Problem 2. Probability of exactly 3 primes in five rolls of a die.
Since the primes are 2, 3 and 5 (one does not count since a prime must have two distinct divisors),
there is a 50 percent chance of a prime on any single roll of a die.
Since throws of a die are independent, the simple multiplication rule applies:
the probability of the three primes coming first followed by two non-primes is:
\[
\Pr(PPPNN) = 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 0.03125 \quad \text{(or 1/32)}
\]
Any other arrangement of 3 primes and 2 non-primes will have the same probability, and there are
\(\binom{5}{3}\) combinations:
\[
\frac{5!}{3! \times 2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} = 10
\]
Hence the desired probability is 0.3125 or 31.25 percent (or 10/32)
Note that if the problem were to find the probability of 3 or more primes in five rolls, we would
also consider the chance of
- getting exactly 4 primes: 4/32
- getting exactly 5 primes: 1/32
The probability of getting 3 OR 4 OR 5 primes is: 10/32 + 4/32 + 1/32 = 15/32.

Problem 4. Pennsylvania Lottery
The daily number has three digits, so can run from 000 to 999 for a total of 1000 possible numbers.
There is a 10 percent chance of getting a number less than 100 on any given night.
Given independence of draws on successive nights, the probability of getting a number less than 100 six
or seven times in seven draws IN A PARTICULAR ORDER (say less than 100 six times in a row, and more
than 99 the final time) is:
For six numbers less than 100 and one greater than 99:
\[
0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.9 = 0.000009
\]
Since there are 7 possible places for the 0.9, we multiply by \(\binom{7}{1}\) to get \(0.0000063\) for the probability of
EXACTLY 6 numbers less than 100.
Since there is only one way to get 7 numbers less than 100 in 7 draws, the probability of this outcome is
\(0.0000001\) (0.1 to the seventh power).
\[
\Pr(6 \; \text{OR 7 numbers under 100}) = \Pr(\text{exactly 6 numbers under 100}) + \Pr(\text{exactly 7 numbers under 100}) = 0.0000064
\]
In EcLS, the function to use is the binomial probability mass function, where you must supply:
the number of successes, the number of trials, and the probability of a success on any one trial.
\(\text{binomial-pmf 6 7 0.1) = 0.0000063 \quad \text{and (binomial-pmf 7 7 0.1) = 0.0000001.}\)
Problem 5. Hunting Moby Dick

There is a probability of $\frac{2}{3}$ that the one small boat sent out daily to harpoon Moby Dick will be sunk on any given day. If you send out 4 small boats on 4 successive days:

You have an expected value of $4 \times \frac{2}{3} = 2.67$ boats sunk over the four days.
The variance of the results will be $4 \times \frac{2}{3} \times \frac{1}{3} = \frac{8}{9}$ and the SD = $\sqrt{\frac{8}{9}} = 0.9428$ -- these results should be expressed in your report to Captain Ahab that we expect to lose about $2.67 \pm 0.94 = 1.73$ to $3.61$ boats every four days.

The problem actually asked is the probability of losing 3 or 4 boats which equals the probability of losing EXACTLY 3 plus the probability of losing EXACTLY 4.

The probability of the four days seeing three losses (L = loss, N = no loss) in exactly the order LLLN is

\[(\frac{2}{3} \text{ cubed}) \times \frac{1}{3} = \frac{8}{81}.
\]

Since the “no loss” can come on any of the four days, the probability of exactly 3 losses is:

\[C(4,1) \times \frac{8}{81} = 4 \times (8/81) = \frac{32}{81}\]

There is only one way to get four losses, and the probability of this happening is $(\frac{2}{3} \text{ to the fourth power}) = \frac{16}{81}$

Adding the two, the probability of losing three or four boats is $\frac{32}{81} + \frac{16}{81} = \frac{48}{81}$

[The text answer 16/27 is the same result, but the above makes the procedure clearer].

The EcLS command (binomial-dist 4 0.666667) results in the output of all probabilities:

<table>
<thead>
<tr>
<th>N [# heads]</th>
<th>P (X = N)</th>
<th>P(X &lt;= N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0123</td>
<td>0.0123</td>
</tr>
<tr>
<td>1</td>
<td>0.0988</td>
<td>0.1111</td>
</tr>
<tr>
<td>2</td>
<td>0.2963</td>
<td>0.4074</td>
</tr>
<tr>
<td>3</td>
<td>0.3951</td>
<td>0.8025</td>
</tr>
<tr>
<td>4</td>
<td>0.1975</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Moments of the binomial distribution (N = 4, p = 0.667)

Expected value = 2.6667  S.D. = 0.9428
Skewness = -0.3536  Kurtosis = 2.6250

What is the chance of fewer than three boats being lost? (The answer, 0.4074, may be read from the table, but it may be useful to compute it as an exercise).
Problem 6. Multiple Guess Test.

The text gives the correct formula to calculate the result; you can use (binomial-dist 100 0.5) to get the result that the chance is less than one-hundredth of one percent. Note that the probability of getting more than 75 heads is the same as the probability of getting less than 25 tails, and that the printout of the cumulative distribution function does not hit .0001 until 31 or fewer tails.

The histogram of the results indicates how nearly the binomial distribution approaches the normal; the normal distribution with expected value 50 and SD = 5 may be used to calculate probabilities as well. Note that the EV of the binomial = Np = 0.5 * 100 = 50 and the variance of the binomial = Np (1-p) = 100 * 0.5 * (1 – 0.5) = 25, so SD = 5.

Consider the chance of getting 40 heads or fewer. The CDF printout from (binomial-dist 100 0.5) reports that this is 0.0284; the normal approximation to the binomial would require you to

-- standardize the critical value of 40 heads: Z = (40 – EV) / SD = (40 – 50) / 5 = -2.
-- look up the standardized value of 40 in table A-3-1: Pr (Z < minus 2) = 0.228

The normal approximation does not give the exact value computed, but is close enough for most practical purposes.

Problem 8. Lefties. (Numbers changed)

If 30 percent of the population is left-handed, what is the probability of a sample of N people containing fewer than 20 percent lefties?

Let N = 5 (less than 20 percent of 5 = less than 1 means zero people), 10 (less than 2 = zero or one), 20 (less than 4 = 0, 1, 2 or 3), 100 (less than 20 = 19 or less) and 1000 (less than 200 = 199 or less).

Rather than doing this by hand, use EcLS and issue the command:

(binary-cdf 0 5 .3) for the first case.
(binary-cdf 1 10 .3) for the second case = about 15 percent
(binary-cdf 3 20 .3) for the third case
(binary-cdf 19 100 .3) for the fourth case
(binary-cdf 199 1000 .3) for the final case = zero percent

As the sample size increases, the probability of making a big mistake drops substantially.