

## **A Hot Mess: Author's Response**

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My responses to commentary by G. Barış Bağci, "On Norton's No-Go Result," and Andrew Jordan and Sreenath Manikandan, "Some Like it Hot," on John D. Norton, "A Hot Mess," *Inference: International Review of Science*. Vol. 4, Issue 3.

Professor Bağci and I differ only on technical issues in the derivation of the no-go result. It will be convenient to explore them first. Then I will address the differences in the fundamentals that separate me from Professors Jordan and Manikandan.

### **On Professor Bağci**

The no go result applies to thermodynamically reversible processes implemented in molecular scale systems. In ordinary thermodynamics, a reversible process is one that passes through a sequence of states, such that the states come arbitrarily close to states in equilibrium with each other. These last equilibrium states are of equal thermodynamic entropy  $S$ . The process can only proceed if there is a very slight entropy increase along the sequence of actual states. If they were to realize the equality of entropy, then the process would be frozen. There would be no entropic forces to advance the process.

Professor Bağci considers a special case not mentioned in the main article but treated by me elsewhere. If our system is in thermal equilibrium with a large heat bath at temperature  $T$  with which it exchanges heat but no work, then the condition of increasing entropy of the total system is expressed by a condition of decreasing free energy  $F$  of the system, excluding the heat bath.

They are equivalent conditions. In both cases, a reversible process is characterized by the fact that each of its stages comes arbitrarily close to states of the same entropy of the total system or of the same free energy of the system excluding the heat bath, in the general and special cases respectively.

The no-go result arises when we now add the fact that thermal systems have molecular constitutions. A new effect, thermal fluctuations, enables the process to move among its states.

This second process is active even if the stages of the process achieve the states of equal entropy or free energy. In this case, the probability  $W$  that fluctuations deliver the system to any particular stage is given by the familiar formulae: the inverted Boltzmann formula  $W = \exp(S/k)$  or the version of it that Professor Bađci uses,  $W = \exp(-F/kT)$ . Since the entropy  $S$  or free energy  $F$  of each of the stages is the same, all the stages are of equal probability. Fluctuations dominate and move the process with equal probability to each of its stages, undoing the possibility of any assured completion.

Professor Bađci's concern is how we are to arrive at the constancy of free energy  $F$  over all the stages. The answer is that it is built in from the start through the definition of a reversible process. The constancy comes from ordinary thermodynamics. If the free energies of the stages of a process are equal, then the sum of thermodynamic forces acting in each stage is zero. Each stage is in static equilibrium and the process does not advance. When that equality is implemented in statistical physics, it requires only that the mean value of the sum of these forces is zero. Fluctuations around the mean will produce momentary, non-zero driving forces that will advance or retard the process.

In place of this reasoning, Professor Bađci tries to derive the constancy of the free energy from a power series expansion that relates the free energy of an equilibrium state to that of the non-equilibrium states to which it fluctuates. As he recounts, the effort fails and for the reason he indicates: the fluctuation states involved are not small fluctuations amenable to power series analysis. I am not clear on the details of Professor Bađci's application of this power series expansion to the case at hand. His analysis considers an equilibrium state and fluctuations from it. Once we move to statistical physics, all the stages become non-equilibrium fluctuation states. There is, however, a single equilibrium state from which they arise as fluctuations.

Since fluctuations can cause the process to migrate over all its stages, these stages can be taken together to form a single larger thermal system at equilibrium. The individual stages are then states to which the system migrates through fluctuations. One might try to use Professor Bađci's power series expansion to recover the free energies and probabilities of the various stages. However a more straightforward method is simply to apply directly to the fluctuation states the free energy analog of Boltzmann's  $S = k \ln W$ . It is the canonical expression for free energy mentioned above. It is given more fully as  $F = -kT \ln Z$ , where  $Z$  is the partition integral over the subvolume of phase space corresponding to the fluctuation state. It is also the probability of the fluctuation state up a constant, normalizing factor.

A simple example illustrates how the combined stages of the process form one larger system at thermal equilibrium. It is perhaps the simplest possible example: a Brownian particle in a Petrie dish of water. When the dish is level horizontally, all positions of the particles are

states of equal entropy or free energy. To have a process that moves the particle across the dish, one must tilt the dish, introducing a dissipative entropy gradient. If we now add in the molecular character of the thermal system, we no longer need to tilt the dish to move the particle. Thermal fluctuations lead it to jiggle about in the celebrated effect of Brownian motion. Over a long period of time, the particle will fully explore the dish and with equal probability in all its parts. Considered over this long time scale, the Brownian particle is behaving like a single molecule gas. If our desired process is to move the particle assuredly from one side of the dish to the other, we must tilt the dish enough to overcome the Brownian motion that would deflect the particle away from the final position sought. This results in added, entropy creating dissipation whose magnitude can be computed from the formula in the no-go result using the probability of completion sought.

As long as we consider just a microscopically visible particle, the overall effect is small. It will take a long time for the Brownian particle to explore the whole dish. However as we consider smaller particles approaching molecular scales, the motion becomes more rapid and the amount of tilting needed becomes significant. In the extreme case of a molecule in air, mere tilting no longer suffices. A dissipative process that creates considerable amounts of entropy must be employed to confine the particle. For a quantitative analysis of these last processes, see (Norton, 2013, Section 10) “Simplest Illustration of the No-go Result: Bead on a wire.”

## **On Professors Jordan and Manikandan**

These correspondents view Landauer’s principle as “easily derivable from basic principles of thermodynamics and statistical physics.” Yet we shall see that the argument they then offer for it goes beyond these basic principles. The goal is to establish that an  $N$  state memory device holding random data is thermodynamically like a single-particle gas in a chamber of volume  $N$ . The isothermal, reversible compression of the single particle gas to unit volume passes entropy of  $k \ln N$  to the heat bath. If the analogy holds, the erasure of the memory device is associated with the same passage of entropy.

The difficulty is that the two systems are not thermodynamically analogous. The uncompressed gas has an entropy that is  $k \ln N$  greater than that of its compressed state. The memory device holding random data has the same entropy as the erased state. Purely thermodynamic reasoning cannot override this difference. To presume otherwise is to commit the misattribution of entropy described in the main article.

To bridge the gap, Professors Jordan and Manikandan follow the standard literature in employing an argument that goes beyond ordinary thermodynamics. They prescribe an erasure procedure envisaged by Landauer and others in this literature. It begins by thermalizing the

memory device, as described in the main article. This is an irreversible expansion that is the real source of the  $k \ln N$  entropy created in the erasure procedure. The expansion is needed since otherwise, they argue, the erasure must employ different procedures according to the content of the memory device. These different procedures require, they argue further, a computer or its operator to have some memory of the data to be erased, which would then merely displace the data but not erase it.

This key argument is not a principled, thermodynamic argument. Rather, it is speculation about how all possible erasure devices must operate. It relies on intuitions tutored by the particular way that modern digital computers happen to be set up and operated. In support they quote Landauer in his original paper remarking on how a computer operates “in most instances.” Mere familiarity is not necessity. The argument fails to demonstrate that all erasure processes must operate in this way. If one is willing to proceed in this loose, speculative manner and neglect fluctuations, one can imagine protocols that do not require such memory records. For illustrations, see Norton (2011, Section 5.2). Whether these are admissible or not does not really matter. The positive but unmet burden of proof rests on proponents of Landauer’s principle. The principle remains now as it was when Landauer first proposed it: a speculation without general proof, based on loose plausibility arguments. Over half a century later, can we not do better?<sup>1</sup>

This is not the most important of our differences. My view differs from that of Professors Jordan and Manikandan in the import of fundamental principles. They regard thermal fluctuation in molecular scale processes as eliminable distractions. All it takes to control them arbitrarily well is a suitably ingenious design of the process. Then thermodynamically reversible processes can be implemented in molecular scale systems. I view thermal fluctuations—thermal noise—as an ineliminable feature of molecular scale processes that derive from a principle, Boltzmann’s  $S = k \ln W$ . They arise inevitably from the molecular constitution of all thermal processes. Their entropically costly suppression is unavoidable in principle if we are to bring any process to completion on molecular scales. Even then, completion can only be assured probabilistically. The relation between the entropy creation required and the probability of completion is quantified in the no-go result. The result is quite general and applies to any process whatever at molecular scales. It is all too easy to underestimate the reach of this last statement.

My concern, expressed in the main article and elsewhere, is that the Landauer principle literature has ignored this ineliminable source of entropy creation, rendering much of their analyses spurious. Professors Jordan and Manikandan provide an illustration of this neglect. They describe a molecular-scale, erasure process to demonstrate their claim that the Landauer limit is achievable in spite of thermal fluctuations. The process they describe is familiar and a

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<sup>1</sup> On the experimental tests of the Landauer principle, I have expressed my disappointment in how little the Berut et al. experiment actually shows in Norton (2013, Section 3.7).

standard in the literature. I gave an extended analysis of it for the one bit case in my Norton (2005, Section 2), but without considering fluctuations. These considerations can be added now. We will see that, as a matter of principle, the entropy creation required to suppress fluctuations precludes realization of the Landauer limit.

On their account, Professors Jordan and Manikandan's erasure process consists of four processes or steps chained together, each completing before the next starts. The second, third and fourth of these are executed reversibly. They are:

2. Isothermal, reversible  $N$ -fold compression of a one-particle gas.
3. Reversible raising of barriers to divide a chamber into  $N$  sections.
4. Reversible removal of the mechanism producing the compressive force to its original configuration.

Since each step is to proceed reversibly, its components are thermal and arbitrarily close to equilibrium with the other components. The no-go result applies to each step individually. Probabilistic completion of each will require creation of thermodynamic entropy in accord with the no-go result formula.

In the case of steps 3 and 4, it is hard to say more since few details are provided for their implementation. However these details are not needed. All that matters is that, in both steps, the erasure process design calls for moving some system through a sequence of states, arbitrarily close to equilibrium states of equal entropy, to some desired end state. The no-go result applies.

More details are provided for step 2. The particle is assumed electrically charged and it is compressed into  $1/N$ th of the chamber volume by a suitably manipulated electric field. All fields inside computers are thermal systems and they exhibit thermal fluctuations. Einstein (1909) demonstrated the necessity of these fluctuations in passing. He showed that without them, one could have processes in which the thermal energy of a particle could be converted completely into thermal radiation, in violation of the second law. The thermal fluctuations in the field were needed to enable thermal equilibrium to be achieved and the second law preserved. Similarly, any device used to manipulate the field is also a thermal system and will exhibit thermal fluctuations. Might there be some electric circuitry involved? Such circuits are subject to Johnson–Nyquist noise, which is the electrical engineer's term for thermal fluctuations in circuitry.

To achieve reversibility, the mean of the compressive force provided by the field and supporting circuitry must match almost exactly the mean of the compression resisting force of the one-particle gas. Superimposed over these means will be the fluctuations that will indifferently advance and retard the process of compression. The size of the fluctuations in the one-particle gas has already been computed in the main text. A thermal particle with three degrees of freedom has a mean energy of  $1.5kT$  with an RMS spread of  $1.22kT$ . That is, its

energy will be fluctuating widely, at least over the interval  $0.28kT$  to  $2.72kT$ . The gains and losses in energy derive from rapid energy exchanges with any system with which it interacts, including the heat bath and the compressing field. To suppress these fluctuations and assure probabilistic completion of the compression, a mean compressive force significantly greater than the mean resistive force must be applied. The ensuing imbalance of forces is dissipative and leads to the creation of entropy. Once again, the no-go result gives the precise relation between the entropy created and the probability of completion.

To give a more complete analysis of the fluctuations in the electric field and its supporting circuitry would be complicated and possibly prohibitively so. However we can see how fluctuations disrupt the compression in a much simpler case in which the gas is compressed in a cylinder by a weighted piston. For a balance of mean forces to be achieved, the weight on the piston must be very small, since the thermal motions of a single particle exert a very weak pressure. Thermal fluctuations in a very light piston lead it to jump about in motions comparable to the thermal motions of the particle in the one-particle gas. Completion of the compression can only be secured probabilistically by employing a weightier piston. The balance of mean forces is lost and the compression is dissipative, creating entropy. I have calculated this case in greater detail in Norton (2011, Section 7; 2017). To preclude confusion, a popular but failed attempt at escaping the effects of fluctuations is to imagine that the one-particle gas is compressed by a very slowly moving, very massive piston. As I explain in Norton (2013, Section 7), such a system is far from thermodynamic equilibrium, so the process is not thermodynamically reversible.

The practical experience of nanotechnology is that all nano-scale devices are beset by thermal fluctuations, electrical noise or what ever you like to call it. Suppressing their disruptive effects by thermodynamically dissipative processes is a controlling consideration in design. For Professors Jordan and Manikandan this is merely a technical nuisance. As far as matters of thermodynamic principle are concerned, they say, dissipation is only necessitated when logically irreversible processes are required. They are, I have argued, mistaken. The existence of thermal fluctuations and the high thermodynamic cost of suppressing them is unavoidable. It is grounded in a most important principle of thermal and statistical physics, Boltzmann's  $S = k \ln W$ . What is notable is that the formula is independent of temperature. So whatever benefits we may gain by cooling our systems, the amount of entropy creation  $S$  depends only on the probability of process completion and not its temperature.

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