2)$

$L' = L$

$N' = \beta(N - \frac{v}{c}Y)$

$\frac{\partial}{\partial \zeta} = \frac{\partial}{\partial z}$

$\frac{\partial}{\partial \xi} = \beta\frac{v}{c^2}\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial x}$

↓

$$\underbrace{\frac{1}{c}\left(\beta\frac{\partial}{\partial t} + \beta v\frac{\partial}{\partial x}\right)\beta\left(Y - \frac{v}{c}N\right)} + \frac{1}{c}\frac{u_y}{\cancel{\beta(1-u_x v/c^2)}}\cdot\overbrace{\cancel{\beta(1-u_x v/c^2)}\rho}^{\rho'} = \frac{\partial L}{\partial z} - \beta\left(\frac{v}{c^2}\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right)\beta\left(N - \frac{v}{c}Y\right)$$

$$\frac{1}{c}\beta^2\frac{\partial Y}{\partial t} - \beta^2\frac{v}{c^2}\frac{\partial N}{\partial t} + \beta^2\frac{v}{c}\frac{\partial Y}{\partial x} - \beta^2\frac{v^2}{c^2}\frac{\partial N}{\partial x} + \frac{1}{c}\rho u_y = \frac{\partial L}{\partial z} - \beta^2\frac{v}{c^2}\frac{\partial N}{\partial t} + \beta^2\frac{v^2}{c^3}\frac{\partial Y}{\partial t} - \beta^2\frac{\partial N}{\partial x} + \beta^2\frac{v}{c}\frac{\partial Y}{\partial x}$$

cancel

cancel

$$\therefore\ \frac{1}{c}\underbrace{\beta^2\left(1 - \frac{v^2}{c^2}\right)}_{1}\frac{\partial Y}{\partial t} + \frac{1}{c}\rho u_y = \frac{\partial L}{\partial z} - \underbrace{\beta^2\left(1 - \frac{v^2}{c^2}\right)}_{1}\frac{\partial N}{\partial x}$$

$$\boxed{\therefore\ \frac{1}{c}\left\{\frac{\partial Y}{\partial t} + \rho u_y\right\} = \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}}$$

$$\boxed{\rho' = \frac{\partial X'}{\partial \xi} + \frac{\partial Y'}{\partial \eta} + \frac{\partial Z'}{\partial \zeta}}$$

$X' = X$ $\quad \frac{\partial}{\partial \xi} = \beta\frac{v}{c^2}\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial x}$ $\quad \rho' = \beta(1 - \frac{u_x v}{c^2})\rho$

$Y' = \beta(Y - \frac{v}{c}N)$ $\quad Z' = \beta(Z + \frac{v}{c}M)$

$$\beta\rho - \beta\frac{u_x v}{c^2}\rho = \beta\frac{v}{c^2}\frac{\partial X}{\partial t} + \beta\frac{\partial X}{\partial x} + \beta\frac{\partial Y}{\partial y} - \beta\frac{v}{c}\frac{\partial N}{\partial y} + \beta\frac{\partial Z}{\partial z} + \beta\frac{v}{c}\frac{\partial M}{\partial z}$$

$\frac{v}{c^2}\times$ one of maxwell's equations - cancel!

divide by β

↓

$$\boxed{\rho = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z}}$$

Puzzle:
why does AE treat case of sources of case of source free eqns in different sections?

§7 Theory of Doppler's Principle and Aberration

§8 Transformation of the Energy of Light Rays. Theory of Radiation Pressure Exerted on Perfect Mirrors

General method for solving problems in electrodynamics of moving bodies

Set up problem in easy to solve case of rest (e.g. light sources at rest)	Transform $\Longrightarrow$	case of moving bodies Read off result!

- Doppler's principle
- Aberration
- Energy of light complex in different frames
- Light reflected off moving mirror $\rightarrow$ radiation pressure

§10 Dynamics of (Slowly Accelerated) Electron

Why slowly? Eliminate energy loss to radiation

<u>Goal</u>: theory leaves maxwell electrodynamics untouched. But dynamics of ordinary bodies will be affected. mass becomes velocity dependent.
Express as "longitudinal", "transverse" mass

Charge ε momentarily at rest in k

$$\mu \frac{d^2\xi}{d\tau^2} = \varepsilon X' \quad \mu \frac{d^2\eta}{d\tau^2} = \varepsilon Y' \quad \mu \frac{d^2\zeta}{\partial \zeta^2} = \varepsilon Z'$$

$$\frac{d^2\xi}{d\tau^2} = \frac{dw_\xi}{d\tau} = \underbrace{\frac{dt}{d\tau}}_{\beta} \cdot \frac{d}{dt}\left(\frac{w_x - v}{1-\frac{vw_x}{c^2}}\right)$$

$$= \beta \underbrace{\frac{1}{1-\frac{vw_x}{c^2}}}_{\beta^2 \text{ since } w_x = v} \frac{dw_x}{dt} + \beta \underbrace{(w_x - v)}_{\substack{0 \\ \text{since } w_x = v}} \frac{d}{dt}\left(\frac{1}{1-\frac{w_x v}{c^2}}\right)$$

$$= \beta^3 \frac{d^2x}{d\tau^2}$$

$$\frac{d^2\eta}{d\tau^2} = \frac{dw_\eta}{d\tau} = \underbrace{\frac{dt}{d\tau}}_{\beta} \frac{d}{dt}\left(\frac{w_y}{\beta(1-\frac{vw_x}{c^2})}\right) = \underbrace{\frac{\beta}{\beta(1-\frac{vw_x}{c^2})}}_{\beta^2} \cdot \frac{dw_y}{dt} + \beta \overset{0}{w_y} \frac{d}{dt}\left(\frac{1}{\beta(\,)}\right)$$

$$= \beta^2 \frac{d^2y}{d\tau^2} \quad \text{and similarly for } \frac{d^2\zeta}{d\tau^2}$$

Charge ε moving at $\underline{v} = (v,0,0)$ in K

$$\underbrace{\mu \beta^3}_{\text{longitudinal mass}} \frac{d^2x}{dt^2} = \varepsilon X \quad \mu\beta^2 \frac{d^2y}{dt^2} = \varepsilon\beta\left(Y - \frac{v}{c}N\right)$$

$$\underbrace{\mu\beta^2}_{\text{transverse mass}} \frac{dz}{dt^2} = \varepsilon\beta\left(Z + \frac{v}{c}M\right)$$

Einstein uses

Force = mass × Acceleration $\Longrightarrow$ moving bodies have different masses (resistance to acceleration) in direction of motion & transverse to direction motion

Hence: Longitudinal, Transverse mass

$\Downarrow$

Later:
Better choice is

$$\text{Force} = \frac{d}{dt}(\text{mass} \times \text{velocity}) \Longrightarrow$$ Same mass in both directions

$$\text{mass} = \frac{\text{rest mass}}{\sqrt{1 - v^2/c^2}}$$

Final Results | Kinetic energy of mass at v = Work to (slowly) accelerate charge to v $= \mu c^2 \left\{ \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right\}$

Three properties of electron's motion amenable to experimental test.