ALBERT EINSTEIN

OUT OF MY LATER YEARS

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AN ELEMENTARY DERIVATION OF THE EQUIVALENCE OF MASS AND ENERGY

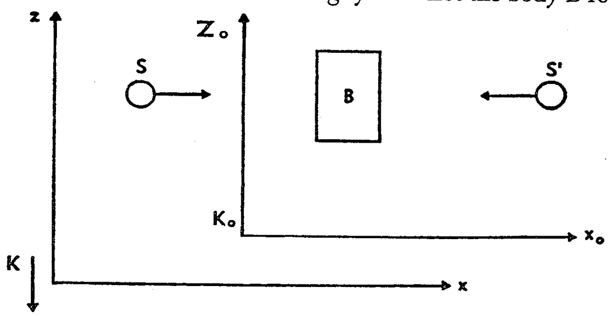
THE FOLLOWING DERIVATION of the law of equivalence, which has not been published before, has two advantages. Although it makes use of the principle of special relativity, it does not presume the formal machinery of the theory but uses only three previously known laws:

(1) The law of the conservation of momentum.

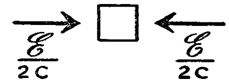
(2) The expression for the pressure of radiation; that is, the momentum of a complex of radiation moving in a fixed direction.

(3) The well known expression for the aberration of light (influence of the motion of the earth on the apparent location of the fixed stars—Bradley).

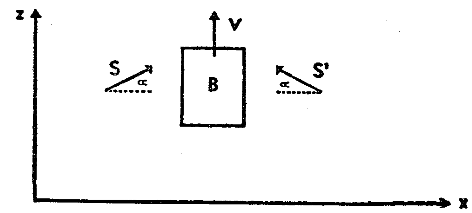
We now consider the following system. Let the body B rest



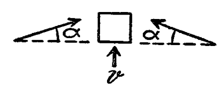
freely in space with respect to the system K_0. Two complexes of radiation S, S′ each of energy $\frac{E}{2}$ move in the positive and negative x_0 direction respectively and are eventually absorbed by B. With this absorption the energy of B increases by E. The body B stays at rest with respect to K_0 by reasons of symmetry.



Now we consider this same process with respect to the system K, which moves with respect to K_0 with the constant velocity v in the negative Z_0 direction. With respect to K the description of the process is as follows:



The body B moves in the positive Z direction with velocity v. The two complexes of radiation now have directions with respect to K which make an angle α with the x axis. The law of aberration states that in the first approximation $\alpha = \frac{c}{v}$, where c is the velocity of light. From the consideration with respect to K_0 we know that the velocity v of B remains unchanged by the absorption of S and S′.



Now we apply the law of conservation of momentum with respect to the z direction to our system in the coordinate-frame K.

I. *Before the absorption* let M be the mass of B; Mv is then the expression of the momentum of B (according to classical mechanics). Each of the complexes has the energy $\frac{E}{2}$ and hence, by a well known conclusion of Maxwell's theory, it has the momentum $\frac{E}{2c}$. Rigorously speaking this is the momentum of S with respect to K_0. However, when v is small with respect to c, the momentum with respect to K is the same except for a quantity of second order of magnitude ($\frac{v^2}{c^2}$ compared to 1). The z-component of this momentum is $\frac{E}{2c}$ sin α or with sufficient accuracy (except for quantities of higher order of magnitude) $\frac{E}{2c}$ α or $\frac{E}{2} \cdot \frac{v}{c^2}$. S and S′ together therefore have a momentum $E\frac{v}{c^2}$ in the z direction. The total momentum of the system before absorption is therefore

$$Mv + \frac{E}{c^2} \cdot v$$

II. *After the absorption* let M′ be the mass of B. We anticipate here the possibility that the mass increased with the absorption of the energy E (this is necessary so that the final result of our consideration be consistent). The momentum of the system after absorption is then

$$M'v$$

We now assume the law of the conservation of momentum and apply it with respect to the z direction. This gives the equation

$$Mv + \frac{E}{c^2} v = M'v$$

or

$$M' - M = \frac{E}{c^2}$$

This equation expresses the law of the equivalence of energy and mass. The energy increase E is connected with the mass increase $\frac{E}{c^2}$. Since energy according to the usual definition leaves an additive constant free, we may so choose the latter that

$$E = Mc^2$$