

EINSTEIN'S
"FUSION OF THE
WAVE AND EMISSION
THEORIES"

HPS 2814
22 March 2023

- Einstein's Road to 1909
- Quantum Theory's Road to 1909
- Einstein's "fusion"



Einstein's Road to 1909

A year after the light-quantum paper, Einstein shows that Planck's law works only if the energy of "resonators" are actually quantized and not merely discretized for counting purposes.

12. *Zur Theorie
der Lichterzeugung und Lichtabsorption;
von A. Einstein.*

In einer letztes Jahr erschienenen Arbeit¹⁾ habe ich gezeigt, daß die Maxwellsche Theorie der Elektrizität in Verbindung mit der Elektronentheorie zu Ergebnissen führt, die mit den Erfahrungen über die Strahlung des schwarzen Körpers im Widerspruch sind. Auf einem dort dargelegten Wege wurde ich zu der Ansicht geführt, daß Licht von der Frequenz ν lediglich in Quanten von der Energie $(R/N)\beta\nu$ absorbiert und emittiert werden könne, wobei R die absolute Konstante der auf das Grammolekül angewendeten Gasgleichung, N die Anzahl der wirklichen Moleküle in einem Grammolekül, β den Exponentialkoeffizienten der Wienschen (bez. der Planckschen) Strahlungsformel und ν die Frequenz des betreffenden Lichtes bedeutet. Diese Beziehung wurde entwickelt für einen Bereich, der dem Bereich der Gültigkeit der Wienschen Strahlungsformel entspricht.

Damals schien es mir, als ob die Plancksche Theorie (der Strahlung²⁾) in gewisser Beziehung ein Gegenstück bildete zu meiner Arbeit. Neue Überlegungen, welche im § 1 dieser Arbeit mitgeteilt sind, zeigten mir aber, daß die theoretische Grundlage, auf welcher die Strahlungstheorie von Hrn. Planck ruht, sich von der Grundlage, die sich aus der Maxwellschen Theorie und Elektronentheorie ergeben würde, unterscheidet, und zwar gerade dadurch, daß die Plancksche Theorie implizite von der eben erwähnten Lichtquantenhypothese Gebrauch macht.

In § 2 der vorliegenden Arbeit wird mit Hilfe der Lichtquantenhypothese eine Beziehung zwischen Voltaeffekt und lichtelektrischer Zerstreuung hergeleitet.

1) A. Einstein, Ann. d. Phys. 17. p. 132. 1905.

2) M. Planck, Ann. d. Phys. 4. p. 561. 1901.

Einstein's Road to 1909

In 1907 Einstein uses his quantum account to answer questions in the specific heat of solids at low temperatures – a problem that was unresolved by Planck.

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9. Die Plancksche Theorie der Strahlung und die Theorie der spezifischen Wärme; von A. Einstein.

In zwei früheren Arbeiten¹⁾ habe ich gezeigt, daß die Interpretation des Energieverteilungsgesetzes der schwarzen Strahlung im Sinne der Boltzmannschen Theorie des zweiten Hauptsatzes uns zu einer neuen Auffassung der Phänomene der Lichtemission und Lichtabsorption führt, die zwar noch keineswegs den Charakter einer vollständigen Theorie besitzt, die aber insofern bemerkenswert ist, als sie das Verständnis einer Reihe von Gesetzmäßigkeiten erleichtert. In der vorliegenden Arbeit soll nun dargetan werden, daß die Theorie der Strahlung — und zwar speziell die Plancksche Theorie — zu einer Modifikation der molekular-kinetischen Theorie der Wärme führt, durch welche einige Schwierigkeiten beseitigt werden, die bisher der Durchführung jener Theorie im Wege standen. Auch wird sich ein gewisser Zusammenhang zwischen dem thermischen und optischen Verhalten fester Körper ergeben.

Wir wollen zuerst eine Herleitung der mittleren Energie des Planckschen Resonators geben, die dessen Beziehung zur Molekularmechanik klar erkennen läßt.

Wir benutzen hierzu einige Resultate der allgemeinen molekularen Theorie der Wärme.¹⁾ Es sei der Zustand eines Systems im Sinne der molekularen Theorie vollkommen bestimmt durch die (sehr vielen) Variablen P_1, P_2, \dots, P_n . Der Verlauf der molekularen Prozesse geschehe nach den Gleichungen

$$\frac{dP_v}{dt} = \Phi_v(P_1, P_2, \dots, P_n), \quad (v = 1, 2, \dots, n)$$

und es gelte für alle Werte der P , die Beziehung

$$(1) \quad \sum \frac{\partial \Phi_v}{\partial P_v} = 0.$$

¹⁾ A. Einstein, Ann. d. Phys. 17. p. 182, 1905 u. 20. p. 199, 1905.

Einstein's Road to 1909

Laue's letter to Einstein, 1906: agreed that "radiant energy can only be emitted and absorbed in certain finite quanta" as long as it is understood that the quantization is "not a characteristic of electromagnetic processes in a vacuum, but of the absorbing or emitting material" and that "radiation does not consist of light quanta ... but only behaves during energy exchange with matter as though it did."



Einstein's Road to 1909

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*Won Nobel in 1914 for X-Ray diffraction,
four years before Planck*

Einstein's Road to 1909

A change in Einstein's thinking: Planck's theory complements rather than contradicts his own theory.

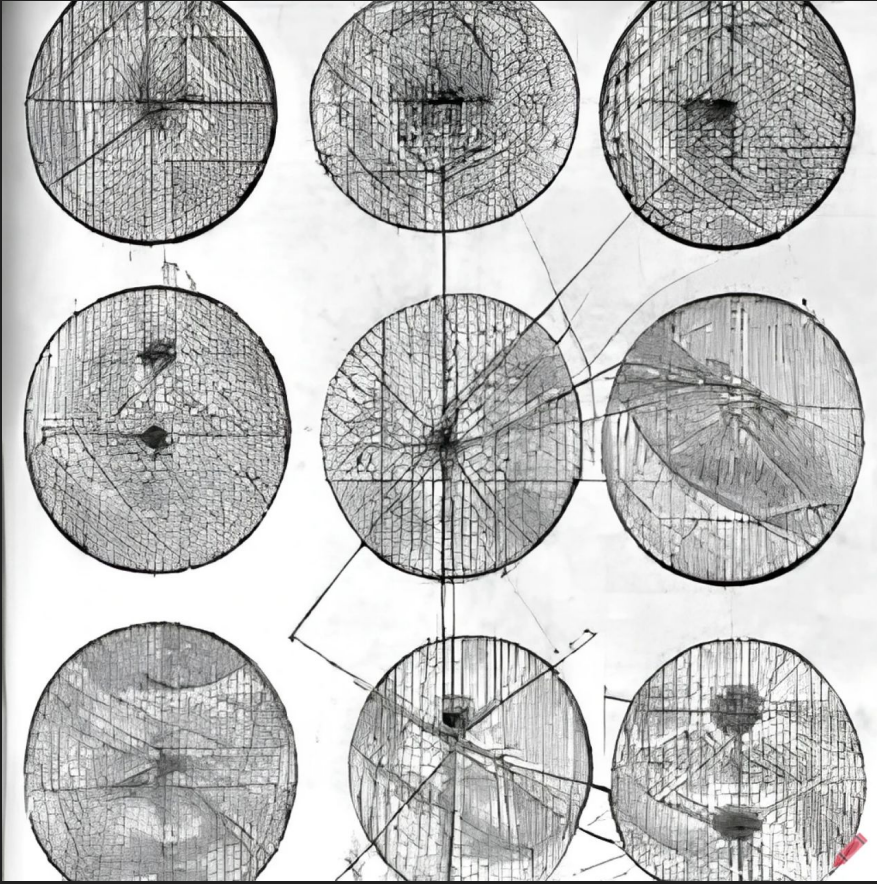
However: "In my opinion the above considerations do not at all disprove Planck's theory of radiation; rather, they seem to me to show that with his theory of radiation Mr. Planck introduced into physics a new hypothetical element: the hypothesis of light quanta."

But Planck in 1907: "I am not looking for the meaning of the elementary quantum of action (light quantum) in the vacuum, but at the places of absorption and emission, and assume that vacuum processes are exactly described by Maxwell's equations"

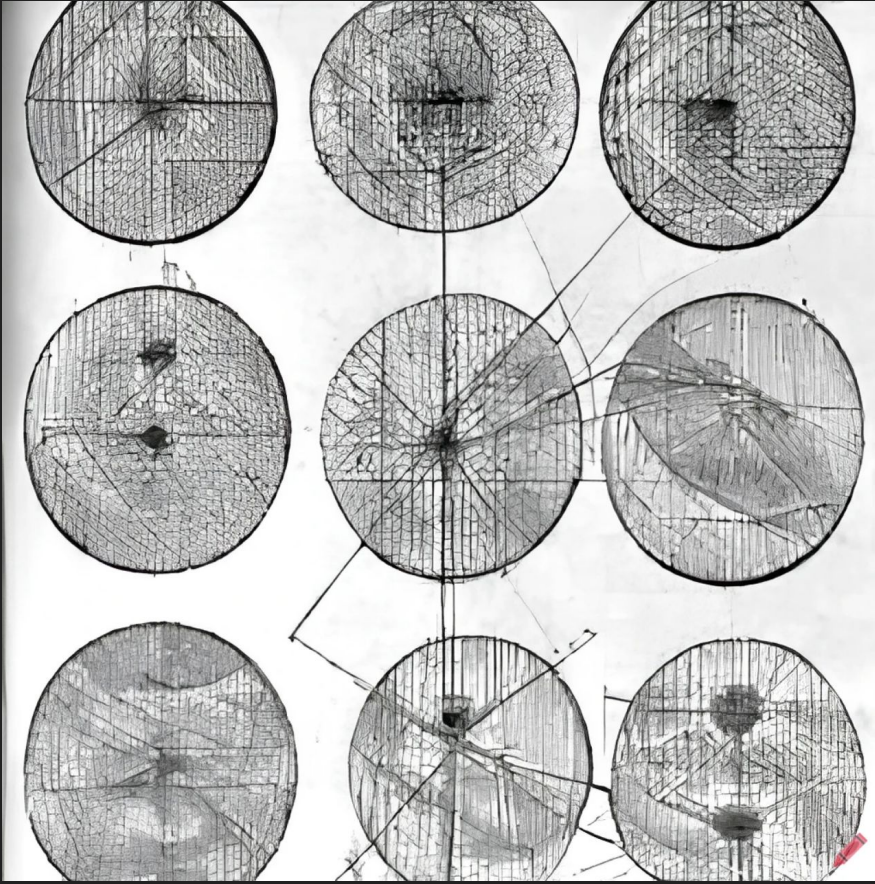
Einstein's Road to 1909

1907 failed his Habilitation. In 1908 passed with the thesis "Consequences for the constitution of radiation of the energy distribution law of black-body radiation". This does not survive.

Left Patent office in 1909 and appointed Associate Professor at Zurich (finalized May, appointed October).



**QUANTUM
THEORY'S ROAD
TO 1909**



QUANTUM THEORY'S ROAD TO 1909

Dall-e prompt "Old Quantum Theory in 1909"

Quantum Theory's Road to 1909

Ehrenfest: Discreteness of Radiation

1905 "Über die physikalischen Voraussetzungen der Planck'sehen Theorie der irreversiblen Strahlungsvorgänge"

1906 "Zur Planckschen Strahlungstheorie"



Quantum Theory's Road to 1909

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Portrait by Harm Kamerlingh Onnes, 1920

Quantum Theory's Road to 1909

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Über die physikalischen Voraussetzungen der Planck'schen Theorie der irreversiblen Strahlungsvorgänge

von

Paul Ehrenfest.

(Vorgelegt in der Sitzung am 9. November 1905.)

1.

H. A. Lorentz hat durch eine dimensionelle Betrachtung gezeigt, welche Vorsicht angewendet werden muß, wenn man die Sätze der Thermodynamik (z. E. den Kirchhoff'schen Satz von der Universalität der Hohlraumstrahlung) von Naturkörpern auf fingierte Systeme ausdehnt. Es war zu erwarten, daß die Anwendung dieser dimensionellen Betrachtung einen klaren Einblick in die physikalischen Voraussetzungen liefern würde, die der Planck'schen Theorie der Hohlraumstrahlung zu Grunde liegen. Speziell ergab sich da die Frage: Was sind die — voneinander unabhängigen — Hypothesen, die diese Theorie befähigen, zu jeder Temperatur eine Hohlraumstrahlung von eindeutig festgelegter Energieverteilung zu liefern?

2.

Wir schicken zunächst die wörtlichen Zitate einiger Stellen der Planck'schen Abhandlungen voraus, in denen Planck den Grundgedanken seiner Theorie darstellt. Hiebei sei es gestattet, für die Zitate folgende Abkürzungen zu gebrauchen (die zitierten Abhandlungen sind insgesamt in den Annalen der Physik erschienen):

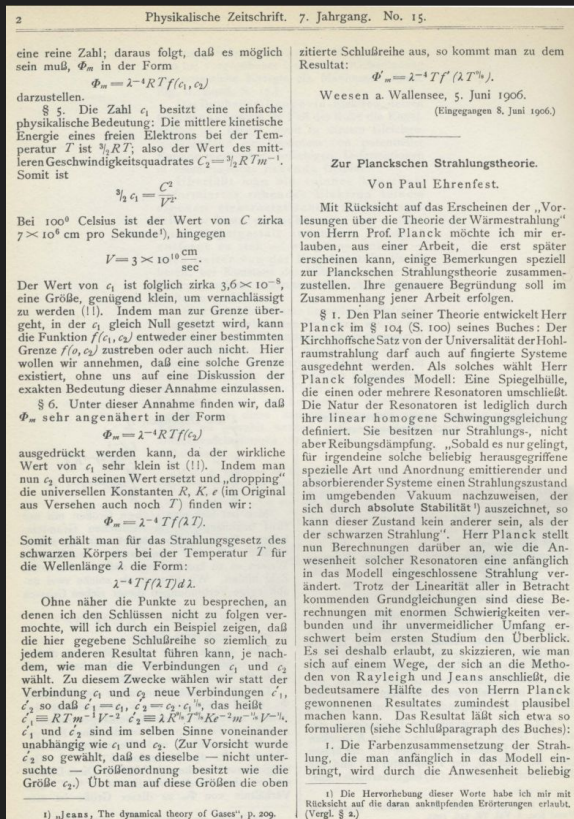
Quantum Theory's Road to 1909

Ehrenfest: Discreteness of Radiation:

1905 "Über die physikalischen Voraussetzungen der Planck'schen Theorie der irreversiblen Strahlungsvorgänge"

1906 "Zur Planckschen Strahlungstheorie"

Neither cite or mention Einstein



Quantum Theory's Road to 1909

In a series of papers, Jeans concluded that classical theory inevitably leads to the Rayleigh-Jeans law.

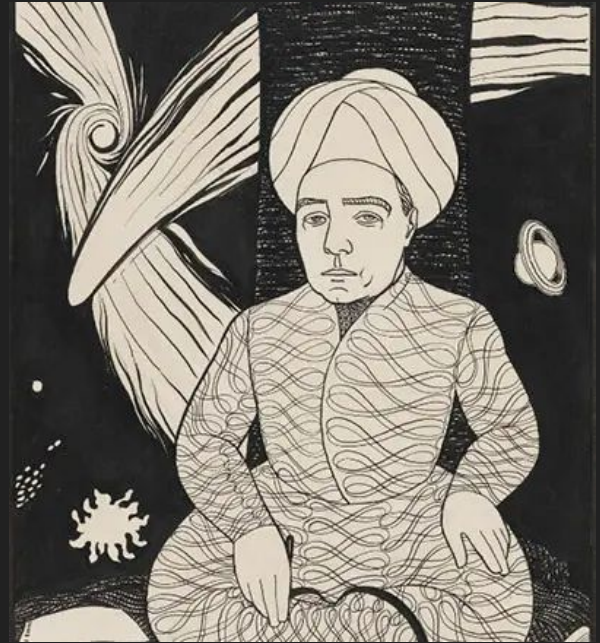
Implication: Planck's Law requires a departure.



Quantum Theory's Road to 1909

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Implication: Planck's Law requires a departure.



"Sir James Jeans and the Mysterious Universe," caricature by Powys Evans, 1932 (National Portrait Gallery, London)

Quantum Theory's Road to 1909

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names also are incorrectly printed, and the references given at the end of the chapter on mine illumination mostly refer to ventilation. On p. 86 the student is taught to load a hole "with nitroglycerine by pouring from a tin cup upon the fuse with its cap and covering the mass with water." Evidently the Coal Mines Regulation Act has no analogue in a country where, as the authors point out, "each new camp, untrammelled by tradition to keep it in the rut of prejudice, displays its genius for organization and absorbs the latest devices, tried and true."

The Practical Photographer. (Library Series.) Edited by Rev. F. C. Lambert, No. 16, *Pictorial Composition*, Pp. xx+84. No. 17, *Animal Photography*, Pp. xxiv+84. (London: Hodder and Stoughton, 1905.) Price 1s. net.

In the first of these books the editor gives an interesting account of the pictorial work of Bernard Allen, illustrating it with six excellent reproductions of his well-known worker's studies. Among the other sections of the book, which are written by various authors, those on the principles of composition, by Arthur Barchet, and some notes on composition in landscape, by Horace Mumsey, will be found of great practical value. In these the pen and ink sketches showing the several methods of producing balance in pictures call for special attention. Other articles, such as that on the arrangement of the foreground, are well worth perusing. Numerous well reproduced illustrations, serving as examples of good and bad composition, accompany the text. The second of the above books appeals to another class of photographers, for, with the exception of the editor's article on the pictorial work of Viscount Maitland, it is devoted to the photography of animals. Like the former book, numerous authors have contributed to the text and a very wide range of points of view is included. It is written on the same practical lines, and is accompanied by fifty-five well selected illustrations. Both volumes will add to the value of this useful library series.

Determination des Espèces minérales. By L. M. Grandjean. Pp. 184. (Paris: Gauthier-Villars, n.d.) Price 2 fr. francs.

In this little book, which is a publication of the "Encyclopédie scientifique des Aide-Mémoire," the author has apparently attempted to devise a royal road for the determination of a mineral species. For this purpose he has compiled a number of lists of the more common minerals arranged according to physical characters, viz. crystal-system, colour, streak, density, &c., and has supplemented these with some instructions on microscopic analysis and chemical examination in the dry way. Such lists are certainly of great value for determination purposes, but, as regards the more common minerals, at any rate, it would be a mistake to encourage the student to rely upon any methodical scheme of determination to the neglect of an acquisition of a thorough knowledge of the characters of the individual species. For many minerals, especially with imperfectly crystallized specimens, we fear these tables would prove an uncertain guide in the absence of any observations of the optical characters or of chemical examination in the wet way. In Brush and Penfield's standard work on determinative mineralogy it is true that no account is taken of the optical characters, but sufficient importance is given to chemical tests in the wet way. The tables are not altogether free from errors and misprints; thus a selenite rate is attributed to sodalite, rhodocite is described as a carbonate, and the density of wollastonite is given as 5.5 on one page and 7.5 on another. The book concludes with a list of 600 minerals with their principal characters, viz. density, hardness, &c.

NO. 1855, VOL. 72]

LETTERS TO THE EDITOR.

[The Editor does not hold himself responsible for opinions expressed by his correspondents. Writers can be understood to return, or to correspond with the writers of rejected memoranda intended for this or any other part of NATURE. No notice is taken of anonymous communications.]

The Dynamical Theory of Gases and of Radiation.
I am glad to have elicited the very clear statement of his view which Mr. Jeans gives in NATURE of April 27. In general outline, it corresponds pretty closely with that expressed by O. Reynolds in a British Association discussion at Aberdeen (NATURE, vol. cxvii, p. 234, 1883). The various modes of molecular motion are divided into two sharply separated groups. Within one group, including the translatory modes, equipartition of energy is supposed to establish itself within a small fraction of a second; but between the modes of this group and those of vibration included in the other group, equipartition may require, Mr. Jeans thinks, millions of years. Even if minutes were substituted for years, we must admit, I think, that the law of equipartition is reconciled with all that is absolutely proved by our experiments upon specific heat, which are, indeed, somewhat rough in all cases, and especially imperfect in so far as they relate to what may happen over long intervals of time.

As I have already suggested, it is when we extend the application of the law of equipartition to the modes of athermal vibration that the difficulties thicken, and this extension we are bound to make. The first question is as to the consequences of the law, considered to be applicable after which, if necessary, we may inquire whether any of these consequences can be evaded by supposing the equipartition to require a long time for its complete establishment. As regards the first question, two things are at once evident. The energy in any particular mode must be proportional to θ , the absolute temperature. And the number of modes corresponding to any finite space occupied by the radiation, is infinite. Although this is enough to show that the law of equipartition cannot apply in its entirety, it will be of interest to follow out its consequences a little further. Some of them were discussed in a former paper, the argument of which will now be repeated with an extension designed to determine the coefficient as well as the law of radiation.

As an introduction, we consider the motion of a stretched string of length l , vibrating transversely in one plane. If v be the velocity of propagation, f the number of subdivisions in any mode of vibration, the frequency f is given by

$$f = \frac{v}{2l} \quad (1)$$

A passage from any mode to the next in order involves a change of only $\frac{1}{2}v$ in the value of f , and hence, if ϵ denote the kinetic energy of a single mode, the law of equipartition requires that the kinetic energy corresponding to the interval $\frac{1}{2}v$ shall be

$$\frac{2\epsilon}{v} \Delta f \quad (2)$$

In terms of λ the wave-length, (2) becomes

$$\frac{2\epsilon}{\lambda} \Delta \lambda \quad (3)$$

This is for the whole length of the string. The longitudinal density of the kinetic energy is accordingly

$$\frac{2\epsilon}{\lambda^2} \Delta \lambda \quad (4)$$

In each mode the potential energy (on the average) equal to the kinetic, so that if we wish to reckon the whole energy, (4) must be doubled. Another doubling ensues when we abandon the restriction to one plane of vibration; and finally for the total energy corresponding to the interval from λ to $\lambda + \Delta \lambda$ we have

$$8\epsilon \Delta \lambda \quad (5)$$

When we proceed to three dimensions, and consider the vibrations within a cube of side l , subdivisions may occur in three directions. In place of (5)

$$f = \frac{v}{2l} \sqrt{p^2 + q^2 + r^2} \quad (6)$$

where q , r , &c. may assume any integral values. The next step is to ascertain what is the number of modes which corresponds to an assigned variation of f .

If the integral values of q , r , &c. be regarded as the x , y , z co-ordinates upon the Law of Composite Radiation, *Phil. Mag.*, xix, p. 320, 1860, then

Quantum Theory's Road to 1909

In a series of papers, Jeans concluded that classical theory inevitably leads to the Rayleigh-Jeans law.

Implication: Planck's Law requires a departure.

On the Laws of Radiation.

By J. H. JEANS, M.A., Fellow of Trinity College, Cambridge, Professor of Applied Mathematics in the University of Princeton.

(Communicated by Professor J. Larmor, Sec. R.S. Received October 11,—
Read November 16, 1905.)

1. An attempt to obtain the law of partition of the radiation proceeding from a radiating body calls at the outset for a consideration of the partition of energy between the matter of which the radiating body is composed, and the ether by which it is surrounded. This question has been discussed by Lord Rayleigh* and by the present author.† Assuming that the ultimate state of equilibrium between the energies of matter and ether has been reached, the theorem of equipartition of energy enables us to determine the amount not only of the total energy of the ether, but also of the energy of each wave-length. It is found that at a temperature T , the energy per unit volume of radiation consisting of waves of wave-lengths between λ and $\lambda+d\lambda$ is

$$8\pi RT\lambda^{-4}d\lambda.$$

It is obvious that this law, according to which the energy tends to run entirely into waves of infinitesimal wave-length, cannot be the true law of partition of the radiant energy which actually occurs in nature. The law is obtained from the supposition that a state of statistical equilibrium has been arrived at between the energies of different wave-lengths and that of matter; the inference to be drawn from the failure of this law to represent natural radiation is that in natural radiation such a state of equilibrium does not obtain. An analogous situation presents itself in the theory of gases. According to the theorem of equipartition of energy, the energy of a gas will ultimately be almost entirely absorbed by the modes of internal vibration of its molecules, whereas it is known that in nature only a very small fraction of the energy is possessed by these internal vibrations. Thus we are led to suppose that there is not a state of equilibrium between the internal vibrations of the molecules and their energy of translation; we find that the transfer of energy from the translational to the vibrational degrees of freedom is so slow that the latter degrees never acquire their full share of energy, as given by the theorem

* "On the Dynamical Theory of Gases and Radiation," *Nature*, May 18, July 13, 1905.

† "On the Partition of Energy between Matter and Ether," *Phil. Mag.* [6], vol. 10, p. 97.

Quantum Theory's Road to 1909

In a lecture in Rome 1908 Lorentz shows that his electron theory leads to the Rayleigh-Jeans law. One can avoid this only by showing that the equipartition theorem does not apply to the infinite number of vibrational modes of the ether. However, Jeans showed that equipartition does apply to all modes.

Called for experimental tests to determine between Planck and Rayleigh-Jeans law. Faced criticism for this.

Responding to Lorentz's appeal to the experimentalists, Lummer and Pringsheim, observed that if the Rayleigh-Jeans law were correct, a metal plate at room temperature would glow in the dark.

LE PARTAGE DE L'ÉNERGIE ENTRE LA MATIÈRE PONDÉRABLE ET L'ÉTHÉR¹⁾.

H. A. LORENTZ.

Le problème sur lequel j'aurai l'honneur de vous présenter quelques réflexions, est celui de la distribution de l'énergie entre la matière et l'éther, en tant que cette distribution s'opère par l'émission et l'absorption de la chaleur rayonnante et de la lumière. Depuis Kirchhoff, les physiciens s'en sont souvent occupés, d'abord en se fondant sur les principes généraux de la thermodynamique, et plus tard en introduisant des idées empruntées à la théorie cinétique de la matière, à la théorie électromagnétique de la lumière, et à la théorie des électrons.

Pour fixer les idées, il conviendra de préciser la question. Figurons nous à cet effet qu'une enceinte ayant la forme d'un parallélépipède rectangulaire, dont les faces intérieures sont parfaitement réfléchissantes, contient un corps pondérable M, qui se trouve à une certaine distance des parois, et supposons que l'éther, le milieu universel qui transmet la lumière et les actions électromagnétiques, remplit l'espace entier à l'intérieur de cette enceinte, pénétrant même les particules dont le corps pondérable se compose.

Kirchhoff a montré que, dans ces circonstances, si le corps M est maintenu à une température déterminée T, il s'établira un état d'équilibre dans lequel l'éther est traversé dans toutes les directions par les rayons émis par la matière pondérable. Ces rayons, incessamment réfléchis par les parois, ne tarderont pas à rencontrer de nouveau le corps M. Ils finiront par en être absorbés, mais la perte que subirait ainsi l'énergie de l'éther se trouve compensée par l'émission de nouvelles ondes

1) Conférence faite au 4^e Congrès international des Mathématiciens.

Quantum Theory's Road to 1909

In a letter to Lorentz in reply to this objection in the Fall of 1908, Planck for the first time stated that the energy of a resonator can only take on the discrete values $h\nu$.

At the end of 1908, senior physicists Planck, Lorentz, and Wien came to accept what Einstein, Jeans, and Ehrenfest had been emphasizing: old physics leads to the Rayleigh-Jeans law and the Planck law ushers a new physics.





Einstein's "fusion"

- Section 1 responds to Ritz and section 2-3 to Jeans.
- (Rayleigh-)Jeans law "is not compatible with the facts".
- Section 4 discusses a "certain logical imperfection" in Planck's theory.

Doc. 56
ON THE PRESENT STATUS OF THE RADIATION PROBLEM
by A. Einstein
[*Physikalische Zeitschrift* 10 (1909): 185-193]

This journal has recently published expressions of opinion by Messrs. H. A. Lorentz¹, Jeans², and Ritz³ which offer good insight into the present status of this extremely important problem. In the belief that it would be of benefit if all those who have seriously thought about this matter would communicate their views, even if they have not been able to arrive at a final result, I would like to communicate the following. [4]

1. The simplest form in which we can express the laws of electrodynamics established so far is that presented by the Maxwell-Lorentz partial differential equations. In contrast to Mr. Ritz³, I regard the forms containing retarded functions as merely auxiliary mathematical forms. The reason I see myself compelled to take this view is first of all that those forms do not subsume the energy principle, while I believe that we should adhere to the strict validity of the energy principle until we shall have found important reasons for renouncing this guiding star. It is certainly true that Maxwell's equations for empty space, taken by themselves, do not say anything, that they only represent an intermediary construct; but, as is well known, exactly the same could be said about Newton's equations of motion, as well as about any theory that needs to be supplemented by other theories in order to yield a picture for a complex of phenomena. What distinguishes the Maxwell-Lorentz differential equations from the forms that contain retarded functions is the circumstance that they yield an expression for the energy and the momentum of the system under consideration for any instant of time, relative to any unaccelerated coordinate system. With a theory that operates with retarded forces it is not possible to describe the instantaneous state of a system at all without using earlier states of the system for this description. [5]

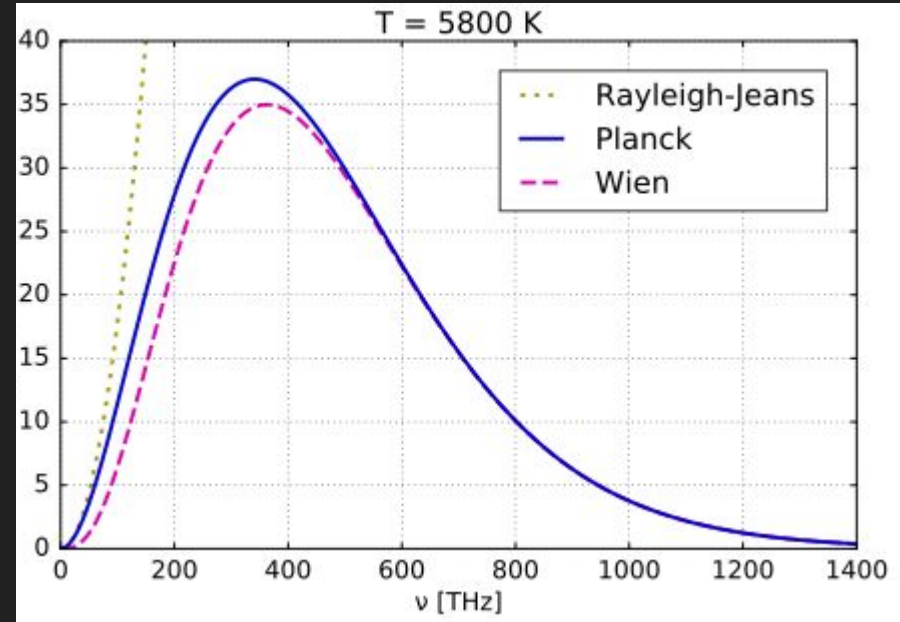
¹H. A. Lorentz, *Phys. Zeit.* 9 (1908): 562-563. [1]

²J. H. Jeans, *Phys. Zeit.* 9 (1908): 853-855. [2]

³W. Ritz, *Phys. Zeit.* 9 (1908): 903-907. [3]

Einstein's "fusion"

- Starts with the definition of the probability of a state as the limiting value of time averages and calculates entropy using the Boltzmann relation.
- Argues that Boltzmann and Planck's definition of the probability of complexions reduce to and make sense only in light of his definition.
- And again presses the point that this leads to Rayleigh-Jeans not Planck.



Einstein's "fusion"

Einstein's step:



Einstein's fusion is guided by "Two considerations ... distinguished by their simplicity".

Focus only on the first (cf. Brett for the second)

5. It is simple to see the way in which one could modify the foundations of the Planck theory in order to have the Planck radiation formula truly result from the theoretical foundations. I will not present the pertinent derivations here but will rather just refer to my papers on this subject.¹

The result is as follows: One arrives at the Planck radiation formula if one

1. adheres to equation (I) between resonator energy and radiation pressure, which Planck derived from Maxwell's theory²;
2. modifies the statistical theory of heat by the following assumption: A structure that is capable of carrying out oscillations with the frequency ν , and which, due to its possession of an electric charge, is capable of converting radiation energy into energy of matter and vice versa, cannot assume oscillation states of any arbitrary energy, but rather only such oscillation states whose energy is a multiple of $h \cdot \nu$. Here h is the constant so designated by Planck, which appears in his radiation equation.

Einstein's "fusion"

- Important methodological step: Use observed values of entropy to calculate W and therefore "statistical properties" of radiation.
- In contrast to the 1905 paper, makes use of the full Planck law instead of Wien regime.
- To clarify these matters, we will try to proceed in the opposite direction than that taken by Mr. Planck in his radiation theory. We consider Planck's radiation formula as correct[49] and ask ourselves whether some conclusion about the constitution of radiation can be inferred from it

Einstein's "fusion"

- In section 6, Einstein wants to calculate the mean square energy fluctuation in black-body radiation due to the Planck law.
- What he wants is an expression for the mean square of the energy fluctuation.
- On to the board!

