Quiz 4 Answer key

Fall 2013 Math 0400

---

1. (a) [2 points] In how many ways can a six-letter security password be formed from letters of the English alphabet (26 letters) if no letter is repeated?

(b) [3 points] How many different signals can be made by hoisting two yellow flags, four green flags, and three red flags on a ship's mast at the same time?

Solution: (a) There are 26 letteres in the alphabet. $P(26,6) = \frac{26!}{(26-6)!} = \frac{26!}{20!} = 21 \cdot 22 \cdot 23 \cdot 24 \cdot 25 \cdot 26 = 165,765,600$.

(b) It is a permutation of $2+4+3=9$ objects, not all distinct. The number of permutations is $\frac{9!}{2! \cdot 4! \cdot 3!} = 1260$

Answer: $R = \$242.23$

2. Let S be any sample space, and let E, F, and G be any three events associated with the experiment. Describe the following events using the symbols $\cap$, $\cup$, and ^{c}.

(a) [3 points] The event that both E and F occur.

(b) [2 points] The event that E occurs but neither of the events F or G occurs.

Solution: (a) $E \cap F$

(b) $E \cap (F \cup G)^c$

3. [5 points] Grade Distributions. The grade distribution for a certain class is shown in the following table:

Table 1: Grade Distributions

Grade	A	B	C	D	F
Frequency of Occurence	12	15	13	7	3

Find the empirical probability distribution associated with these data.

Solution: The sample space is $S = \{A, B, C, D, F\}$, $n(S) = 12 + 15 + 13 + 7 + 3 = 50$,

$n(A) = 12$, $n(B) = 15$, $n(C) = 13$, $n(D) = 7$, $n(F) = 3$. Then $P(A) = \frac{12}{50} = 0.24$, $P(B) = \frac{15}{50} = 0.3$, $P(C) = \frac{13}{50} = 0.26$, $P(D) = \frac{7}{50} = 0.14$, $P(F) = \frac{3}{50} = 0.06$.

Table 2: Grade Distributions

Grade	A	B	C	D	F
Frequency of Occurence	11	14	12	7	3
Probability distribution	0.24	0.3	0.26	0.14	0.06

bonus problem [5 points extra] Two fair dice are rolled. Find the probability that the sum s of the numbers appearing uppermost satisfies the condition $5 \leq s \leq 7$. Simplify your answer.

Solution: The sample space S is a set of all pairs (elementary enents) of integers (k, m), where $1 \leq k \leq 6$ and $1 \leq m \leq 6$.

So, $S = \{(1,1), (1,2), (1,3), ..., (6,5), (6,6)\}$, $n(S) = 36$.

Let the event E_5 contains all pairs that have a sum s satisfying the condition $5 \leq s \leq 7$. Then

$$E = \{(1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (5,1), (5,2), (6,1)\}.$$

$n(E) = 15$. Then $P(E) = \frac{15}{36} = \frac{5}{12}$.