Einstein. Footnote added later to 1905 paper.
Lorentz transformation may also be derived by the condition that $x^2+y^2+z^2=c^2t^2$ transforms into $\xi^2+\eta^2+\zeta^2=c^2\tau^2$

Derived only up to constant multiple $\varphi(v)$

Show this with the simpler case of

$$\xi^2 = c^2\tau^2 \xrightarrow[\text{transforms}]{} x^2 = c^2t^2$$

Step 1. The transformation is linear, so its most general form is

$$\tau = \varphi(t+bx) \quad \xi = \psi(x-vt) \quad \cdots(1)$$

constants, but may be functions of v

we must have this since $\xi=0$ corresponds to $x=vt$

Step 2. Substitute these equations into $\xi^2=c^2\tau^2$

$$\xi^2=c^2\tau^2 \longrightarrow \psi^2(x-vt)^2 = c^2\varphi^2(t+bx)^2$$

$$\psi^2x^2 - \underbrace{\psi^2 2v\,xt} + \psi^2v^2t^2 = c^2\varphi^2t^2 + \underbrace{c^2\varphi^2 2bxt} + c^2\varphi^2b^2x^2$$

If this is to reduce to $x^2=c^2t^2$, the "xt" terms must cancel

$$-\psi^2 v = c^2\varphi^2 b$$

$$\boxed{b = -\frac{v}{c^2}\cdot\frac{\psi^2}{\varphi^2}} \quad \cdots(2)$$

Step 3 Use the expression for b to simplify further

After elimination of xt terms, $\xi^2 = c^2\tau^2$ becomes

$$\psi^2 x^2 + \psi^2 v^2 t^2 = c^2\varphi^2 t^2 + c^2\varphi^2 b^2 x^2$$

$$\left(\psi^2 - c^2\varphi^2 b^2\right)x^2 = \left(c^2\varphi^2 - \psi^2 v^2\right)t^2$$

$$= c^2\left(\varphi^2 - \psi^2 \frac{v^2}{c^2}\right)t^2$$

$$\therefore \cancel{\psi^2}\underbrace{\left(1 - c^2\frac{\varphi^2}{\psi^2} b^2\right)}_{b^2 = \frac{v^2}{c^4}\cdot\frac{\psi^4}{\varphi^4} \;\rightarrow\; \left(1 - \frac{v^2}{c^2}\cdot\frac{\psi^2}{\varphi^2}\right)} x^2 = \cancel{\psi^2}\, c^2\left(\frac{\varphi^2}{\psi^2} - \frac{v^2}{c^2}\right)t^2$$

$$\therefore \left(1 - \frac{v^2}{c^2}\cdot\frac{\psi^2}{\varphi^2}\right)x^2 = c^2\left(\frac{\varphi^2}{\psi^2} - \frac{v^2}{c^2}\right)t^2$$

$$\therefore \frac{\psi^2}{\varphi^2}\cancel{\left(\frac{\varphi^2}{\psi^2} - \frac{v^2}{c^2}\right)}x^2 = c^2\cancel{\left(\frac{\varphi^2}{\psi^2} - \frac{v^2}{c^2}\right)}t^2$$

$$\frac{\psi^2}{\varphi^2}x^2 = c^2 t^2 \xrightarrow[\text{if } \psi^2 = \varphi^2]{\text{becomes}} x^2 = c^2 t^2$$

$$\boxed{\psi = \pm\varphi} \quad \cdots (3)$$

choose "+" so that τ and t both increase in the same direction

Step 4 Substitute (2) and (3) into (1)

$$\tau = \varphi(v)\left(t - \frac{v}{c^2}x\right) \qquad \xi = \varphi(v)(x - vt)$$