Sept. 17, 2007

Chem. 1410

Problem Set 3, due Sept. 24, 2007

Do the following problems from Engel; these are *not* to be handed in for grading; solutions will be distributed via .pdf.

Chapter 4: 4.5, 4.10, 4.15, 4.17, 4.23

The following three problems are to be handing in for grading:

(1) **Uncertainty Product.** Consider a quantum mechanical wavefunction of the form:

$$\psi(x) = \frac{1}{\left[2\pi\sigma^2\right]^{1/4}} \exp\left(\frac{-x^2}{4\sigma^2}\right) \qquad [1]$$

a) Verify that $\psi(x)$ is normalized properly in the quantum mechanical sense, i.e., show that $\int_{-\infty}^{\infty} \left|\psi(x)\right|^2 dx = 1$.

Note:

i) Since $\psi(x)$ is a real-valued function, $\psi(x) = \psi^*(x)$ here.

ii) $\frac{1}{\left[2\pi\sigma^2\right]^{1/2}} \int_{-\infty}^{\infty} dx \exp\left(\frac{-x^2}{2\sigma^2}\right) = 1$ for any real number σ.

b) Show that *<x>=0*, and *<p>=0* for this $\psi(x)$.

Note: $< A > \equiv \int_{-\infty}^{\infty} dx \psi^*(x) \hat{A} \psi(x)$, where $\hat{A}$ is the quantum mechanical operator corresponding to observable A.

c) Show that $< x^2 > = \sigma^2$ here.

Note: $\frac{1}{\left[2\pi\sigma^2\right]^{1/2}} \int_{-\infty}^{\infty} dx \exp\left(\frac{-x^2}{2\sigma^2}\right) x^2 = \sigma^2$ for any real number σ.

d) Show that $<p^2>=\alpha\hbar^2/\sigma^2$ for the wavefunction considered in Eq. 1, where α is a numerical coefficient that you are to compute.

Hint: All of the non-zero integrals needed to evaluate α are given above.

e) For a system represented by a general state function $\psi(x)$, the "uncertainty" in a quantum mechanical observable *A* is defined as $\delta A \equiv \sqrt{<(A-<A>)^2>}$. Thus, using the results obtained above, show that for $\psi(x)$ in Eq. 1:

$$\delta x \delta p = \sqrt{\alpha}\hbar$$

f) It can be shown (you do not have to do this here!) that $\delta x \delta p \geq \hbar/2$ for *any* physically admissible state function. Is the result obtained in part e) for the particular (Gaussian) state function specified in Eq. 1 consistent with the fundamental inequality just stated?

(2) **Probability Distribution for a Particle in a Box.** A particle of mass *m* moves in a one dimensional box whose width is *L*. Suppose that the particle is prepared in eigenstate *n* of the box (with *n=1,2,3,…*). Deduce a formula for the probability that a measurement of the system will find the particle in the *middle third* of the box. Evaluate your formula explicitly for the cases that n=1 (the ground state of the system) and n=3. What is the limiting value of this probability as $n \to \infty$?

(3) **Engel, P4.25.**