Albert Einstein

"On the Inertia of Energy Required by the Relativity Principle"

Annalen der Physik

23 (1907), pp. 371-384

Notes by John D. Norton

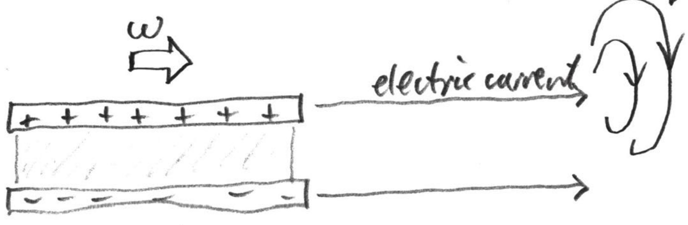
September 2015

Introductory puzzle NOT mentioned by Einstein

Trouton & Noble 1903

The other second order ether drift experiment

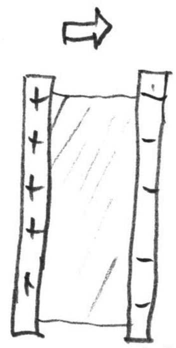
charged capacitor moves at w in ether



parallel

magnetic field created with energy

$\sim\left(\frac{w}{c}\right)^2$ Electro-static energy of capacitor



perpendicular

No current

∴ No magnetic field energy term.

Energy parallel > Energy perpendicular

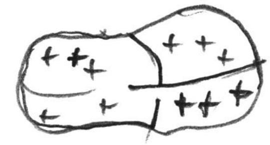
Expect capacitor to turn to this orientation

capacitor suspended by wire ⟹ No turning observed

How do we <u>now</u> explain this?

Main Results

1. Kinetic energy of stressed body



$$K = \underbrace{\left[\frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2\right]}_{\text{familiar term}} + \text{term due to stress in the body}$$

$$\Delta E = \frac{v^2}{c^2}\frac{1}{\sqrt{1-v^2/c^2}}\left(\text{Rest volume}\right)\left(\text{Normal stress in direction motion}\right)$$

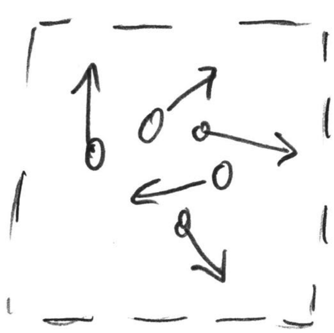
Energy NOT due to elastic deformation

2. Rigid bodies require signals to propogate at speeds > speed light. [(novel?) energy argument]

   Possibility of signals arriving before they leave

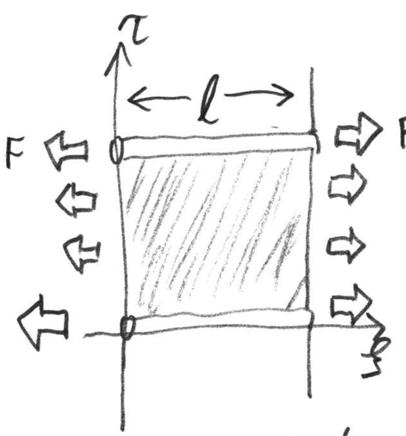
   "impossible"

3. Collection of moving masses : Kinetic energy of each mass contributes to Rest mass of the total system



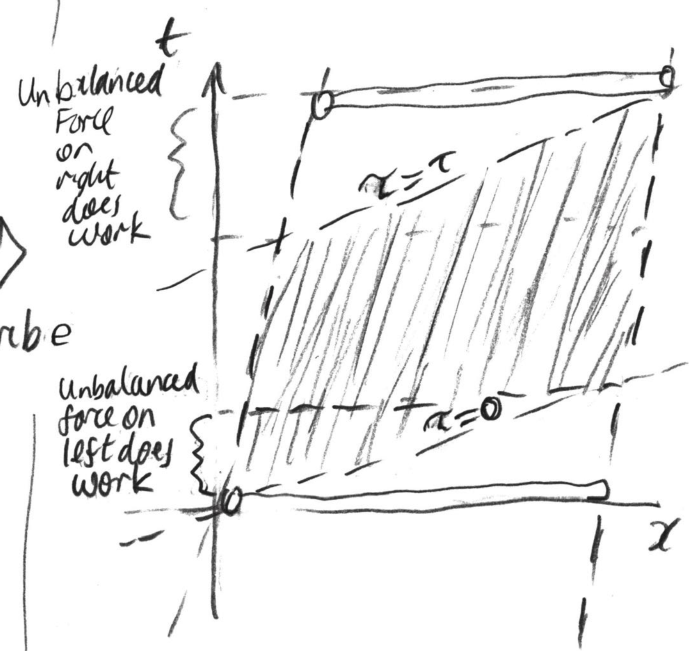
Simplified version of result 1

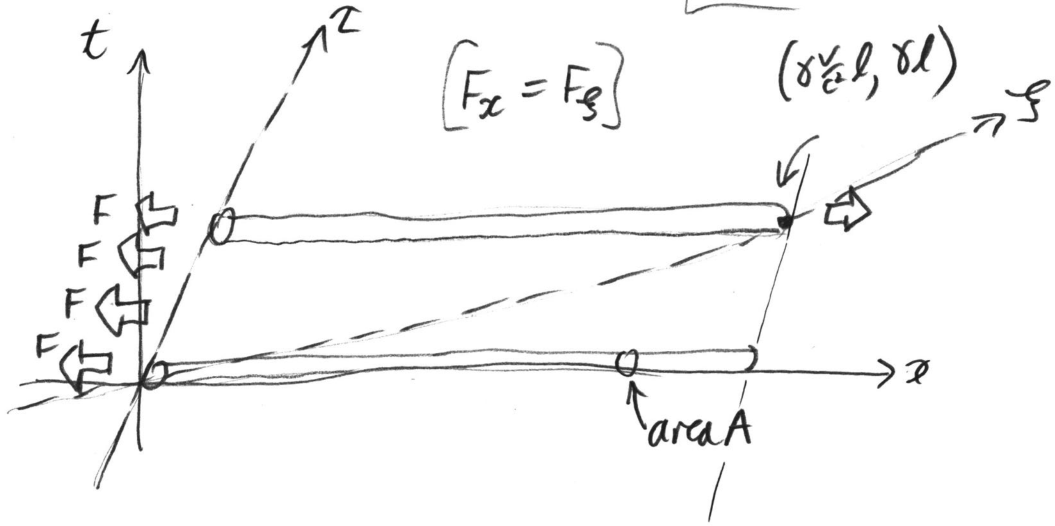
In $\mathcal{R}(\xi, \eta, \zeta, \tau)$, rod at rest



Rod in tension between times $\tau = 0$, $\tau = \tau$ ONLY

Redescribe





$\tau = 0$
$\xi = l$

$$t = \gamma(\tau + v/c^2\, \xi) = \gamma \frac{v}{c^2} l$$

$$x = \gamma(\xi + v\tau) = \gamma l$$

$$\gamma = 1/\sqrt{1 - v^2/c^2}$$

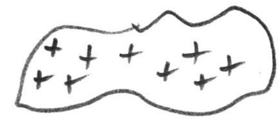
$$\begin{pmatrix}\text{work done} \\ \text{on rod} \\ \text{during} \\ \text{time} \\ \gamma v/c^2\, l \\ \text{of unbalanced} \\ \text{force}\end{pmatrix} = -F \cdot v \cdot \gamma \frac{v}{c^2} l$$

$$= -\frac{F}{A} \cdot Al \cdot \frac{v^2}{c^2} \cdot \gamma$$

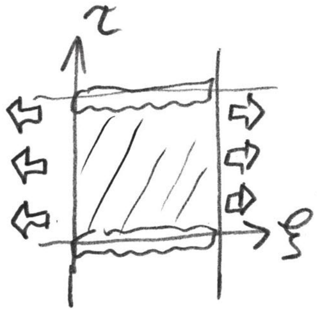
$$= \begin{pmatrix}\text{Normal} \\ \text{stress}\end{pmatrix}\begin{pmatrix}\text{Rest} \\ \text{volume}\end{pmatrix} \frac{v^2/c^2}{\sqrt{1 - v^2/c^2}} = \underset{\text{energy change in rod}}{\Delta E}$$

Einstein's more general case

$K(\xi, \eta, \zeta, \tau)$ Rest frame of electrically charged body



charge density ρ'



External field present only in $\tau = 0$ to $\tau = T$

Net force in x-direction

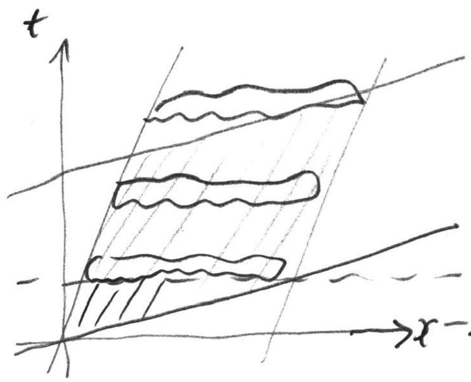
$$= \int X' \frac{\rho'}{4\pi} \, d\xi \, d\eta \, d\zeta$$

$$= 0$$

Equilibrium

X' = x-component of $\underset{\sim}{E}$

Transform to $K(x, y, z, t)$ where body moves at v in x-direction



same imbalance of forces here

compute energy during brief period of unbalanced force: (Long, messy integration)

$$\Delta E = -\frac{(v/c)^2}{\sqrt{1 - v^2/c^2}} \sum \xi K_\xi$$

ξ component of force on rest volume element $d\xi d\eta d\zeta$

$$= \frac{X_1' \rho'}{4\pi} \cdot d\xi d\eta d\zeta$$

proper position in body

sum over all volume elements

Long Term Significance

[Energy density represented as 0-0 component (STRESS-energy tensor)]

Lorentz transformation
mixes energy density & stresses

$$T = \left[\begin{array}{c|ccc} \text{Energy density} & \multicolumn{3}{c}{\text{Energy flux} = \text{momentum density}} \\ \hline \text{Energy flux} & \text{Normal stress} & \multicolumn{2}{c}{\text{Shear stress}} \\ \| & & \text{Normal stress} & \\ \text{momentum density} & \text{Shear stress} & & \text{Normal stress} \end{array}\right]$$

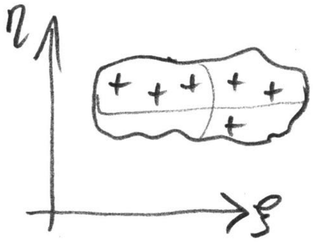
Rest frame fluid under pressure $\Longleftarrow$ boost in x-direction $\Longrightarrow$ ** Term Linear in rest energy density, pressure **

$$T = \left[\begin{array}{c|ccc} \text{rest energy density} & & 0 & \\ \hline & \text{Pressure} & & \\ 0 & & \text{Press} & \\ & & & \text{Press} \end{array}\right]$$

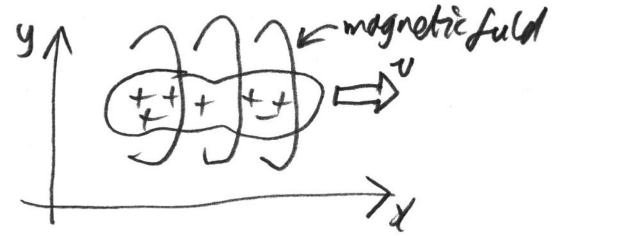
$$T = \left[\begin{array}{c|c} \text{Energy density} & \\ \hline & \end{array}\right]$$

Apparent Anistropy of e-m self energy of charged body

Body at rest in $k(\xi, \eta, \zeta, \tau)$



Body moves at v in $+x$ direction of $K(x, y, z, t)$



Electric field $(X', Y', Z') \neq 0$

magnetic field $(L', M', N') = 0$

Electrostatic system

From 1905 special relativity paper

$$X = X' \qquad L = L'$$
$$Y = \beta\left(Y' + \tfrac{v}{c}N'\right) \qquad M = \beta\left(M' - \tfrac{v}{c}Z'\right)$$
$$Z = \beta\left(Z' - \tfrac{v}{c}M'\right) \qquad N = \beta\left(N' + \tfrac{v}{c}Y'\right)$$

$$\beta = 1/\sqrt{1 - v^2/c^2}$$

Total electromagnetic self-energy in K $= E_e = \dfrac{1}{8\pi}\displaystyle\int X^2 + Y^2 + Z^2 + L^2 + M^2 + N^2 \, dx\,dy\,dz$

Substitute k frame values in formula

$$dx = \sqrt{1 - v^2/c^2}\, d\xi = \tfrac{1}{\beta} d\xi \qquad X = X' \qquad L = L'$$

$$\left.\begin{aligned} Y &= \beta\left(Y' + \tfrac{v}{c}N'\right) = \beta Y' \\ N &= \beta\left(N' + \tfrac{v}{c}Y'\right) = \beta \tfrac{v}{c} Y' \end{aligned}\right\} \quad Y^2 + N^2 = \beta^2\left(Y'^2 + \left(\tfrac{v}{c}Y'\right)^2\right) = \frac{1 + (v/c)^2}{1 + (v/c)^2} Y'^2$$

similar $\quad Z^2 + M^2 = \dfrac{1 + (v/c)^2}{1 - (v/c)^2} Z'^2$

$$\boxed{E_e = \frac{1}{8\pi}\int \frac{1}{\beta}\left[X'^2 + \frac{1 + (v/c)^2}{1 - (v/c)^2}\left(Y'^2 + Z'^2\right)\right] d\xi\, d\eta\, d\zeta}$$

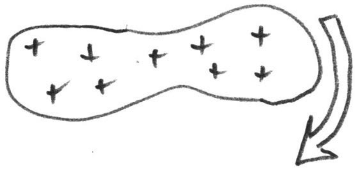
Self-energy of moving charged body varies with orientation w.r.t motion

JDN (Einstein does not say this) Effect sought in Trouton-Noble experiment

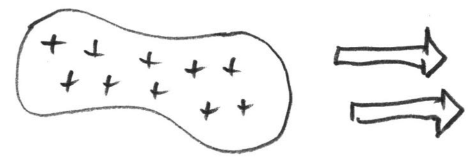
Einstein's Worry

contradiction with principle of relativity

Rest frame $\Longrightarrow$ Frame in which body moves uniformly



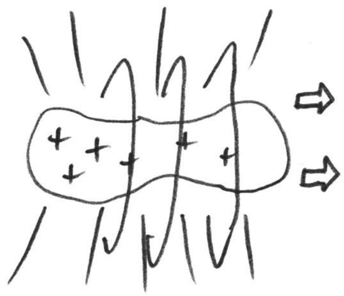
charged body can rotate "infinitely slowly"



charged body cannot rotate infinitely slowly but will settle in lowest energy state

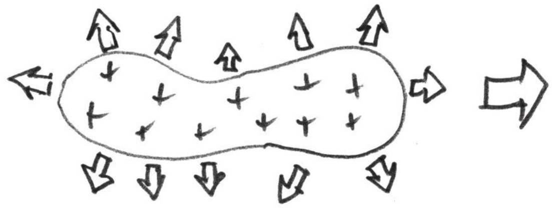
Einstein's Solution

Energy of moving body due to electric charge =



Energy of its electromagnetic field

+



Energy associated with stresses induced by charges

$$E_e = \underbrace{\frac{1}{8\pi}\int \frac{1}{\beta}\left[X'^2 + \frac{1+(v/c)^2}{1-(v/c)^2}\left(Y'^2+Z'^2\right)\right] d\xi\, d\eta\, d\zeta}_{\text{Anisotropic}}$$

$$\Delta E = \underbrace{-\frac{v^2}{c^2}\beta\frac{1}{4\pi}\int \xi X' \rho'\, d\xi\, d\eta\, d\zeta}_{\text{Anisotropic}}$$

$$\rho' = \frac{\partial X'}{\partial \xi}+\frac{\partial Y'}{\partial \eta}+\frac{\partial Z'}{\partial \zeta}$$

sum is Isotropic!

$$E_e + \Delta E = \underbrace{\left[\frac{1}{8\pi}\int X'^2+Y'^2+Z'^2\, d\xi\, d\eta\, d\zeta\right]}_{\text{Rest energy of electrostatic field } E_s} \frac{1}{\sqrt{1-v^2/c^2}}$$

$$= \frac{\left(\frac{E_s}{c^2}\right)c^2}{\sqrt{1-v^2/c^2}}$$

$\boxed{\text{Electric charge add mass } E_s/c^2 \text{ that dilates with speed isotropically like ordinary mass}}$

Evaluating $E_e + \Delta E$

$$\Delta E = -\frac{v^2}{c^2}\,\beta\,\frac{1}{4\pi}\underbrace{\int_{\text{all space}} \xi X'\left(\frac{\partial X'}{\partial \xi}+\frac{\partial Y'}{\partial \eta}+\frac{\partial Z'}{\partial \zeta}\right)d\xi\, d\eta\, d\zeta}_{\text{integrate by parts}}$$

integrate by parts

$$\int_{\text{all space}} \underbrace{\left(\frac{\partial}{\partial \xi}\xi X'^2+\frac{\partial}{\partial \eta}\xi X'Y'+\frac{\partial}{\partial \zeta}\xi X'Z'\right)} - X'^2 - X'\xi\frac{\partial X'}{\partial \xi} - Y'\xi\frac{\partial X'}{\partial \eta} - Z'\xi\frac{\partial Z'}{\partial \xi}\; d\xi\, d\eta\, d\zeta$$

By Gauss' theorem equal to an integral over an infinitely distant surface of vector

$(\xi X'^2, \xi X'Y', \xi X'Z')$

$\sim 1/\text{radius}^4$ or $1/\text{radius}^3$

$\therefore$ vanishes

$$-Y'\xi\frac{\partial^2\varphi}{\partial\eta\,\partial\zeta} = -Y'\xi\frac{\partial^2\varphi}{\partial\zeta\,\partial\eta} = Y'\xi\frac{\partial Y'}{\partial\xi}$$

$$Z'\xi\frac{\partial Z'}{\partial\xi}$$

$$(X', Y', Z') = \left(-\frac{\partial\varphi}{\partial\xi}, -\frac{\partial\varphi}{\partial\eta}, -\frac{\partial\varphi}{\partial\zeta}\right)$$

$$= \int_{\text{all space}} -X'^2 \underbrace{- \xi X'\frac{\partial X'}{\partial \xi} - \xi Y'\frac{\partial Y'}{\partial \xi} - \xi Z'\frac{\partial Z'}{\partial \xi}}_{-\frac{1}{2}\xi\frac{\partial}{\partial\xi}\left(X'^2+Y'^2+Z'^2\right)}\; d\xi\, d\eta\, d\zeta$$

integrate by parts again

OVER

!

$$= \int_{\substack{\text{all}\\\text{space}}} -X'^2 - \frac{1}{2}\frac{\partial}{\partial\xi}\left(\xi\left(X'^2+Y'^2+Z'^2\right)\right) + \frac{1}{2}\left(X'^2+Y'^2+Z'^2\right)\cancelto{1}{\frac{\partial\xi}{\partial\xi}} \; d\xi\, d\eta\, d\zeta$$

$$\int_{\substack{\text{all}\\\text{space}}} \frac{\partial}{\partial\xi}\left(\xi\left(X'^2(\xi,\eta,\zeta) + \dots\right)\right) d\xi\, d\eta\, d\zeta$$

$$= \int_{\substack{\text{all}\\\eta,\zeta}} \xi\left(X'^2(+\infty,\eta,\zeta) + \dots\right) - \xi\left(X'^2(-\infty,\eta,\zeta)\right)\, d\eta\, d\zeta$$

$$= \int_{\substack{\text{all}\\\eta,\zeta}} 0 \; d\eta\, d\zeta = 0$$

$$= -\frac{1}{2}\int_{\substack{\text{all}\\\text{space}}} X'^2 - Y'^2 - Z'^2 \; d\xi\, d\eta\, d\zeta$$

combining:

$$\Delta E = \frac{v^2}{c^2}\beta\,\frac{1}{8\pi}\int_{\substack{\text{all}\\\text{space}}} X'^2 - Y'^2 - Z'^2 \; d\xi\, d\eta\, d\zeta$$

$$E_e + \Delta E = \overbrace{\frac{1}{8\pi}\cdot\frac{1}{\beta}\int X'^2 + \left[\frac{1+(v/c)^2}{1-(v/c)^2}\right](Y'^2+Z'^2)\,d\xi\,d\eta\,d\zeta}^{E_e}$$

$$+ \underbrace{\frac{v^2}{c^2}\,\frac{1}{8\pi}\,\beta\int X'^2 - Y'^2 - Z'^2\,d\xi\,d\eta\,d\zeta}_{\Delta E}$$

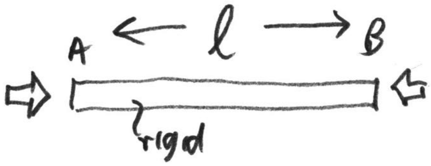
$$\frac{1}{8\pi}\,\beta\int \underbrace{\underset{\substack{\uparrow\\ \left(1-\frac{v^2}{c^2}\right)}}{\frac{1}{\beta^2}}\left(X'^2 + \left[\frac{1+(v/c)^2}{1-(v/c)^2}\right]\left[Y'^2+Z'^2\right]\right)}\,d\xi\,d\eta\,d\zeta$$

$$\left(1-\frac{v^2}{c^2}\right)X'^2 + \left(1+\frac{v^2}{c^2}\right)\left[Y'^2+Z'^2\right] = \left(X'^2+Y'^2+Z'^2\right) - \frac{v^2}{c^2}\left(X'^2-Y'^2-Z'^2\right)$$

$$\therefore\ E_e + \Delta E = \beta\,\underbrace{\frac{1}{8\pi}\int X'^2+Y'^2+Z'^2\,d\xi\,d\eta\,d\zeta}_{E_s}$$

Impossibility of rigid rods / signals at $v > c$

In rest frame of rod

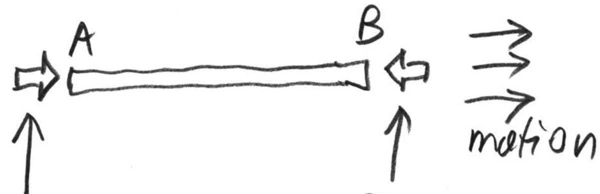


Forces act for very short time at ends of rod

⇓

No change in rest state rod

In frame in which rod moves at v



Force acts EARLIER by time $\frac{v}{c^2}\beta l$

Force acts later

Rod's motion unchanged

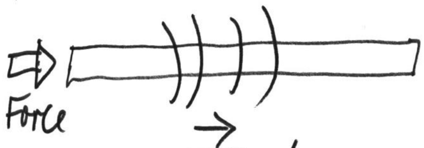
Force at A does work ⇒ Supplies energy to rod BUT Rod's state does not change in any way.

$\therefore$ Energy of rod $\neq$ function $\left(\begin{matrix}\text{state of}\\ \text{rod}\end{matrix}\right)$

Hence rigidity untenable

(Novel?) Dynamical argument

Kinematic causal argument



Effect propagates at W

view in frame that moves at $+v$

$$W' = \frac{W - v}{1 - \frac{Wv}{c^2}}$$

signal moves at W' covers distance l in time T

$$W' = \frac{l}{T} = \frac{W - v}{1 - \frac{Wv}{c^2}}$$

$$\therefore T = \frac{l\left(1 - \frac{Wv}{c^2}\right)}{W - v}$$

If $W > C$

$$= \frac{l}{W - v} \cdot \left(1 - \frac{W}{c} \cdot \frac{v}{c}\right)$$

greater than 1

$v < c$ can be brought close enough to c so that $\left(1 - \frac{W}{c} \cdot \frac{v}{c}\right) < 0$

$$\boxed{T < 0}$$

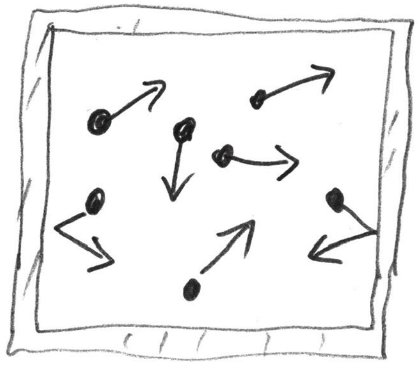
"effect ... precedes cause (accompanied by act of will, for example"

"... does not contain a contradiction from a purely logical point of view ...

conflicts so absolutely with the character of all experience, that the impossibility of the assumption ... proved ..."

Energy of a system consisting of mass points moving force-free

The obvious question.



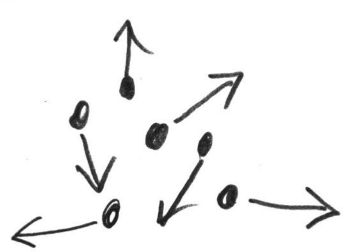
Each molecule of an ideal gas with rest mass μ has total energy $\mu c^2/\sqrt{1-v^2/c^2}$

Does this total energy contribute to the rest mass of the gas system?

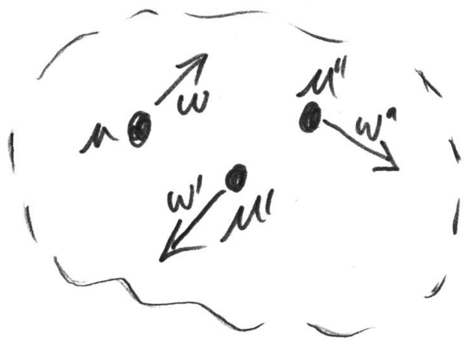
... Not quite the question Einstein answers.

Gas pressure $\Longrightarrow$ stress in chamber wall $\Longrightarrow$ Energy term ΔE if gas + chamber moves as a whole.

Einstein just considers system of moving masses WITHOUT containment



$k(\xi,\eta,\zeta)$ is the rest frame in the sense that total momentum $=0$



$$\sum \frac{\mu w_\xi}{\sqrt{1-(w/c)^2}} = 0$$

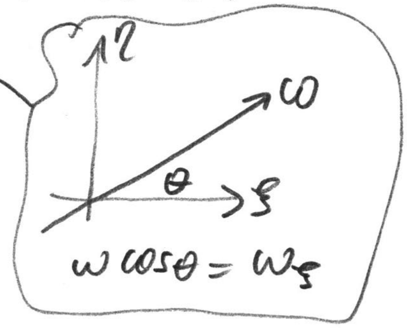
$$\sum \frac{\mu w_\eta}{\sqrt{1-(w/c)^2}} = 0$$

$$\sum \frac{\mu w_\zeta}{\sqrt{1-(w/c)^2}} = 0$$

In $K(x,y,z)$ ... k moves at v in $+x$ direction:

Energy ε of each mass ("easily determined")

$$\varepsilon = \mu c^2 \frac{1+\frac{vw\cos\theta}{c^2}}{\sqrt{1-v^2/c^2}\sqrt{1-w^2/c^2}}$$



$\therefore$ Total energy $E = \sum_{\substack{\text{all}\\\text{masses}}} \varepsilon = \frac{1}{\sqrt{1-v^2/c^2}}\left[\sum \mu c^2 \cdot \frac{1}{\sqrt{1-w^2/c^2}}\right]$

$$+\frac{v}{\sqrt{1-v^2/c^2}}\underbrace{\left[\sum \frac{\mu w\cos\theta}{\sqrt{1-w^2/c^2}}\right]}_{0 \text{ by momentum condition}}$$

$\left(\begin{array}{l}\text{Total energy}/c^2\\ \text{in (momentum}=0\text{)}\\ \text{frame}\end{array}\right) = \left(\begin{array}{l}\text{rest}\\ \text{mass of}\\ \text{total system}\end{array}\right)$ ***

$$\boxed{E = \frac{\left[\dfrac{\sum \frac{\mu c^2}{\sqrt{1-w^2/c^2}}}{c^2}\right] c^2}{\sqrt{1-v^2/c^2}}}$$