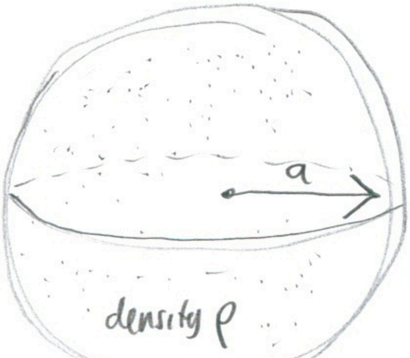
Newtonian cosmologies.
Dynamics of a sphere of matter of radius a, uniform density ρ



motion of point on surface

$$\left[\underbrace{\ddot{a}}_{\frac{d^2a}{dt^2}} = -\underbrace{\frac{4\pi G\rho a^3}{3}}_{\text{constant mass } M \text{ of spher}} \cdot \frac{1}{a^2} = -\frac{4\pi G\rho}{3} \cdot a\right]$$

Integrate with respect to t. But first multiply both sides by $\dot{a}$ ... make integration easier

$$\dot{a}\ddot{a} = -\frac{4\pi G\rho}{3} a\dot{a} = -\underbrace{\frac{4\pi G\rho a^3}{3}}_{\substack{\text{constant} \\ \text{w.r.t. } t}} \cdot a^{-2} \cdot \dot{a}$$

$$\int \dot{a}\ddot{a}\,dt = \int \frac{1}{2}\frac{d}{dt}(\dot{a})^2 dt = \frac{1}{2}\dot{a}^2 + \text{constant}$$

$$\int a^{-2}\dot{a}\,dt = \int a^{-2}\frac{da}{dt}dt = \int a^{-2}da = -a^{-1} + \text{constant}$$

$$\frac{1}{2}\dot{a}^2 + \text{constant} = \frac{4\pi G\rho a^3}{3} \cdot a^{-1} - \underbrace{\frac{4\pi G\rho a^3}{3} \cdot \text{constant}}_{\text{constant}}$$

$$\therefore \boxed{\dot{a}^2 = \frac{8\pi G\rho a^2}{3} \quad \text{or} \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}}$$

constants $\to 0$ since we stipulate $\dot{a} = 0$, when $a = 0$