Bayes problems
Dr. McGahagan - Stat 1040

**Problem 1.** What is the relation between AND, OR and IF?

Specifically, note that in rolling 2 dice (Green and Red, with the Red die being rolled first)
Consider the two events:
1. The two dice show a total of 10
2. The red die shows a 6

A. the prior probabilities:

\[ P(\text{total of 10}) = \]
\[ P(\text{red is a 6}) = \]

B. the posterior probabilities:

\[ P(\text{total of 10} \mid \text{red is a 6}) = \]
\[ P(\text{red is a 6} \mid \text{total of 10}) = \]

C. the join probability:

\[ P(\text{total of 10 AND red is a 6}) = \]

Question 1: Are the two events (total of 10, red is a 6) independent? How do you know?

Question 2: Are the two events exclusive?

Question 3: Can you find a formula for how to get one prior probability from the other?

If it were not possible to directly compute \( P(\text{red is a 6} \mid \text{total of 10}) \), but you were given all the other probabilities above, how could you do so?
Problem 2. We are concerned about large-scale theft from our office supplies at the University of Pittsburgh (all branches, with 10,000 employees). Our concern is not with the occasional pen or box of paper clips, but with computers and copier machines going missing. We think that up to 100 employees may be in on the heists. We decide to give everyone a lie detector test; the lie detector operator assures us that the test is very reliable, and will correctly identify the guilty 90 percent of the time, but because the innocent may be nervous about their taking a few pens, they may test as guilty 20 percent of the time. We want to find the probability someone is guilty given than they fail the lie detector test.

Symbolically, \( P(+ \mid \text{Guilty}) = 0.90 \) and \( P(+ \mid \text{Not guilty}) = 0.20 \)

We also have \( P(\text{Guilty}) = 100/10,000 = 0.01 \) and \( P(\text{Not guilty}) = 0.99 \)

We want to find \( P(\text{Guilty} \mid +) \)

Solution 1: Create a table:

<table>
<thead>
<tr>
<th></th>
<th>GUILTY (G)</th>
<th>NOT GUILTY (NG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test indicates guilt (+)</td>
<td>100</td>
<td>9,900</td>
</tr>
<tr>
<td>Test indicates innocent (-)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total number</td>
<td>100</td>
<td>9,900</td>
</tr>
</tbody>
</table>

Entries in the table will be the joint probabilities \( P(\text{+ and Guilty}), P(\text{+ and Not guilty}), \) etc

Solution 2: use Bayes' theorem:

\[
P(\text{G \mid +}) = \frac{P(\text{+ and G})}{P(+)} = \frac{P(\text{+ \mid G}) P(G)}{P(\text{+ AND G}) + P(\text{+ and NG})}
\]

Hint: your final answer should be \( P(\text{G \mid +}) = 90 / 2070 \)
Royal Oak problem (modified)

“Royal Oak” was the roulette like game presented by de Moivre in a text problem: 32 slots, choose a number, bet £ 1. If you are right, you win £ 27 (and get your £ 1 back); otherwise, you lose the pound.

1. What is the expected value of your winnings after 32 plays?

What is the expected value of one play of the game?

2. Create a box model for the game (peek ahead to the “bomber” problem on the next page of the text)

What is the mean of the tickets you place in the box?

What is their standard deviation?

3. Consider carefully the alternate bet that the “master” is willing to make: betting even money that any given number will turn up at least once in 22 plays of the game.

Calculate the chance that it will.

4. Explain why this does NOT mean that the game is fair to the players. (What is the difference between the master's bet and the players' bet?)

5. Suppose the master is willing to bet that the same number will turn up exactly three times in the 22 plays of the game -- but only if you put up £ 50 to his £ 1. Should you take the bet? What are the odds of the master winning?
Arrangements:

We have a few people in search of a limited number of offices:
let's call them Ron and Rick and John and Herman and Michelle and Newt and Mitt.
(Any similarity to actually living people is purely coincidental). The offices they expect to have are:
President, Vice President, State, Treasury, Defense, Attorney General and Secretary of Energy.

1. How many arrangements are there for possible assignments of 7 people to 7 offices?
   (hint: factorials come naturally here)

2. Suppose the field of candidates is much larger (in reality of course the president might well not
   appoint many of his competitors to office, and would search a wider field of candidates). Let there be
   the same 7 offices, but 2000 contenders for the offices, with 1000 men and 1000 women. The choice is
   made randomly from among the contenders.

   What is the probability of selecting exactly 4 women and 3 men for the seven offices?

Hints:
1. Assume that the odds of anyone being chosen stay the same on each draw,
2. you must consider two things:
   a. The probability of selecting any SINGLE arrangement of 4 women and 3 men, say:
      WWWWMMM
      Simple multiplication rule will work under assumption (1).
   b. The number of arrangements. Apply the binomial formula: (total)! / (successes)! (failures)!
      (where you can define success as selecting a woman -- or as selecting a man. The result will be
      the same.