W. Thomson, "On an Absolute Temperature scale..."
1848

Carnot
1824

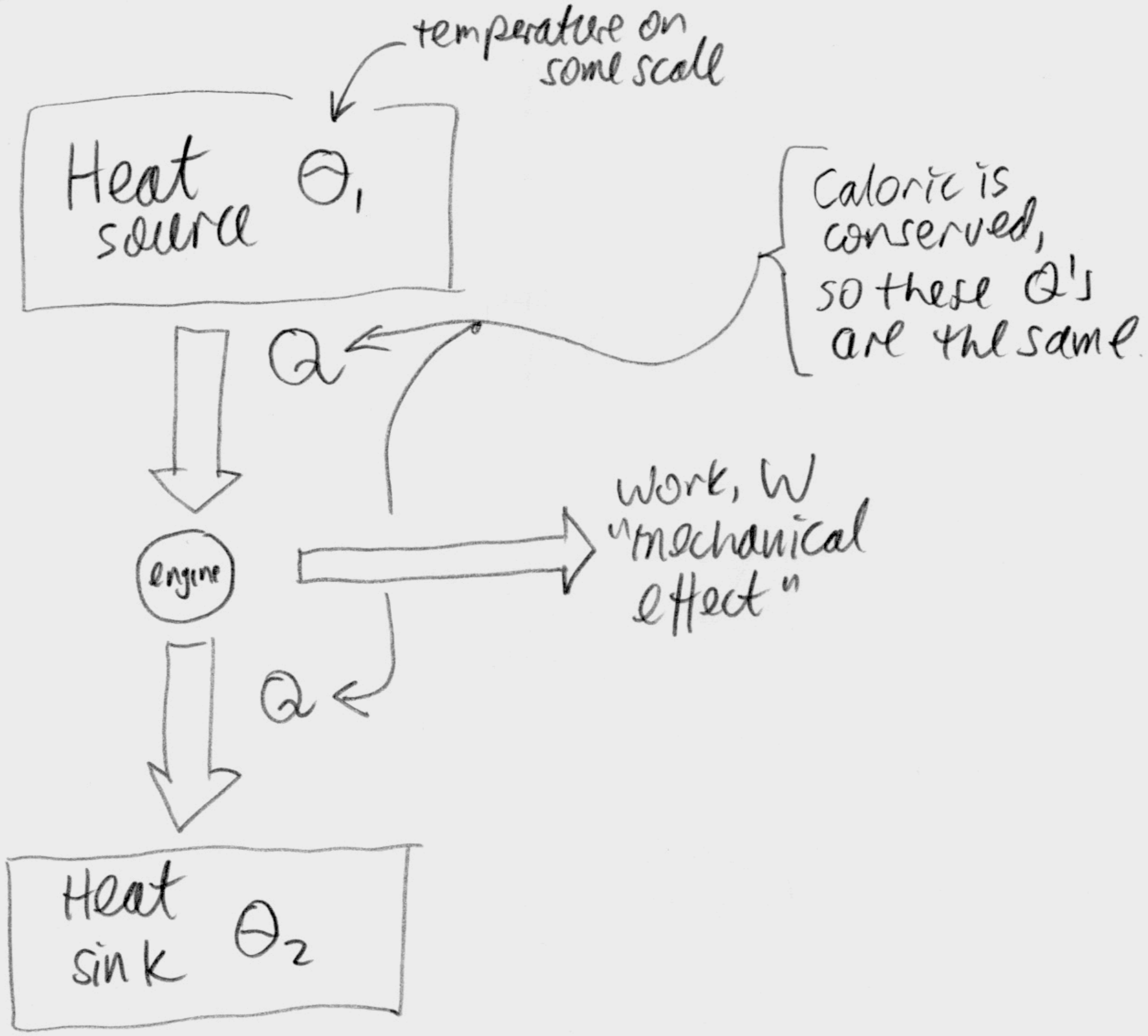
Maximum $\dfrac{\text{Work}}{\text{Heat}}$ in heat engine fixed by Temperatures of heat source and sink

independent of material, design heat engine etc.

Thomson
1848

Independently measureable quantities ⇒ Define temperature independent of materials

Carnot's model of a heat engine

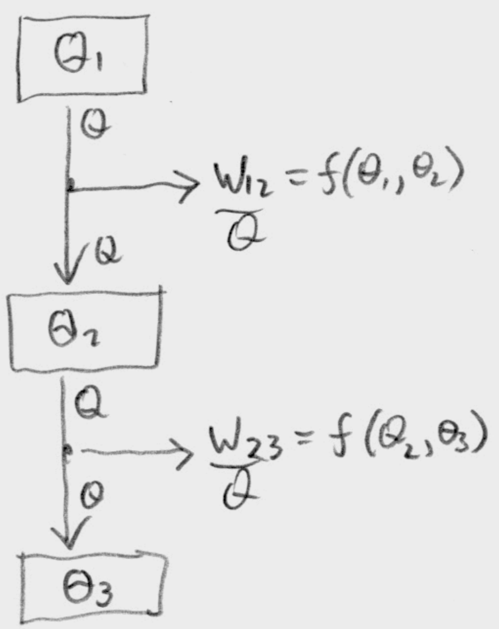


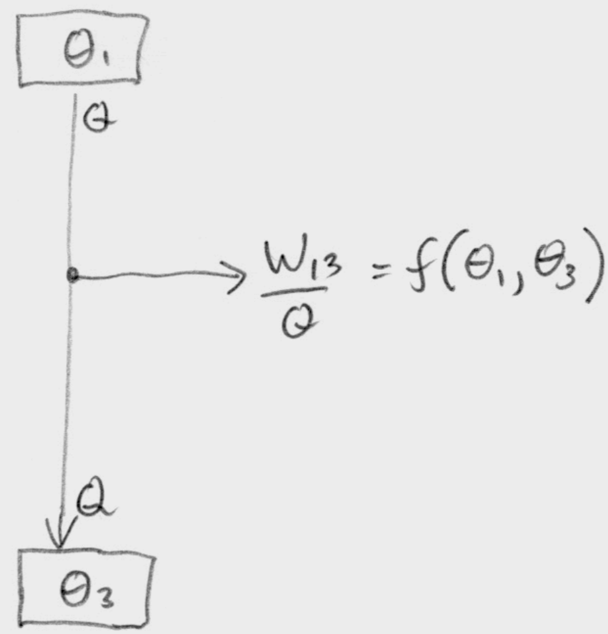
$$\boxed{W/Q = f(\Theta_1, \Theta_2)}$$ for a reversible engine

$\frac{W}{Q} \propto \Delta T$ absolute temperature difference is the only possibility

Proof is NOT Thomson's

For reversible engines:





$$W_{12} + W_{23} = W_{13}$$

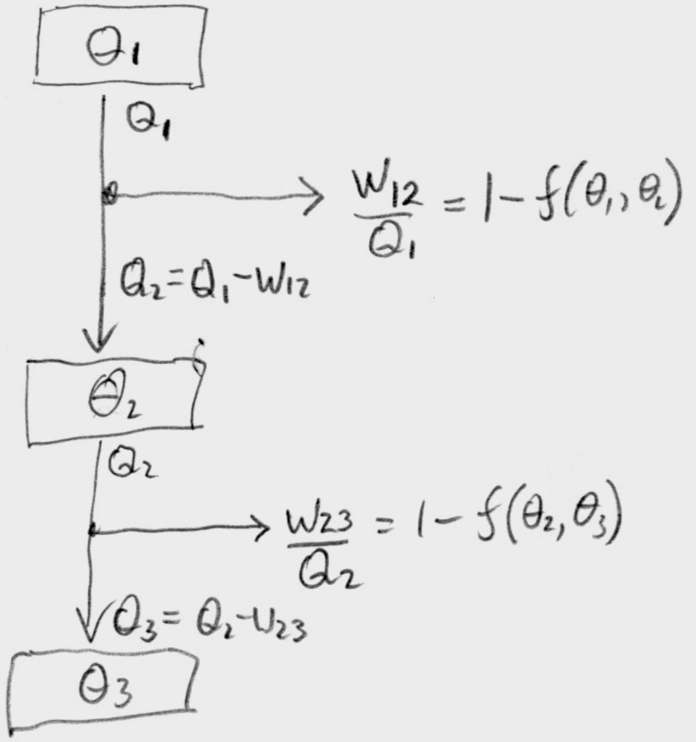
Fix as reference temperature state

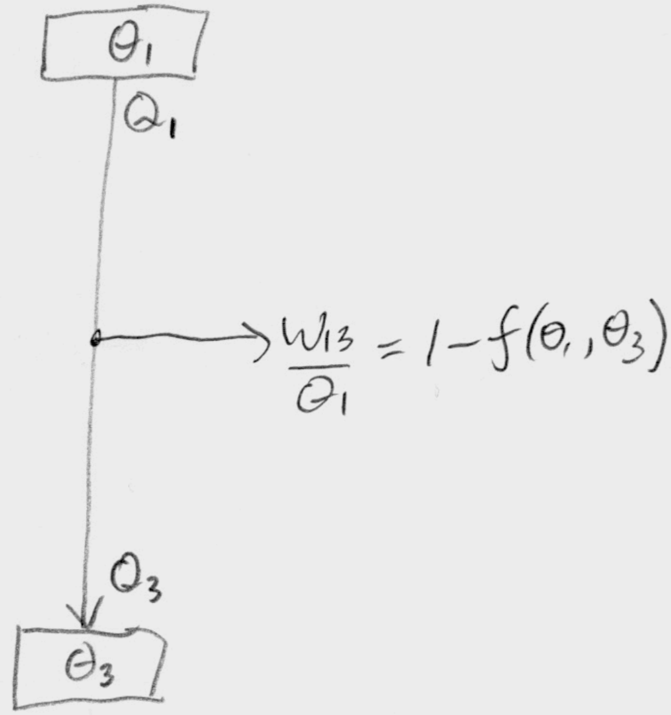
$$\therefore f(\theta_1, \theta_2) + \underbrace{f(\theta_2, \theta_3)}_{\text{"}T_2\text{"}} = \underbrace{f(\theta_1, \theta_3)}_{\text{"}T_1\text{"}}$$

$$\boxed{\therefore \frac{W_{12}}{Q} = f(\theta_1, \theta_2) = T_1 - T_2}$$

Derivation modified for case of heat & work are interconvertible

For reversible engines





Hence

$$\frac{Q_2}{Q_1} = f(\theta_1, \theta_2) \quad \frac{Q_3}{Q_2} = f(\theta_2, f\theta_3) \quad \frac{Q_3}{Q_1} = f(\theta_1, \theta_3)$$

Fix as reference temperature state

$$f(\theta_1, \theta_2) \cdot \underbrace{f(\theta_2, \theta_3)}_{\text{“}T_2\text{”}} = \underbrace{f(\theta_1, \theta_3)}_{\text{“}T_1\text{”}}$$

$$\boxed{\frac{Q_2}{Q_1} = f(\theta_1, \theta_2) = \frac{T_1}{T_2} \qquad \frac{W_{12}}{Q_1} = 1 - f(\theta_1, \theta_2) = 1 - T_1/T_2}$$