Sept. 24, 2007

Chem. 1410

Problem Set 4, due Oct. 1, 2007

Do the following problems from Engel; these are *not* to be handed in for grading; solutions will be distributed via .pdf.

Chapter 7: P7.1, P7.2, P7.4

The following three problems are to be handing in for grading:

(1) **Position Matrix Elements of the Harmonic Oscillator**

Consider a 1D harmonic oscillator of mass *m* and force constant *k* (hence with angular frequency $\omega = \sqrt{k/m}$). Denote the unit-normalized energy eigenfunctions associated with this system as $\psi_n(x)$, $n = 0,1,2,...$ Deduce a formula for the integrals $I_j = \int_{-\infty}^{\infty} dx \psi_j(x) x \psi_0(x)$ for the cases that j=0,1,2 [Hint: the cases j=0 and j=2 are particularly easy!]

Notes:

i) The following Gaussian integral identity should prove useful: $\int_{-\infty}^{\infty} dx G(x) x^2 = \sigma^2$, where $G(x) \equiv \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-x^2 / 2\sigma^2)$.

ii) It turns out that the integrals I_k (often termed "position matrix elements") determine the intensity of the spectroscopic transitions from state 0 to state j of a harmonic oscillator when it absorbs light.

(2) **Particle in a 2D Box.** Consider a particle of mass *m* moving in a two-dimensional square box with side length L. The potential energy is zero inside the box and infinite outside.

(a) Which of the following functions are allowed (unnormalized) energy eigenfunctions of this system? For those that are energy eigenfunctions, what is the associated energy eigenvalue? For those that aren't, explain why not.

$$(i)\,\sin(\pi x / L)\sin(3\pi y / L); 0 < x < L, 0 < y < L$$
$$(ii)\,\sin(\pi x / L)\sin(3\pi y / 2L); 0 < x < L, 0 < y < L$$
$$(iii)\,\cos(\pi x / L)\sin(4\pi y / L); 0 < x < L, 0 < y < L$$

(b) Drawn an energy level diagram indicating the three lowest energy levels for this system and the appropriate degeneracy of each level. Make sure the values of the energy levels are clearly noted.

(3) **1D Harmonic Oscillator and Basic Quantum Measurement Principles.** Consider a particle of mass *m* moving in a one-dimensional harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$.

(a) Write down the formula for the normalized ground state eigenfunction and sketch it. Write down the formula for the corresponding energy eigenvalue.

(b) The same as (a), but for the first excited state. [Be sure to identify any nonstandard symbols that appear in your eigenfunction formula.]

(c) Suppose the particle is prepared in the superposition state $\psi = 0.949\varphi_0 + 0.316\varphi_1$, where φ_0, φ_1 are the normalized ground and first excited state energy eigenfunctions considered in parts (a) and (b).

(i) What is the probability that a measurement of energy will give yield the ground state energy eigenvalue? The first excited state eigenvalue? The second excited state eigenvalue?

(ii) What is the expectation value of energy for a particle prepared in the state ψ? Give your answer in units of $\hbar\omega$ (where, as usual, $k \equiv m\omega^2$).