GOALS, CONSTRAINTS, AND TRANSPARENT ASSIGNMENT
A FIELD STUDY OF THE UEFA CHAMPIONS LEAGUE

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Abstract. We analyze a matching mechanism developed to solve a complex constrained-assignment problem, where the entire randomization needs to be transparent to an outside observer. The application (the UEFA Champions League tournament draw) is high stakes; participants’ expected outcomes can shift by millions of euro depending on a particular realization of the draw. Moreover, for a centralized assignment with nontrivial constraints, the draw brings in a huge audience, with at least a million following along from around the world. After quantifying the constraint effects, we turn to a normative assessment: Are there better, fairer procedures? Relying upon a combination of theory, structural estimation, and simulation, we outline a quantitative methodology aimed at assessing the transparent assignment procedure developed by UEFA. Our analysis indicates that the mechanism comes quantitatively close to a constrained-best in fairness terms. Moreover, we demonstrate that while substantially better mechanisms do not exist given the current constraint structure, it is possible to substantially reduce matching distortions by only marginally slacking the constraints.

1. Introduction

Managers are typically the party tasked with deciding upon workable solutions to assignment problems. For example, consider a choice over a monthly scheduling procedure to allocate workers to a set of shifts, taking into account constraints on the required mix of worker skills, shifting availabilities, previous allocations, etc. While managers might be able to quantify their design objectives (for example, ex ante fairness for allocation to over/under-demanded positions), implemented assignment procedures are often ad hoc or informal (turn-taking, drawing names from a hat, asking for volunteers) rather than the outcome of an explicit optimization over the process. One reason for this may be that assignment problems—particularly those with nontrivial constraints—are inherently challenging to optimize over. However, another important institutional factor for
many assignment problems is the need for the mechanism to appear fair to the various stakeholders (employees, the media, regulatory agencies, etc.). Even qualitative descriptions of an objective-optimizing algorithm can be complex, particularly to lay people. Moreover, the process of realizing randomized assignments through an algorithm can be opaque, as if produced by a black box. Consequently, managers might subordinate other design objectives to ensure that the resulting assignment process is transparent. Still, it is natural to wonder what the losses from such choices are; whether there might be alternative assignment procedures that better-achieve a manager’s objectives, while maintaining transparency.

Our paper conducts an analysis of a constrained assignment with huge public scrutiny, hundreds of thousands following the assignment procedure, and with millions of euros at stake from the realizations. The developed mechanism provides a solution to a combinatorically complex constrained-assignment problem that prioritizes transparency of the process, conducting all randomizations through simple urn draws. After characterizing theoretical properties of the chosen mechanism, we outline how a recent market-design tool (Budish et al., 2013) allows us to quantify the loss from this transparent design, relative to an optimal one. Focusing on ex ante fairness we show that to all practical extents the developed procedure is very close to a constrained best.

Our application is a public drawing of football-team pairs in the Union of European Football Association’s (UEFA) Champions League (UCL). The UCL is one of the most successful pan-European ventures, and certainly the one with the most enthusiasm from the general public. The tournament brings together football clubs from across the continent (and beyond) that normally play within their own country-level associations. Selection into the competition is limited to the highest-performing clubs from each nation, where a series of initial qualifying rounds whittle the number of participating teams down to 32 group-stage participants. From there, half of the clubs advance to a knockout stage that begins with the Round of 16 (R16), followed by four quarter-finals (QF), two semi-finals (SF), and a final (F) that determines a European champion. Outside of the World Cup, the UCL final game is one of the most-watched global sporting events, eclipsing even the viewership of the Superbowl in the United States.

Because the UCL is under a magnifying glass—from the teams, the fans, and the media—UEFA has a clear interest in creating impartial and meritocratic assignment rules for the tournament, with credible randomizations. While a fair public drawing could be trivially designed in many situations (for instance, matching teams through urn draws without replacement) the tournament’s design problem is complicated due to three constraints
imposed on the R16 match pairs: (i) Each pairing must be between a group winner and a group runner-up (the bipartite constraint). (ii) Teams that played one another in the prior group stage cannot be matched (the group constraint). (iii) Teams from the same national association cannot be matched (the association constraint). These three constraints provide some degree of meritocratic seeding (bipartite), promote variability in the realized matchups (group), and maintain an international character to the tournament (association).

The chosen randomization assembles each R16 team pairing through a dynamic public draw of balls from an urn, where the draw composition is adapted dynamically by a computer in order to respect the constraints. Our paper sets out to analyze the properties of this ad hoc assignment mechanism, using the tools of market design: theory, estimation, and simulation (Roth, 2002).

We first theoretically characterize the simple to follow (but combinatorically complex) UEFA draw procedure. Next, we quantitatively analyze the constraint effects, showing that the association exclusions generate large distortions, of up to two million euro. Of particular analytical interest are the indirect effects of the matching constraints, where spillovers from other team-pair exclusions disproportionately affect the likelihood of other teams’ matches. A natural question is whether an alternative procedure exists that respects the constraints but results in a fairer randomization. Using an objective function focused on disparate treatment of otherwise equally treated teams, we use the Budish et al. (2013) result to relax (without loss of generality) the complex problem of looking for randomizations over constrained assignments; instead focus on the far-more-tractable problem of finding expected assignments that satisfy the constraints. The conclusion from our analysis of the UEFA assignment rule across the past sixteen years shows that even though marginal improvements are possible, the tournament’s procedure comes very close to achieving the same outcomes as optimal designs.

While our search for a superior constrained-assignment procedure suggests only minimal scope for improvement (and large potential costs from forgoing the transparent procedure UEFA developed), a related question is the extent to which better outcomes are possible from slacking the imposed constraints. As a constructive exercise, we conclude the paper by showing that a relatively small relaxation of the association constraint can substantially reduce the distortions. Importantly, this can be done with only minimal adjustments to the current procedure, retaining its transparent design.

As an assignment rule, our application’s design stands out as one that focuses on fuzzier attributes: simplicity to lay people, transparency in the process, etc. The analytical tools we bring to bear are certainly capable of deriving an optimal procedure for any given
objective. However, rather than as a constructive design tool, our analysis instead focuses on quantifying the losses from a design tailored towards transparency.

In terms of the paper’s organization, Section 2 provides a brief review of related literature. In Section 3, we describe the application and discuss the imposed constraints. In Section 4, we formalize the chosen procedure and indicate some theoretical results. In Section 5, we describe the data used in estimating a simulation model for the tournament; through this model we then derive quantitative features of the actual procedure, then turn to counterfactual procedures. Finally, Section 6 concludes.\(^1\)

\[\text{2. Literature review}\]

Our paper contributes to two main strands of economic literature: market design and tournaments. While there is large theoretic literature on the incentive effects of tournaments (see Prendergast, 1999) our paper is more closely related to a growing body of applied work exploiting public and well-structured sports-tournament outcomes as naturally occurring experiments. In recent years, data from football through cricket and golf has been used to provide evidence supporting both standard theory (Walker and Wooders, 2001; Chiappori et al., 2002; Palacios-Huerta, 2003) and behavioral biases (Bhaskar, 2008; Apesteguia and Palacios-Huerta, 2010; Pope and Schweitzer, 2011; Foellmi et al., 2016).

Where the literature on sports tournaments has been centered around various positive aspects of individual’ behavior, our paper instead emphasizes normative features of the tournament itself. In this sense, our work is more closely related to the market design literature, and a handful of applied papers examining well-structured environments. Key examples here are: Fréchette et al. (2007), demonstrating the problem of inefficient unravelling in a decentralized market through US college football bowls; Anbarci et al. (2015), designing a fairer mechanism for penalty shootouts in football tournaments; Baccara et al. (2012), investigating spillovers and inefficiency in a faculty office-assignment procedure; and Budish and Cantillon (2012), studying the superiority of a manipulable mechanism to the strategy-proof mechanism using data from a business school’s course assignment procedure. In each, quantitative market-design methodologies are developed through a combination of theory with a structural analysis of the application. Similarly, our paper approaches the assignment design question through a combination of theory tools, estimation techniques, and simulation within the application (see Roth, 2002).

\(^1\)The paper’s Appendix presents proofs together with additional (theoretical and empirical) results for interested readers. Full data, programs, and the Online Appendix are available at https://sites.google.com/site/martaboczon.
The main normative insights for our application are made possible through the core theorem in Budish et al. (2013)—which shows that a consideration of the expected assignment matrix is sufficient for an analysis of the assignment design problem, as long as the constraints satisfy a biheirarchy separability condition. This result allows us to show near-optimality of the existing UEFA assignment rule. To our knowledge our paper is one of the first ones to apply this market-design tool in a normative assessment of a procedure in the field. While our tournament setting is of standalone interest, the focus on constraints in matching is related to topics in school choice such as the implementation of affirmative-action constraints (for example, Dur et al., 2016).

Lastly, our paper introduces a novel design consideration to a literature that is primarily focused on fairness, efficiency, and strategy proofness (see Abdulkadiroğlu and Sönmez, 2003, and references thereof). In our setting instead of manipulation by participants, the main design consideration is that of minimizing the potential for deceptive behavior on the part of principals, removing any opaque or manipulable features in the randomization. This leads to an additional and nontrivial challenge: designing a procedure that incorporates substantial combinatoric complexity through the constraints, where the randomization procedure is transparent and credible to the general public. In this sense, the paper is related to Akbarpour and Li (forthcoming) who examine a designer’s credibility problems in implementing auction rules.

3. Application Background

The UCL is the most prestigious worldwide club competition in football. Its importance within Europe is similar to that of the Superbowl in the United States, though with stronger global viewership figures (details below). The tournament is played annually between late June and the end of May by the top teams from 57 national associations across Europe, featuring most of the sports’ star players.

See also Hylland and Zeckhauser (1979) and Bogomolnaia and Moulin (2001), each of which considers assignments with individual choice, whereas our paper focuses on alternative randomization procedures from the designer’s point of view.

While our paper primarily serves as an application for the Budish et al. (2013) result, it also contributes to a literature on optimal tournament design. For example, see Dagaev and Sonin (2018), Guyon (2018, 2015), Ribeiro (2013), Scarf and Yusof (2011), Scarf et al. (2009), and Vong (2017).

See also Bó and Chen (2019) on the importance of simplicity and transparency in a historical random assignment for civil-servants in Imperial China.
In the 2017 season, 6.8 million spectators attended UCL matches, with more than 65,000 at the final game.\textsuperscript{5} In addition to the in-stadium audience the tournament has vast media exposure through television and the Internet. The UCL final game is globally the most-watched annual sporting event, where the 2015 final had an estimated 400 million viewers across 200 countries, with a live audience of 180 million. For comparison, Super Bowl viewership in the United States ranged from 103 to 112 million over the past three years.\textsuperscript{6}

Revenue for UEFA is primarily generated (98.5 percent) by selling broadcasting and commercial rights for its club and national team competitions (such as the UCL, the UEFA Europa League, and the UEFA Super Cup). In the 2017 season, 50 percent of the generated revenue (2.8 billion euro) was distributed to the UCL participants as either prize money for tournament progression (60 percent) or financial benefits distributed through a market pool (40 percent).\textsuperscript{7}

Fixed payments of 12.7 million euro (as of the 2017 season) were received by each of the 32 clubs that qualified for the group stage, where an additional performance bonus of 1.5 (0.5) million euro was awarded for a group stage win (draw). Beyond the group stage, teams are given additional payments for reaching each of the subsequent tournament stages. Teams that advanced to the R16 (in the 2017 season) garnered an additional 6 million euro, quarter-finalists 6.5 million more, semi-finalists 7.5 million more, and finalists a further 11 million (with the winner receiving a 4.5 million bonus). While prize money is fixed, benefits based on the market-pool share are in proportion to the value of the TV market for each club’s games. Among the biggest winners of the 2017 season were Juventus, the runner-up with tournament revenue of 110 million euro (a 46:54 split between prize money and market pool), Leicester City, a quarter-finalist with 82 million euro (a 40:60 split), and the 2017 European champion Real Madrid with 81 million euro (a 64:36 split).

Introduced in 1955 as a European Champion Club’s Cup (and consisting only of the national champion from each association) the tournament has evolved over the years to

\textsuperscript{5}Since each UCL season spans across two calendar years, for clarity and concision we refer to a particular season by the year of its final game; so 2019 would indicate the 2018–19 season.

\textsuperscript{6}Furthermore, the UCL proves a massive success on social media. In the 2017 season, the UCL official Facebook page became the worlds’ most followed for a sporting competition with 63 million fans, 300 million video views, and 98 million interactions over the 2017 UCL final.

\textsuperscript{7}The first-order beneficiaries are clubs participating in the group stage and onward. The remaining UEFA revenue is distributed among second- and third-order beneficiaries, which are clubs participating in the qualifying rounds and non-participating clubs, respectively. The solidarity payments made to the latter teams are distributed via national associations and allocated for the most part to youth training programs.
admit multiple entrants from each national association (at most five).\textsuperscript{8} The last major change to the tournament’s design took place in the 2004 season. As such, in our empirical analysis we focus on the 2004–19 seasons (the last completed season at the time of writing).

Since the 2004 season, the UCL consists of a number of pre-tournament qualifying rounds followed by a group and then a knockout stage, similar in format to the World Cup, but played concurrently with the national associations’ leagues. In the group stage, 32 teams are divided into eight groups of four.\textsuperscript{9} Beginning in September each team plays the other three group members twice (once at home, once away). At the end of the group stage in December, the two lowest-performing teams in each group are eliminated, while the group winner and runner-up advance to the knockout stage. The knockout stage (except for the final game) follows a two-legged format, in which each team plays one leg at home, one away. Teams that score more goals over the two legs advance to the next round, where the remaining teams are eliminated.\textsuperscript{10}

The focus of our paper is on the assignment problem of matching the 16 teams at the beginning of the knockout phase into eight mutually disjoint pairs.\textsuperscript{11,12} If the problem consisted simply of matching two equal-sized sets of teams under the bipartite constraint, the assignment could be conducted with two urns (one for group winners, one for runners-up) by sequentially drawing team pairs without replacement. However, the presence of the group and association constraints prohibits such a simple mechanism for two reasons. First, after drawing a team to be partnered, the urn containing eligible partner draws must not contain any directly excluded teams. Second, a match with a non-excluded partner must not force an excluded match at a later point in the draw. While the first concern is easy to address, the second one requires a more-complicated combinatoric inference.

\textsuperscript{8}For more details regarding the format changes, see Table 3 in the Appendix.
\textsuperscript{9}Prior to the 2015 season, seeding in the group stage was entirely determined by the UEFA club coefficients calculated based on clubs’ historical performance, with the titleholder being automatically placed in Pot 1. Starting from the 2016 season, the titleholder together with the champions of the top-seven associations based on UEFA country coefficients are placed in Pot 1. The remaining teams are seeded to Pots 2-4 based on club coefficients. The eight groups are then assembled by making sequential draws from the four pots with a restriction that teams from the same association cannot be placed in the same group, enforced in a similar way to the R16 match we detail in the paper.
\textsuperscript{10}Technically, the scoring rule is lexicographic over total goals, and goals away from home. A draw on both results in extra time, followed by a penalty shootout if a winner is still not determined.
\textsuperscript{11}In principle, a similar analysis could be conducted for the group-stage assignment problem. However, for tractability purposes, we focus on the R16 draw in isolation.
\textsuperscript{12}The quarter- and semi-final draws are free from seeding as well as the association constraint, and as such are conducted in a standard fashion by drawing balls from an urn without replacement.
For illustration, consider the following dynamic draw from two urns. The first urn contains teams $A$, $B$ and $C$; the second teams $d$, $e$ and $f$. Suppose that the match-ups $Ad$ and $Be$ are directly excluded by the constraints. An initial draw from the first pot selects team $A$; as such, the process directly excludes $d$ as a potential match partner for $A$. Assume $f$ is selected from the second urn (containing $e$ and $f$) to create the $Af$ pair. In the second round of the draw, suppose $C$ is chosen from the first urn; since $C$ has no directly excluded partners, a draw from the second urn could be over $d$ and $e$. However if $Cd$ were formed, $B$ would have no valid partner in the third round, as $Be$ is directly excluded. This implies that for the process to work, in the second round $C$ must be indirectly excluded from matching to $d$.

Although this logic is easy to follow in a three-to-three matching, with eight teams on each side and many more constraints, the combinatorics become involved. While matchings could be formed via fully computerized draws, UEFA instead opts to make the randomization transparent through urn draws. The dynamic draw procedure UEFA developed randomizes the R16 tournament matching as follows: (i) eight blue balls representing eight runners-up are placed in the first urn and one runner-up ball is drawn without replacement; (ii) a computer algorithm determines the maximal feasible set of group winners that could match with the drawn runner-up given the constraints and all previous draws; (iii) white balls representing the feasible group winner matches are placed into a second urn, where one is drawn; (iv) a pairing of the two drawn teams (one winner and one runner-up) is added to the aggregate R16 matching. This procedure repeats until all eight matches are formed. In what follows, we refer to the above algorithm as the constrained dynamic $R$-to-$W$ draw, where $W$ and $R$ indicate the sets of group winners and runners-up, respectively.

Key features of the UEFA procedure are its simplicity and transparency with respect to the interim draw, and each specific urn randomization. While representatives from each R16 team attend the draw, the event also attracts substantial attention from the media and general public. The 20-minute draw ceremony is streamed live by UEFA over the Internet and broadcast by many national media companies. Examining the viewership figures for the Internet stream, the most recent UCL R16 draw for the ongoing 2020 season attracted 970,000 viewers on UEFA.tv alone.\footnote{A full rerun of the 2020 UCL R16 draw ceremony is available on UEFA’s YouTube channel.} While this figure is small relative to game-day TV audiences, it is a huge viewership when considering centralized assignment.

In the absence of the association constraint, the tournament has 14,833 possible R16 matchings, where each same-nation exclusion significantly reduces the number of valid
Across the 16 most-recent seasons, the number of valid assignments ranged from 2,988 in the 2009 season through 6,304 in 2011 to 9,200 in 2006. We graph the relationship between the number of possible matchings and the number of same-nation exclusions implied by the association constraint in Figure 1. While the number of valid assignments is not purely a function of the number of exclusions (it depends on their arrangement too) the relationship in question can be approximated by a linear function that decreases by 1,400 matchings for each same-nation exclusion.

4. Theory for the Current Procedure

We now turn to the theory for the current procedure, where we describe a generalized version of the dynamic mechanism used by UEFA to draw assignments satisfying the constraints. After characterizing the effective lottery over matchings induced by the dynamic draw, we show that the randomization in question is distinct from simpler static assignments. A political constraint also excludes Russian teams from being drawn against Ukrainian teams. In what follows, we re-interpret this restriction as an extended association constraint.

In order to calculate the number of valid assignments in each season we proceed in three steps. First, we create a set of all assignments satisfying the bipartite constraint, where this set has $8! = 40,320$ possible matchings. In the second step, we delete the 25,487 assignments that violate the group constraints. Finally, from the remaining 14,833 assignments, we delete those that violate the association constraints.

See Table 6 in the Appendix for the constraints in the seasons 2004–19.
implementations, as well as from some dynamic variants. Finally, we demonstrate that
the constraints enforced by UEFA satisfy the Budish et al. (2013) separability condition,
which allows us to simplify our search for better assignment mechanisms on the more-
computationally tractable space of expected assignments.

4.1. **Constrained Dynamic Draw.** Let $W = \{w_1, w_2, ..., w_K\}$ and $R = \{r_1, r_2, ..., r_K\}$ denote
the sets of group winners and runners-up, respectively. Let $V$ be the set of all possible
perfect (exhaustive one-to-one) matchings between $W$ and $R$. We examine a random as-
signment mechanism $\psi: 2^{V} \rightarrow \Delta V$ that takes as input $\Gamma \subseteq V$ (a set of admissible matchings)
and provides as output a probability distribution over the elements of $\Gamma$. A generalized
version of the dynamic draw employed by UEFA proceeds as follows:

**Algorithm** ($\Gamma$-constrained $R$-to-$W$ dynamic draw). Given an input set of admissible match-
ings $\Gamma \subseteq V$, the algorithm selects a matching $\psi(\Gamma)$ in $K = |W|$ steps.

**Initialization:** Set $R_0 = R$, and $\Gamma_0 = \Gamma$.

**Step-k:** (for $k = 1 \text{ to } K$)

(i) Choose $R_k \in R_{k-1}$ through a uniform draw over $R_{k-1}$;

(ii) Choose $W_k \in W_k := \{w \in W | \exists V \in \Gamma_{k-1} \text{ s.t. } R_k w \in V\}$ (the set of admissible partners for
$R_k$) through a uniform draw;

(iii) Define the currently unmatched runners-up $R_k = R_{k-1} \setminus \{R_k\}$, and valid assignments
given the current draw $\Gamma_k = \{V \in \Gamma_{k-1} | R_k W_k \in V\}$.

**Finalization:** After $K$ steps the algorithm assembles a vector of $K$ runner-up–winner pairs,
$v = (R_1 W_1, ..., R_K W_K)$, where the realization of $\psi(\Gamma)$ is $\{R_1 W_1, R_2 W_2, ..., R_K W_K\} \in \Gamma$.

This dynamic procedure as an interim output produces a sequence of $K$ matches, $v$. In
order to characterize the probability of a specific matching $V$ we define: (i) $P(\Gamma)$, the set
of possible sequence permutations for matching $V$; and (ii) $W_k(v)$, the set of admissible
match partners for runner-up $R_k$ selected at Step-$k$($i$) in the permutation $v{17}$

**Proposition 1.** Under the $\Gamma$-constrained $R$-to-$W$ draw the probability of any matching $V \in \Gamma$
is given by

$$Pr\{\psi(\Gamma) = V\} = \frac{1}{K!} \sum_{v \in P(\Gamma)} \prod_{k=1}^{K} \frac{1}{W_k(v)}.$$  

{17}That is, for the permutation $v = (R_1 W_1, ..., R_K W_K)$ the set of partners at step $k$ is $W_k(v) := \{w \in W | \exists V \in \Gamma \text{ s.t. } R_k w \in V \text{ and } \land_{j=1}^{k-1} (R_j W_j \in V)\}$.  


Proof. For any matching \( V \in \Gamma \), each of the \( K! \) possible permutations of \( V \) has strictly positive probability, so \( \Pr \{ V \} = \sum_{v \in P(V)} \Pr \{ v \} \), which can be rewritten using the chain rule as
\[
\Pr \{ V \} = \sum_{v \in P(V)} \prod_{k=1}^{K} \left( \Pr \{ R_k | v_{k-1} \} \cdot \Pr \{ W_k | R_k, v_{k-1} \} \right),
\]
where \( v_{k-1} \) denotes the matches selected in steps 1 through \( k-1 \). Since the randomization is fair at each step, \( \Pr \{ R_k | v_{k-1} \} \) simplifies to \( \frac{1}{K-k+1} \) and \( \Pr \{ W_k | R_k, v_{k-1} \} = \frac{1}{W_k(v)} \).

Proposition 1 shows that the mechanism’s probability distribution over \( \Gamma \) requires \( K! \times |\Gamma| \) calculations. Even though the cardinality of \( \Gamma \) can be substantially lower than \( K! \), the exact computation of \( \Pr \{ V \} \) involves between \( K! \) and \( (K!)^2 \) steps, and can be taxing even for our application with \( K = 8 \).

Though the above assignment rule is combinatorically involved, the UEFA draw procedure has three useful features. First, all draws are conducted using an urn, and thus each individual randomization is fair/uniform. Second, the number of possible realizations in each draw is always less than eight, implying that each individual draw is easy to comprehend, with a clear urn composition. Finally, all nontrivial elements of the draw conducted opaquely by the computer (the calculation of the set of valid partners at each step) can be checked ex post, and thus the draw procedure is fully verifiable by more-sophisticated viewers. Consequently, as long as the urn draws are conducted fairly and publicly,\(^\text{18}\) it is not possible for the designer to cherry pick realizations, inoculating the mechanism against corruption on the part of managers and principals.

Given the characterization in Proposition 1, one question is the extent to which the above calculation can be simplified. Defining two randomization mechanisms as distinct if they induce different probabilities over the matchings in \( V \), we can show that:

**Proposition 2.** The \( \Gamma \)-constrained dynamic \( R \)-to-\( W \) draw is distinct from:

(i) A uniform draw over \( \Gamma \);

(ii) A \( \Gamma \)-constrained dynamic procedure that fairly draws admissible pairs;

(iii) The \( \Gamma \)-constrained \( W \)-to-\( R \) dynamic draw.

**Proof.** See Appendix for counter-examples. \( \square \)

\(^{18}\) Unlike many state lotteries which use mechanical randomization devices to draw urn outcomes, the UEFA draw is conducted by human third-parties (typically famous footballers). Pointing to football fans’ distrust in the process, the human draw has led to plausible allegations of UEFA rigging draws with hot/cold balls (here made by a former FIFA president Sepp Blatter in an interview with Argentine newspaper *La Nacion* on June 13th, 2016).
The first two parts of Proposition 2 are essentially negative results, indicating that our environment is not equivalent to algorithmically simpler fair draws over complete matchings or individual matches, where the third part shows that the procedure is asymmetric. While the main takeaway from Proposition 2 is negative, it does demonstrate three potentially constructive design channels, analogous to the reversal of the proposing sides in the National Resident Matching Program algorithm detailed in Roth and Peranson (1997).\textsuperscript{19}

4.2. Reducing the Assignment Problem’s Dimension.} Above we characterize a generalized version of the dynamic draw employed by UEFA to assemble the R16 matching. Within this family of randomization procedures, the actual UEFA draw defines the admissible matchings \( \Gamma \) via a set of match exclusions \( H \subset R \times W \), where the overall exclusion set \( H = H_A \cup H_G \) is the union of the association-level exclusions \( H_A \) and group-level exclusions \( H_G \). The precise set \( H \) varies across seasons depending on the group-level assignment and the composition of teams in the R16. The admissible matching set for the UEFA implementation of the constrained dynamic draw is defined by

\[
\Gamma_H := \{ V \in \mathcal{V} | V \cap H = \emptyset \},
\]

where the draw induces the random matching \( \psi(\Gamma_H) \).

While our paper first analyzes the effects of constraints in \( H \) on expected draw outcomes, we also examine the extent to which fairer random assignments might exist. To aid us in this endeavor we employ the core result in Budish et al. (2013) that guarantees the existence of an equivalent randomization over assignments in \( \Gamma_H \) for every feasible expected assignment. This allows us to relax the constrained assignment problem over discrete final matchings to one of finding expected assignments allowing for fractional (and continuous) assignment in the analysis.

First, notice that any assignment \( V \) can be rewritten as a matrix \( X(V) \in \{0, 1\}^{K \times K} \) with a generic entry \( x_{ij}(V) = 1 \{ r_i w_j \in V \} \) indicating whether or not runner-up \( r_i \) is matched to winner \( w_j \). Since \( V \) represents a perfect matching between \( R \) and \( W \), \( X(V) \) is a rook-matrix where each row and column have exactly one unit-valued entry with all other entries equal to zero. Second, for any random draw over \( \Gamma_H \), the expected assignment matrix is defined as \( A := \mathbb{E}X(V) = \sum_{V \in \Gamma_H} \Pr\{V\} \cdot X(V) \), with the generic entry \( a_{ij} \) representing the probability of the \( r_i w_j \) match.

An expected assignment matrix \( A \) in our setting satisfies the matching constraints if:

\textsuperscript{19}However, while distinct, we later show that in our particular setting the three draw procedures lead to only marginally different outcomes. Nevertheless, a uniform draw over \( \Gamma \) has a computational advantage for approximating assignment probabilities in fractions of second.
(i) Each entry $a_{ij}$ can be interpreted as the probability of $r_iw_j$ being part of $V$ ($\forall i,j : 0 \leq a_{ij} \leq 1$);

(ii) Excluded entries have zero probability ($\forall r_iw_j \in H : a_{ij} = 0$);

(iii) Each row and column can be interpreted as the marginal probability distribution for the respective team ($\forall i,j : \sum_{k=1}^{K} a_{kj} = \sum_{k=1}^{K} a_{ik} = 1$).

While satisfying these matching constraints is clearly a necessary condition for any expected assignment resulting from a randomization over the feasible assignment set $\Gamma_H$, the following indicates it is also sufficient.

**Proposition 3 (Implementability).** For any expected assignment matrix $A$ satisfying the matching constraints there exists an equivalent randomization in $\Delta \Gamma_H$.

**Proof.** The matching constraints can be grouped into two distinct sets: (i) the union of the singleton constraints and the $K$ row constraints; and (ii) the $K$ column constraints. As such the matching constraints satisfy the bihierarchy condition in Budish et al. (2013, Theorem 1) and the result follows. \(\square\)

An implication of Proposition 3 is that for any maximization problem over $\Delta \Gamma_H$ (a space with $O(K!)$ degrees of freedom), if we can express the objective in terms of expected assignments, then it is without loss of generality to consider maximization over the space of expected assignment matrices satisfying the constraints (a space with $O(K^2)$ degrees of freedom). Therefore, for our specific UEFA application with $K = 8$, the theorem reduces the degrees of freedom from 2,000–10,000 across the sixteen seasons we look at to 30–40.

A trivial corollary to the above is:

**Corollary 1.** Any expected assignment matrix satisfying the matching constraints is implementable by randomizing over a finite collection of $\Gamma_j$-constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw mechanisms $\{\psi(\Gamma_j)\}_{j=1}^{J}$, with each feasible assignment set $\Gamma_j \subseteq \Gamma_H$.

**Proof.** Set $J = |\Gamma_H| < K!$ and for each entry $V_j \in \Gamma_H$ set $\Gamma_j = \{V_j\}$. By Proposition 3, there exists a probability $p_j$ of selecting each admissible matching $V_j$ that induces and expected assignment matrix satisfying the matching constraints. The existence follows trivially by setting $Pr\{\Gamma_j\} = p_j$. \(\square\)

Therefore, the quotas for each element $a_{ij}$ are a min and a max of zero for the excluded singletons; a min of zero and a max of one for the non-excluded singletons; a min and a max of one for the row sum; and a min and a max of one for the column sum.
Table 1. Expected assignment matrix for the 2018 R16 draw

<table>
<thead>
<tr>
<th></th>
<th>Basel</th>
<th>Bayern München</th>
<th>Chelsea</th>
<th>Juventus</th>
<th>Sevilla</th>
<th>Shakhtar Donetsk</th>
<th>Porto</th>
<th>Real Madrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manchester United</td>
<td>0 (H_G)</td>
<td>0.148</td>
<td>0 (H_A)</td>
<td>0.183</td>
<td>0.183</td>
<td>0.155</td>
<td>0.148</td>
<td>0.182</td>
</tr>
<tr>
<td>Paris Saint-Germain</td>
<td>0.109</td>
<td>0 (H_G)</td>
<td>0.294</td>
<td>0.128</td>
<td>0.128</td>
<td>0.155</td>
<td>0.148</td>
<td>0.128</td>
</tr>
<tr>
<td>Roma</td>
<td>0.159</td>
<td>0.151</td>
<td>0 (H_G)</td>
<td>0 (H_A)</td>
<td>0.189</td>
<td>0.160</td>
<td>0.152</td>
<td>0.189</td>
</tr>
<tr>
<td>Barcelona</td>
<td>0.149</td>
<td>0.144</td>
<td>0.413</td>
<td>0 (H_G)</td>
<td>0 (H_A)</td>
<td>0.150</td>
<td>0.144</td>
<td>0 (H_A)</td>
</tr>
<tr>
<td>Liverpool</td>
<td>0.159</td>
<td>0.151</td>
<td>0 (H_A)</td>
<td>0.189</td>
<td>0 (H_G)</td>
<td>0.160</td>
<td>0.152</td>
<td>0.189</td>
</tr>
<tr>
<td>Manchester City</td>
<td>0.156</td>
<td>0.148</td>
<td>0 (H_A)</td>
<td>0.183</td>
<td>0.184</td>
<td>0 (H_G)</td>
<td>0.148</td>
<td>0.183</td>
</tr>
<tr>
<td>Besiktas</td>
<td>0.109</td>
<td>0.105</td>
<td>0.293</td>
<td>0.128</td>
<td>0.128</td>
<td>0.150</td>
<td>0 (H_G)</td>
<td>0.129</td>
</tr>
<tr>
<td>Tottenham Hotspur</td>
<td>0.160</td>
<td>0.152</td>
<td>0 (H_A)</td>
<td>0.189</td>
<td>0.189</td>
<td>0.159</td>
<td>0.151</td>
<td>0 (H_G)</td>
</tr>
</tbody>
</table>

Note: Probabilities derived from a simulation (N = 10^6) of the UEFA draw procedure. Exclusion constraints indicated by the group constraint (H_G) and the association constraint (H_A).

While Proposition 3 facilitates the examination of whether better randomizations might exist, Corollary 1 provides us with a practical tool for implementing a randomization by a pre-draw over the input set of admissible assignments. After drawing a subset of the feasible assignments, the previously outlined dynamic matching procedure can be used to realize a specific matching.

5. Analyzing the Constraint Effects in UCL R16

In this section, we first quantify how the current UEFA mechanism affects expected assignments in the UCL R16, and how the spillovers from others’ exclusions impact otherwise equally treated teams. Second, we examine the normative question of whether a fairer randomization mechanism is possible. Finally, after showing that substantially better mechanisms do not exist given current constraints, we discuss the extent to which substantially fairer outcomes can be achieved through a slight relaxation of the constraint set.

5.1. Distortions in the Current UEFA Mechanism. We start our empirical analysis with an illustrative example from the UCL R16 draw in 2018. The expected assignment matrix, \( \hat{A} \), under the current UEFA draw procedure is given in Table 1. Each row represents a group winner, and each column a runner-up, so the row-\( i \)-column-\( j \) cell indicates the probability that the (\( ij \))-pair is selected within the realized R16 matching.\(^{21}\)

\(^{21}\)We calculate all probabilities with a Monte Carlo simulation of size \( N = 10^6 \). At this size, 95 percent confidence intervals for each probability are within ±0.001 of the given coefficient (see Proposition 4 in the Appendix).
The constraints in the 2018 draw are as follows: First, along the diagonal, the probabilities of each match are zero, reflecting the eight group constraints. Second, seven same-nation matches are excluded reflecting the 2018-specific association constraint. Finally, all rows and columns sum to exactly one, as each represents the marginal match distribution for the respective team through the bipartite constraint.

Despite having fair urn draws at each point in time, the likelihoods of two teams playing each other are not uniform due to asymmetry generated by the association constraint. For illustration, consider Paris Saint-Germain (PSG) in 2018, the second row of Table 1. As PSG is the only French team in the R16 in the 2018 season it has no same-nation exclusions and thus, seven feasible match partners. However, the likelihoods of each of the seven match-ups varies substantially—with the probability of PSG playing Chelsea almost three times larger than that of PSG playing either Basel, Shaktar, or Porto (columns 1, 6, and 7).

In what follows, we quantify the effects of distortions induced by the R16 matching constraints using a structural model of game outcomes estimated using historical (and out-of-sample) data from the 2004–19 UCL seasons. By simulating the R16 draw and all subsequent games within the tournament, we show that tournament matching constraints have major effects on teams’ expected earnings and progression probabilities within the tournament. Finally, we outline a fairness objective function designed to isolate the indirect spillovers from enforcing the constraints, and demonstrate that even though the spillovers are substantial, the UEFA draw procedure comes very close to a constrained-best.

5.2. Data and Estimation of Game-Outcome Model. In order to account for variation in teams’ ability while examining potential effects driven by the tournament’s constraints, we estimate a commonly used structural model for football-game outcomes: the bivariate Poisson (Maher, 1982; Dixon and Coles, 1997).

Model. Let \( S_i \) and \( S_j \) be the random variables indicating the number of goals scored by home-team \( i \) and guest-team \( j \) in a given game. In a bivariate Poisson model with parameters \( (\lambda_1, \lambda_2, \lambda_3) \) the realized scoreline \( (s_i, s_j) \) has a joint probability distribution

---

\(^{22}\)The bipartite and group constraints impose symmetric restrictions, leading to an equal probability of matching with every non-excluded partner. Consequently, without the association constraint, the expected assignment would have a one-in-seven chance for each off-diagonal entry.

\(^{23}\)Note that the constraints are not mutually exclusive and consequently, even though the expected assignment is an \( 8 \times 8 \) matrix, it has 34 degrees of freedom. The expected assignment matrices for the R16 draw in the other 15 seasons can be found in the paper’s Online Appendix.
given by
\[
\Pr(S_i, S_j) = \exp \left\{ - (\lambda_1 + \lambda_2 + \lambda_3) \right\} \frac{\lambda_1^{s_i} \lambda_2^{s_j}}{s_i! s_j!} \sum_{k=0}^{\text{min}(s_i, s_j)} \binom{s_i}{k} \binom{s_j}{k} k! \left( \frac{\lambda_3}{\lambda_1 \lambda_2} \right)^k,
\]
where \( \mathbb{E}[S_i] = \lambda_1 + \lambda_3 \), \( \mathbb{E}[S_j] = \lambda_2 + \lambda_3 \) and \( \text{Cov}(S_i, S_j) = \lambda_3 \).

In our specification we follow Karlis and Ntzoufras (2003) and assume that \( \ln \lambda_1 = \mu^t + \eta^t + \alpha^t_i - \delta^t_j \), \( \ln \lambda_2 = \mu^t + \alpha^t_i - \delta^t_j \), and \( \lambda_3 = \rho^t \). In this specification \( \alpha^t_k \) and \( \delta^t_k \) measure the idiosyncratic offensive and defensive abilities for team \( k \) in season \( t \) (larger values indicating greater ability), while \( \mu^t \) denotes the season-specific constant and \( \eta^t \) the season-specific home-advantage parameter.

We estimate the above model via constrained maximum likelihood separately for each of the sixteen seasons \( t \) from 2004 to 2019. For scale identification we impose two sum-to-zero constraints in each season, forcing \( \sum_k \alpha^t_k = \sum_k \delta^t_k = 0 \). For estimation in season \( t \) we rely on game-level data from the group stage in season \( t \) and the group and knock-out stages (except for the final game which is played on a neutral soil) in seasons \( t - 1 \) and \( t - 2 \).

In Figure 2 we graph the estimated parameters (defense on the horizontal axis, offense on the vertical) for the subset of teams reaching the R16 in the 2004–19 seasons, dropping those that fail to get past the group stage. The strongest teams have large positive values for both the offense and defense parameters; see for example Manchester United in 2010 and FC Barcelona in 2011. Conversely, low-performing teams have either a negative value for the offense parameter (AS Monaco in 2015), the defense parameter (AC Milan in 2011), or both (Rangers in 2006). The large mass of teams are low-to-medium-strength with small but positive values over both the offense and defense parameters.

Complementing the figure, Table 2 presents summary statistics for the estimated offense and defense parameters broken out by the realized stage reached within the tournament. We find that the eight teams that progress to the quarter-finals are stronger both offensively and defensively than the teams eliminated in the R16. Similarly, the four teams

\[\text{This results in a total of 408 game-level observations used in the estimation for the 2004 season, 376 observations for the 2005 season, and 348 observations for each season between 2006 and 2019. The differences in the number of observations across years result from a change to the tournament design in the 2004 season, where a second group stage feeding into the quarter-finals was replaced by the R16. Table 7 in the Appendix provides summary statistics for the UCL game-level outcomes in all seasons between 2002 and 2019.}\]

\[\text{Table 8 in the Appendix provides estimates for the constant term } \mu^t, \text{ the home-advantage parameter } \eta^t, \text{ and the correlation coefficient between the number of goals scored by opposing teams } \rho^t \text{ in the seasons 2004–19.}\]
advancing to the semi-finals have better offensive and defensive performance relative to those knocked out in the quarter-finals. A two-sample Kolmogorov-Smirnov test confirms the pattern, indicating that the empirical distributions of the offense and defense parameters in the R16 and the semi-finals are statistically different ($p = 0.004$ for defense, $p < 0.001$ for offense).

5.3. **Constraint effects in the UCL draw.** In what follows, we separately quantify the total and spillover effects from the constraints on two main metrics for teams’ outcomes: their expected prize money and the probability of reaching the semi-final stages of the competition (and beyond). In summary, we find that:

---

As mentioned in Section 3, in addition to the direct prize money, teams’ revenue stem from the market pool share and in-stadium game attendance. These other earnings scale with team’s progression within the tournament, and their local media-markets. Therefore, our prize-money measure serves as a lower-bound
**Result 1.** The association constraint in the R16 generates substantial effects by: (i) altering expected tournament prizes by millions of euro; (ii) significantly affecting the chances of reaching later stages of the tournament; and (iii) creating spillovers to the matching chances of otherwise equally treated teams.

**Evidence:** We combine our model of the exact draw procedure and the estimated bivariate Poisson model to calculate expected tournament outcomes for each R16 team $i$ in season $t$. We conduct our draw simulations twice, once under the current set of UEFA constraints (valid assignments in $\Gamma_{HG} \cup \Gamma_{HA}$), and once under a counterfactual set of constraints that drops the association constraint entirely (valid assignments in $\Gamma_{HG}$).\(^{27,28}\)

First, from a linear regression on the 256 team-year observations (16 teams across 16 seasons) we find that expected prizes are highly predictive of out-of-sample realized prizes ($p < 0.000$), with a correlation coefficient of $\rho = 0.492$.\(^{29}\) In Figure 3 we illustrate this relationship (conditioning on the actual R16 draw outcomes) for the 2018 and 2019 seasons.\(^{30}\)

Second, in order to quantify the importance of the R16 draw, we assess the difference in expected outcomes for each team between a very good R16 draw, and a very bad one. Our simulations produce expected tournament outcomes conditional on each realized R16 draw $V$, $\mathbb{E}[\pi^t_i|V]$. Therefore, given a large number of simulated draws we can approximate the sampling distribution for $\mathbb{E}[\pi^t_i|V]$ over the random matching $V \in \Gamma_H$. In particular, we can measure the range in expected outcomes between the 90th and 10th percentiles of the corresponding empirical distributions.

---

\(^{27}\)In detail, for both the actual and counterfactual calculations we proceed by first drawing $J = 1,000$ R16 matchings, $\{V_j\}_{j=1}^J$, using the relevant $\Gamma_H$-constrained $\mathcal{R}$-to-$\mathcal{W}$ dynamic draw. For each drawn R16 matching $V_j$ we simulate the remaining tournament outcomes $S = 1,000$ times (the R16 home/away games, quarter- and semi-final home/away games, and the final game on neutral soil). Consequently, each season is simulated one million times.

\(^{28}\)Given the double-loop for the simulation we also calculate accurate metrics for the expected prize money conditional on each R16 draw as $\mathbb{E}[\pi^t_i|V_j] := \frac{1}{S} \sum_{s=1}^S \pi^t_i|V_{js}$. The unconditional expected prize is then calculated as $\mathbb{E}[\pi^t_i|\Gamma_H] := \frac{1}{J} \sum_{j=1}^J \mathbb{E}[\pi^t_i|V_j]$.

\(^{29}\)We fix the prize money structure from the 2017 season across all years for compatibility (see Section 3 for more details regarding the distribution of prize money broken up by the stage of the competition).

\(^{30}\)Overall, the expected prizes calculated under the model explain 41 percent of the total variation in realized prizes. Moreover, the remaining variability matches the intrinsic variability we would expect under the estimated Poisson model. As indicated in Figure 3, the model mechanically over-predicts realized prize money for teams that are eliminated in the R16, and under-predicts for those that win the tournament.
Figure 3. Realized prize money versus simulated prize money

Note: Simulated prize money is calculated given the realized R16 UEFA draws. Red dashed lines indicate fitted linear relationships.

In Figure 4(A) we graph (for each team \( i \) in season \( t \)) the range of expected outcomes over the current draw procedure against the team’s unconditional expected prize money \( \mathbb{E}[\pi_t | \Gamma_H] \) (calculated prior to the draw). The figure indicates a strong link between teams’ overall strength, and their susceptibility to good versus bad R16 draws. For teams most likely to be eliminated at the start of the knockout stage (and so expected to earn less than 20 million euro) the difference between a good (90\(^{th}\) percentile) and bad (10\(^{th}\) percentile) draw does not exceed three million euro. Conversely, for teams of intermediate strength (expecting to earn 25–30 million euro) the difference between a good and a bad R16 realization becomes substantially larger (an effect of approximately five million euro). However, the effects dissipate somewhat for the strongest teams (particularly group winners, who are already seeded through the bipartite constraint), falling back down to a range of approximately three million euro for the top-performing teams.

While Figure 4(A) illustrates the variability in expected prize money induced by the R16 draw realization, Figure 4(B) shows a complementary picture of the effects on another metric: the probability of reaching the competition’s semi-finals. We find that for intermediate teams (those with an ex ante chance of reaching the semi-finals between 0.25 and 0.5) a favorable R16 draw increases the chances of advancing to the semi-finals by approximately 20 percentage points.
Panel (A) shows the R16 draw effect in prize money (the prize money difference between the 90th and 10th percentile over the R16 draw outcomes) against the unconditional outcome. Panel (B) replaces prize money as the outcome metric with the probability of reaching the semi-finals of the competition. Red dashed lines indicate fitted quadratic relationships.

While similar results follow from alternative metrics, henceforth we focus on the prize-money effects. While a monetary unit has obvious benefits of being practical and economically comprehensible, it has the added benefit of also being an objective measure for how the tournament organizers weigh different stages of the competition.

Finally, having demonstrated that the model is predictive of realized prize money outcomes, and that the R16 draw has a large economic effect for the participating teams (particularly those of intermediate-ability), we next quantify the degree to which the association constraint distorts tournament outcomes.

For each team $i$ in season $t$ we calculate the association-constraint effect as the difference in expected prizes between the current draw mechanism and the counterfactual draw where we drop the association constraint entirely:

$$\Delta \pi_i^t := \mathbb{E}[\pi_i^t | \Gamma_{HA \cup HG}] - \mathbb{E}[\pi_i^t | \Gamma_{HG}].$$

Teams with a positive value of $\Delta \pi_i^t$ are those benefiting from the association constraint, whereas those with a negative value are being disadvantaged. Across all 16 UCL seasons,
the association-constraint effect has a standard deviation of 0.3 million euro (it is mean-zero by construction within each season) and a range of 2 million euro: a cost of 0.8 million euro to Arsenal in the 2014 season (eliminated in the R16) and a subsidy of 1.2 million euro to Real Madrid in the 2017 season (went on to become European champion). The effects from enforcing the association constraint are therefore substantial.

In order to validate the measured association-constraint effect, we further demonstrate that it is predictive of realized outcomes even after controlling for teams’ ability. For each season $t$ we construct a zero-to-one ability index for the R16 teams using the estimated bivariate Poisson model. By way of example, for the 2018 season, our ability index runs from Shaktar Donets at 0, FC Basel at 0.134 and Besiktas at 0.233, up to Real Madrid at 0.919, Liverpool at 0.999, and Barcelona at 1. Regressing the realized tournament prizes for each team-year observation on the ability index, we extract the fitted residuals as a measure of the prize-money outcome that is orthogonal to teams’ ability. Figure 5 illustrates the relationship between the association-constraint effect $\Delta \pi_t^i$ on the horizontal axis, and the fitted residuals from the regression of realized prizes on teams’ ability on the vertical axis. Even though the measured association-constraint effect explains only 4.7 percent of the total variation in realized prizes after controlling for ability, our model-generated measure is statistically significant at any conventional significance level ($p < 0.001$).

A similar exercise conducted using a probit model, suggests that both the ability index and the association-constraint effect are significant predictors of teams’ success in the final stages of the competition. While a unit shift in the ability index—moving from the worst to the best team—increases the likelihood of a semi-final appearance by 67.3 percent ($p<0.000$), a measured one-million-euro subsidy from the association constraint increases the likelihood by 19.7 percent ($p = 0.004$).

31 For each season $t$ and team $i$, we calculate the average probability that team $i$ wins a game against each of the other R16 teams that season using the bivariate Poisson model as:

$$\omega_t^i = \frac{1}{15} \sum_{j \neq i} \Pr(S_i > S_j).$$

Next, we re-scale the average probabilities to run from zero to one at the season level as $\tilde{\omega}_t^i = \frac{\omega_t^i - \omega_t^{\text{best}}}{\omega_t^{\text{best}} - \omega_t^{\text{worst}}}$ where $\omega_t^{\text{best}}$ and $\omega_t^{\text{worst}}$ are the best and worst values for $\omega_t^i$ in season $t$.

32 We find that a unit increase in the ability index (i.e. going from the worst to the best team in a given season) increases the expected prize money by 17.8 million euro.

33 The association-constraint effects for reaching later stages of the tournament are diminishing in magnitude. Whereas a one-million-euro association-constraint subsidy increases the chances of reaching the quarter-finals by 31.2 percent ($p = 0.001$), the chances of reaching the final are raised by only 9 percent ($p = 0.086$).
Above, we quantify the total effect of the association constraint, consisting of both the direct effect on team $i$ from the $ij$ exclusion as well as the indirect spillover effect from others’ constraints (i.e., how the $jk$ exclusion affects the chances of the $ik$ match-up, $i \neq j$). For illustration, consider the match probabilities for Real Madrid and Barcelona in the 2018 season (see Table 1, column 8 and row 4, respectively). While the Real-Madrid and Barcelona exclusion directly benefits the two Spanish teams (these are two of the three strongest teams that season according to our ability index), Barcelona additionally profits from substantial spillovers generated by six other same-nation exclusions. In particular, consider Barcelona and two other unconstrained group winners, PSG and Besiktas (each with seven potential match partners, rows 2 and 7, respectively). Even though none of the three aforementioned group winners are constrained from matching with the two lowest-performing teams (Basel and Shaktar Donetsk, columns 1 and 6) Barcelona is 1.35 times more likely to match to either of them than are either PSG or Besiktas, the less-constrained group winners.

Holding constant the matching exclusions, distortions caused by the direct effects are an unavoidable consequence. However, the indirect effects have the potential to be ameliorated through better randomizations. In order to quantify the size of the indirect effects of the association constraint we construct a spillover measure, motivated by the idea of
equal treatment of equals (ETE), designed to comparatively assess the matching chances of otherwise equally treated team pairs.

We regard two teams $i$ and $j$ as otherwise equally treated with respect to third team $k$ if both the $ik$ and $jk$ matches are not directly excluded. As such, by construction the measure only compares team pairs that are either both group winners (members of $W$) or both runners-up ($R$). Given a set of match constraints $H$ the set of otherwise equally treated match comparisons is:

$$\Upsilon_H := \{(ik,jk) \mid i,j \in W, k \in R, ik,jk \not\in H\} \cup \{(ki,kj) \mid k \in W, i,j \in R, ki,kj \not\in H\}.$$

We define the $ik$ and $jk$ match-ups as being more equally treated the smaller the distance between the $ik$ and $jk$ likelihoods. For any expected assignment $A$ our ETE objective measures the average absolute difference between all otherwise equally treated team pairings as:

$$Q(A; H) = \frac{1}{|\Upsilon_H|} \sum_{(ik,jk) \in \Upsilon_H} |a_{ik} - a_{jk}|.$$

In Figure 6, we graph the ETE objective for each season $t$ under the current UEFA draw procedure, against the number of same-nation exclusions. Since the relationship in question is strongly positive, we conclude that the association constraint substantially distorts
match chances of otherwise equally treated teams. Specifically, the estimated slope coefficient from a regression of $Q(\hat{A}^t)$ on the number of same-nation exclusions suggests an average wedge in match-likelihoods of approximately 5 percentage points for every ten exclusions.\textsuperscript{34} This represents a relative swing of up to a third for unconstrained teams. Although this result points to quantitatively large spillovers even after accepting the constraints’ direct effect, we next show that there is only limited scope to ameliorate the spillovers through better randomization procedures.

5.4. Near-Optimality of the Current Procedure. A natural question raised by Result 1 is whether there exists a randomization procedure that generates less distortions than the one currently employed. In this section we provide the following largely negative answer:

**Result 2.** While the UEFA mechanism is not optimal given the constraints, it comes very close to the optimal mechanism when considering the ETE spillover measure.

**Evidence:** We demonstrate Result 2 through Proposition 3 in Section 4.2. The proposition states that a feasible assignment-producing mechanism exists for every feasible expected assignment matrix satisfying the constraints. This result substantially simplifies the problem, by reducing the effective degrees of freedom by two orders of magnitude, allowing for a computationally tractable optimization over the expected assignment $A_t^\star$. The main result we arrive at is that the optimal solution $A_t^\star$ is not substantially better in terms of the ETE metric than the current dynamic draw procedure for any season between 2004 and 2019.\textsuperscript{35}

The optimal expected assignment solves the following optimization problem:

$$A_t^\star := \arg \min_{A} Q(A; H^t),$$

subject to the matching constraints:

(i) $\forall ij \in H^t : a_{ij} = 0$; (ii) $\forall ij : 0 \leq a_{ij} \leq 1$; (iii) $\forall i : \sum_k a_{ik} = \sum_k a_{ki} = 1$,

where $H^t$ denotes the set of association and group match exclusions in season $t$ and $Q(\cdot)$ is the ETE spillover measure.

\textsuperscript{34}In the absence of any same-nation exclusions our measure is zero by construction; we therefore estimate relationships without a constant.

\textsuperscript{35}One potential objection is that we are not using the right objective function $Q(\cdot)$. While we are amenable to suggestions for other objectives, we have additionally tried minimizing the square differences between match probabilities, the differences between the maximal and minimal positive-probability matches for each team, as well as a measure based on the Kullbeck-Leibler divergence. None of these showed economically meaningful gains from optimization, where the interpretation of the objective became harder than the “average difference” in match likelihoods interpretation for our chosen ETE measure.
In Figure 7(A) we graph the ETE spillover measure for the optimal expected assignments $A_t^*$ against the spillover measure for the expected assignments $\hat{A}_t$ under the current procedure, across the 2004–19 seasons. While some improvement is possible across the tournament years, the gains are marginal (on average a less than 10 percent relative reduction in the size of the spillovers).\footnote{The size of the reduction is given by the estimated slope coefficient from a regression of the spillover measure under the optimal randomization against the current procedure for all years $t = 2004, ..., 2019.$
}

Against the small potential benefits, there are large prospective implementation costs associated with modifying the existing procedure, as constructive mechanisms that produce the optimal expected assignments $A_t^*$ are potentially quite complex in comparison to the current procedure.\footnote{See Online Appendix B to Budish et al. (2013) for a construction.
} While Corollary 1 provides a channel through which the optimal mechanism might be implemented—a pre-draw where the organizers randomize the feasible matching set $\Gamma$—even this would be cumbersome and might engender suspicion from observers. Put against this transparency cost, a reduction in the match-up distortions between equally treated teams from a 5 percent average difference under the current procedure to a 4.5 percent difference for an optimal procedure seems marginal.\footnote{While there may exist a simple modification of the current matching mechanism that would improve upon the current procedure, none of the distinct mechanisms detailed in Proposition 2 achieve such an end (see Figures 10–12 in the Appendix). Therefore, simple modifications such as changing the order of the draw to $W$-to-$R$ or replacing sequential team draws with draws of admissible team-pairs do not provide improvements over the current procedure.
}

5.5. **Weakening the Constraints.** One response to Result 2 is to accept the current mechanism and the distortions it generates. In this final section, however, we take a different approach, and examine the extent to which gains can be made by weakening the association constraint. Specifically, we investigate the extent to which the association constraint can be partially relaxed while still protecting the tournament from excessive R16 same-nation match-ups. There are several practicable ways in which this could be accomplished, but in what follows, we focus on a procedure that relaxes the association constraint while only marginally modifying the current draw procedure. Specifically, we study an alternative to the current association constraint where we allow at most one same-nation match in the R16. Therefore, we can continue to use the constrained $R$-to-$W$ dynamic mechanism detailed in Section 4.1, with a sole modification to the (now

\footnote{We should note that the inability to improve upon the expected assignments generated under the UEFA mechanism is not driven by a limited scope in moving the expected assignments under the constraints. Taking the 2018 season as a (fairly representative) example, we can obtain any value for our spillover measure from a minimum of 0.03 to an upper bound of 0.32.
}

25
Figure 7. Spillover measures: Counterfactuals versus actual

Note: Red dashed line indicates fitted linear relationship.

As such, the relaxation retains the desirable features of the current mechanism: the randomization is transparent (fair draws from a small-sample urn) and the more-opaque combinatoric check continues to be fully verifiable at all points during the draw.

Under this relaxation we find that:

**Result 3.** Weakening the association constraint to allow for at most one same-association match in the R16 substantially reduces the distortions, while protecting associations from excessive same-nation match-ups. Moreover, as a secondary effect, weakening the association constraint (mechanically) reduces the number of same-nation games in the later stages of the tournament.

**Evidence:** We start by constructing analog results to those presented in Section 5.1. In Figure 7(B), we illustrate the spillover measure for the UEFA draw procedure with the at-most-one same-association constraint set on the vertical axis against that for the UEFA mechanism under the default constraint set. We find that allowing for a single same-nation match in the R16 decreases the total distortions by more than 70 percent. This is
Next, we define the relaxed-constraint effect as

$$\Delta \tilde{\pi}_t^i := \mathbb{E}[\pi_t^i|\tilde{\Gamma}_{HA,HG}] - \mathbb{E}[\pi_t^i|\Gamma_{HG}],$$

where $\mathbb{E}[\pi_t^i|\tilde{\Gamma}_{HA,HG}]$ is the expected prize for team $i$ in season $t$ under the at-most-one same-association draw, and $\mathbb{E}[\pi_t^i|\Gamma_{HG}]$ is the expected prize in the absence of any association constraints as before.\(^{40}\) In Figure 8, we illustrate the relationship between $\Delta \tilde{\pi}_t^i$ and the association constraint effect $\Delta \pi_t^i$ defined in Section 5.1. We conclude that allowing for a single same-association match in the R16 reduces the total prize distortions by 71 percent.\(^{41}\)

The above demonstrates a substantial reduction in the matching distortions with only a slight relaxation of the association constraint. However, there are presumably nontrivial costs associated with allowing for same-nation R16 matches. Relaxing the association

\(^{40}\)We numerically calculate $\mathbb{E}[\pi_t^i|\tilde{\Gamma}_{HA,HG}]$ via the same Monte Carlo simulation method used in Section 5.1 and described in footnotes 27 and 28.

\(^{41}\)The estimated slope coefficient from a regression of the relaxed-constraint effect on the current-constraint effect, across all R16 teams in all seasons between 2004 and 2019, is equal to 0.29.
constraint as we have done leads to a single same-nation match-up in the R16 in approximately six out of every ten tournaments. In seasons with at least six same-nation exclusions, this ratio increases to seven-in-ten. Since the association constraint is imposed intentionally, UEFA likely has a clear underlying preference for the tournament to be primarily an international competition in its early stages.\footnote{This, however, cannot be determined, since “the identification of design constraints is usually more difficult because they are rarely communicated by the organizers” (Csató, 2018).}

As a final point in favor of our relaxation approach, we show that, perversely, imposing same-nation exclusions at earlier stages of the tournament has the effect of increasing the likelihood of same-nation match-ups in the subsequent rounds. Using our estimated model of goal outcomes and the at-most-one same-association match mechanism, we assess the predicted change in the number of same-nation matches in later stages of the tournament. We find that for every same-association pairing generated in the R16, there is a 0.10 reduction in the same-nation games in later stages of the tournament.\footnote{The effect would be larger if we additionally accounted for the same-nation exclusions in the group stage that precedes the R16.}

In Figure 9 we illustrate these two compensating effects in all seasons between 2004 and 2019.
6. Conclusion

We document a constrained-assignment problem—one with huge public interest, and with millions of euro at stake from the outcome—where the randomization mechanism is primarily focused on transparency and credibility, to both participants and to the general public. This is in contrast to many market-design solutions that are focused on more-quantifiable objectives: efficiency, fairness, strategic compatibility, etc. While many assignment procedures will choose rules that achieve or maximize these theoretical objectives, the resulting algorithms can seem opaque; to the extent that participants can misunderstand inherent features of the design (for example, not using dominant strategies in deferred acceptance, see Rees-Jones and Skowronek, 2018). In such situations, managers and principals may choose to focus instead on a transparent (but ad hoc) procedure; especially, when the designer lacks credibility with the mechanism’s participants.

In the present paper we ask the following questions: What are the losses from a design focused on transparency? How does a particular design compare to an optimal one? In our application, we illustrate how the tools of market design can be brought to bear on these questions. In our assessment of the UEFA draw mechanism we show that the enforced matching constraints significantly distort the tournament’s outcomes. However, we also demonstrate that the chosen procedure is very close to a constrained best in terms of equal treatment of otherwise equally treated participants.

Our methodology relies on a recently introduced market-design tool (Budish et al., 2013) that allows us to simplify the domain of analysis without loss of generality, switching to a consideration of expected assignments. This shift in domain not only allows us to more compactly specify a clear and easy-to-interpret objective, more importantly, it makes optimization possible by reducing the number of degrees of freedom by two orders of magnitude. Through this shift we are able to show that the UEFA mechanism is remarkable: it solves a complex combinatoric problem, but in a way that is both transparent and comprehensible to the general public and fully verifiable by more-sophisticated third parties. Not only that, the developed procedure comes very close to achieving a first-best outcome under the constraints.

References


Roth, Alvin E, “The economist as engineer: Game theory, experimentation, and computation as tools for design economics,” Econometrica, 2002, 70 (4), 1341–78.


Appendix A. Technical Appendix: For Online Publication

A.1. Proof of Proposition 2. The constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw is distinct from: (i) The mechanism that fairly draws over $\Gamma$; (ii) The dynamic mechanism that fairly draws admissible pairs; and (iii) The constrained $\mathcal{W}$-to-$\mathcal{R}$ dynamic draw.

A.1.1. Part (i). Consider a problem of matching $\mathcal{R} = \{a, b, c, d\}$ to $\mathcal{W} = \{e, f, g, h\}$ under the set of constraints $H = \{ae, bf, cg, dh, ah, bg, dg\}$.\footnote{The proof requires at least a $4 \times 4$ market, as a $3 \times 3$ is degenerate with the standard symmetric group constraints and a single asymmetric same-nation exclusion. The proof can obviously be extended to any $n \times n$ market by making the remaining $(n - 3)$ match partners unique through exclusions.} The three resulting feasible matchings are given by $V_1 = \{ag, be, ch, df\}$, $V_2 = \{ag, bh, ce, df\}$, and $V_3 = \{ag, bh, cf, de\}$. Under the fair draw from $\Gamma = \{V_1, V_2, V_3\}$, the probability of $V_1$ is equal to $\frac{1}{3}$. However, under the dynamic $\mathcal{R}$-to-$\mathcal{W}$ mechanism, (and knowing that $a$ is degenerate) the probability of $V_1$ is given by:

$$
\Pr\{V_1\} = \Pr\{be \in V^*\} = \sum_{x \in \mathcal{W}} \Pr\{w_1 = x\} \cdot \Pr\{be \in V^*|w_1 = x\} = \frac{13}{36}.
$$
Hence, the two mechanisms are distinct.\(^{45}\)

A.1.2. Part (ii). Consider a problem of matching \(\mathcal{R} = \{a, b, c, d\}\) to \(\mathcal{W} = \{e, f, g, h\}\) under the set of constraints \(H = \{bf, cg, ch, bg, dh\}\). Among the five resulting feasible matchings, only \(V_1 = \{af, bh, ce, dg\}\) contains the match \(af\).\(^{46}\) Under the dynamic \(\mathcal{W} \rightarrow \mathcal{R}\) mechanism \(\Pr\{V_1\} = \Pr\{af \in V^*\} = \frac{161}{864}\). However, under the dynamic \(\mathcal{R} \rightarrow \mathcal{W}\) mechanism \(\Pr\{V_1\} = \Pr\{af \in V^*\} = \frac{55}{288}\). Hence, the two mechanisms are distinct.

A.1.3. Part (iii). Consider a problem of matching \(\mathcal{W} = \{a, b, c, d\}\) to \(\mathcal{R} = \{e, f, g, h\}\) under the set of constraints \(H = \{bf, cg, ch, bg, dh\}\). Among the five resulting feasible matchings, only \(V_1 = \{af, bh, ce, dg\}\) contains the match \(af\). Under the dynamic draw of pairs \(\Pr\{V_1\} = \Pr\{af \in V^*\} = \frac{59}{308}\). However, under the dynamic \(\mathcal{R} \rightarrow \mathcal{W}\) mechanism \(\Pr\{V_1\} = \Pr\{af \in V^*\} = \frac{55}{288}\). Hence, the two mechanisms are distinct.

A.2. Additional Results.

A.2.1. Simulation Errors.

**Proposition 4.** Simulating the mechanism \(10^6\) times leads to 95 percent confidence intervals smaller than \(\pm 0.001\).

*Proof.* Assignments are independent draws from a fixed distribution with a probability of selecting assignment \(V\) given by \(f(V)\). The probability that the particular match \(ab\) is selected is given by \(p_{ab} = \sum_{V \in M(ab)} f(V)\) where \(M_{ab} := \{V \in \Gamma \mid ab \in \mu\}\) is the set of matchings which include \(ab\). We simulate the vector \(8^2\)-vector \(\hat{p}\) where each element in \(\hat{p}_{ab}\) is calculated from the \(N\) independent simulation assignments \((\hat{V}_i)_{i=1}^N\)

\[
\hat{p}_{ab} := \frac{1}{N} \sum_{i=1}^N 1\{ab \in \hat{V}_i\}.
\]

The vector \(\hat{p}\) has the obvious property that \(E(\hat{p}) = p\). We can use the central-limit theorem to show that \(\sqrt{n}(\hat{p} - p) \xrightarrow{D} N_{64}(0, \Omega)\) where the variance-covariance matrix \(\Omega\) has a generic

\(^{45}\)The proof becomes more cumbersome but still goes through if we removed the \(dg\) exclusion that forces \(ag\) to be degenerate.

\(^{46}\)A similar counterexample can be constructed with the standard group restriction enforced, but would require a \(5 \times 5\) market; We omit it for tractability and instead, focus on a \(4 \times 4\) sub-market.

\(^{47}\)The four other matchings are: \(V_2 = \{ae, bh, cf, dg\}\); \(V_3 = \{ag, bh, ce, df\}\); \(V_4 = \{ag, bh, cf, de\}\); \(V_5 = \{ah, be, cf, dg\}\).
element given by:

$$\omega_{ab,cd} = \Pr\{ab \land cd\} - \Pr\{ab\} \Pr\{cd\},$$

which can be estimated by

$$\hat{\omega}_{ab,cd} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{ab, cd \in \hat{V}_i\}} - \hat{p}_{ab} \hat{p}_{cd}.$$

However, given a simulation-size of $N = 10^6$, a conservative estimates (as $\omega_{ab,ab} \leq \frac{1}{4}$) for the 95 percent confidence interval for each probability $p_{ab}$ is given by $\hat{p}_{ab} \pm \frac{1.96}{2000} = \hat{p}_{ab} \pm 0.001$. □
### Table 3. Format of the post-qualifying stages of the UCL between 1956 and 2019

<table>
<thead>
<tr>
<th>Season(s)</th>
<th>1st knockout phase</th>
<th>Group stage</th>
<th>2nd knockout phase</th>
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<tbody>
<tr>
<td></td>
<td>K1</td>
<td>K2</td>
<td>G1</td>
</tr>
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</tr>
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<td>1968–1991</td>
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<td>2004–2019</td>
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</table>

*Note:* K1 and K2 denote the number of teams competing in the 1st and 2nd knock-out round; G1 and G2 in the 1st and the 2nd round of the group stage; R16 in the R16, QF in the quarter-final, SF in the semi-final, and F in the final game.

### Table 4. Number of teams from each association participating in the UCL R16 by season

<table>
<thead>
<tr>
<th>Season</th>
<th>TOP5</th>
<th>BEL</th>
<th>CYP</th>
<th>CZE</th>
<th>DEN</th>
<th>GRE</th>
<th>NED</th>
<th>POR</th>
<th>RUS</th>
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*Note:* BEL indicates Belgium, CYP Cyprus, CZE Czech Republic, DEN Denmark, ENG England, ESP Spain, FRA France, GER Germany, NED the Netherlands, ITA Italy, POR Portugal, RUS Russia, SCO Scotland, SUI Switzerland, TUR Turkey, UKR Ukraine. TOP5 denotes English, Spanish, French, German, and Italian associations together.
Table 5. Number of same-nation exclusions generated by each national association in the UCL R16 by season

<table>
<thead>
<tr>
<th>Season</th>
<th>TOP5</th>
<th>POR</th>
<th>RUS</th>
<th>Total</th>
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Mean 2.8 2.8 1.0 2.5 1.7 5.5 5.6

Note: (1) indicates a constraint generated by FC Zenit (RUS) and FC Dynamo Kyiv (UKR).

Table 6. Same-nation exclusions in the UCL R16 by season

<table>
<thead>
<tr>
<th>Season</th>
<th>TOP5</th>
<th>POR</th>
<th>RUS</th>
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</table>

Note: In a (m|n)-pair m indicates the number of seeded (group stage winners) teams and n the number of unseeded (group stage runners-up) teams. m × n is the total number of exclusions generated by a given association.
Table 7. Summary statistics for the number of goals scored in the UCL

<table>
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<tr>
<th>Season</th>
<th># Games</th>
<th>Average</th>
<th>Std. Dev.</th>
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<td></td>
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<tr>
<td>2019</td>
<td>124</td>
<td>1.71</td>
<td>1.23</td>
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Note: From the group stage onward except for the final game played on a neutral ground.

Table 8. Estimated bivariate Poisson model coefficients by season

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<tr>
<th>Season</th>
<th>$\mu^t$</th>
<th>$\eta^t$</th>
<th>$\rho^t$</th>
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<td>Mean</td>
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Note: $\mu^t$ denotes the constant term, $\eta^t$ the home-effect parameter, and $\rho^t$ the correlation coefficient between the number of goals scored by the two opposing teams in season between 2004 and 2019.
Figure 10. Spillover measures: Uniform draw over feasible assignments versus current UEFA mechanism (2004–19)

Note: Red dashed line indicates fitted linear relationship.

Figure 11. Spillover measures: Dynamic draw over admissible pairs versus current UEFA mechanism (2004–19)

Note: Red dashed line indicates fitted linear relationship.
Figure 12. Spillover measures: Dynamic $\mathcal{W}$-to-$\mathcal{R}$ mechanism versus current UEFA mechanism (2004–19)

Note: Red dashed line indicates fitted linear relationship.