Problem 1. Abundance, direction of trade and PPFs

Based on Penn World Tables 6.2, for 2003 (US dollars at Purchasing Power Parity, base year 2000)

- Canadian labor force = 17 million
- Canadian capital stock = $ 2,125 billion
- Canadian K/L = $ 125,000
- Canadian GDP per capita: $ 30,000

- Indian labor force = 467 million
- Indian capital stock = $ 3,085 billion
- Indian K/L = $ 6,600
- Indian GDP per capita: $ 3,200

Note that although India actually has a larger capital stock than Canada, its labor force is so much larger that Canada is (by a wide margin) the capital intensive country. We would of course expect Canada to export capital-intensive goods and India to export labor-intensive goods.

Be sure to be able to draw the PPFs, price lines and trade triangles. A trick to make your graph neat is to draw the price line first, as if it were a Ricardian PPF, and then fit the PPF under it. If you place your labor intensive good on the X-axis, your price line for India should show low prices for labor intensive goods reflecting the abundant labor resources (and hence a relatively flat line), for Canada a steep line and higher Y-intercept.

After trade, the international price line will have an intermediate slope, and both production and consumption will adjust in each country.

Alan Deardorff's Glossary of International Economics (http://www-personal.umich.edu/~alandear/glossary/) has a nice diagram under the heading “Trade and Transformation Curve Diagram”; as drawn, the curve would look like the Indian PPF (with labor intensive good on the X-axis); draw a steeper PPF for Canada. Place your mouse arrow over the term “Autarky” to see the impact of trade on India.

Problem 2. Leontief's Paradox.

The “paradox” is that:

- US exports had a K/L ratio of 2.55 million dollars / 182 person-years = $ 14,000 per person-year
- US imports had a K/L ratio of 3.1 million dollars / 170 person-years = $ 18,000 per person-year

If US exports had a K/L ration of 2.55 million dollars / 100 person years = $ 25,500 per person-year, there would be no paradox.

Be aware of the possible explanations of the paradox (factor-intensity reversal, land abundance, unbalanced trade, trade barriers, skill levels, and – as the Indian import of textiles section at the end of the chapter suggests, Harvey Leibenstein's X-inefficiency explanation).
See the handout on percentage changes attached to the previous chapter.
With a Cobb-Douglas production function, we can show that

\[
\Delta (PQ) = \Delta (wL) + \Delta (rK)
\]

will reduce to:

\[
PQ \% \Delta P = wL \% \Delta w + rK \% \Delta r
\]

for each good.

Assuming (as in the text problem) that in the computer industry we have

\[
PQ = 100, wL = 50 \text{ and } rK = 50, \text{ and that the price does not change, we have}
\]

\[
0 \% = 50 \% \Delta w + 50 \% \Delta r \text{ which we will rearrange to give } \% \Delta w = - \% \Delta r
\]

and in the shoe industry, where the price increases by 50 \%, we have

\[
100 (0.5) = 5 \% \Delta w + 95 \% \Delta r
\]

\[
50 = -5 \% \Delta 5 + 95 \% \Delta r \text{ (be sure to watch the signs in this problem !)}
\]

\[
50 = 90 \% \Delta r
\]

or \% \Delta r = 50 / 90 = 0.5556 or 55.6 percent.

Hence \% \Delta w = \text{ negative } 55.6 \text{ percent.}

Workers lose, and capitalists win (return to capital rises by a greater percentage than price), consistently with the predictions of Stolper-Samuelson

Note that the wage share of income in the computer industry was 50 percent of the value of the output
and in the shoe industry only 5 percent, so that the computer industry was labor intensive (presumably skilled
labor intensive), and the shoe industry capital intensive. The exponent on labor in the Cobb-Douglas production
function would have been 0.50 in the computer industry, and 0.05 in the shoe industry.

For another exercise, try the values: in the computer industry \( wL = 40, rK = 60 \), and this time the computer
industry price increases by 10 percent.

Computer industry:

\[
100 (0.1) = 40 \% \Delta w + 60 \% \Delta r
\]

Shoe industry:

\[
0 = 5 \% \Delta w + 95 \% \Delta r
\]

\[
\% \Delta w = -95 / 5 \% \Delta r = 19 \% \Delta r \text{ resubstitute this into the computer industry equation to get:}
\]

\[
10 = -40 * 19 * \% \Delta r + 60 * \% \Delta r
\]

\[
10 = -700 \% \Delta r
\]

\[
\% \Delta r = -1 / 70 = \text{ negative } .0143 \text{ or the return to capital falls by } 1.4 \text{ percent}
\]

\[
\% \Delta w = +19 / 70 = 0.2714 \text{ or the return to labor increases by } 27 \text{ percent.}
\]

A rise in the price of the labor intensive good results in wages rising by a greater percentage, and the return to
capital falling absolutely.
Problem 5. The Stolper-Samuelson Theorem, Graphically

Consult text figures 4-6 and 4-8, where the wage/rental ratio is on the vertical axis and the labor/capital ratio is on the horizontal axis. Note the logic behind the graph:

(In the equation below Lbar = endowment of labor, Kbar = endowment of capital, Kc = capital in computers, Ks = capital in shoes, and similarly for Lc and Ls.

\[
\begin{align*}
\frac{L_{\text{bar}}}{K_{\text{bar}}} &= \frac{L_c + L_s}{K_{\text{bar}}} = \frac{L_c}{K_{\text{bar}}} + \frac{L_s}{K_{\text{bar}}} = \frac{L_c * K_c}{K_{\text{bar}}} + \frac{L_s * K_s}{K_{\text{bar}}}
\end{align*}
\]

The relative demand for labor is the weighted average of the L/K ratios in each industry, with the weights being:

\[
\text{Kc/Kbar and Ks/Kbar (the fraction of total capital in each industry)}
\]

There is some adjustment in each industry toward more capital intensive forms of industry if the wage/rental ratio rises (that is, if labor is relatively more expensive, businesses throughout the economy will try to use less of it in relation to capital). These are shown as light blue lines in the text figures. Note that the position of these lines is determined by the technology of each industry, so these lines will not shift if the price of a good changes.

Note also that the capital intensive industry will have a lower L/K ratio, so it will be the bottom light blue curve.

But an increase in the price of shoes (as in the text version of problem 3 above) will shift the proportion of capital in the industry where the price has risen – Ks will increase, as well as of course Ls. The weighted average of the two light blue curves will have a greater weight Ks/Kbar on the labor-capital ratio – which means the relative demand curve will shift down towards the Ls/Ls curve.

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Stolper-Samuelson Theorem

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[Graph showing the wage/rental ratio on the y-axis and the labor/capital ratio on the x-axis, with light blue lines representing adjustments in each industry, and the relative demand curve shifting down.]
Problem 5 note.

On the graph above, the industry L/K curves are in blue, with the equation for the shoe industry being \( w/r = 50 - L_s/K_s \) and the equation for the computer industry being \( w/r = 100 - L_c/K_c \). At any L/K ratio, the wage/rental ratio in the computer industry would be greater than the wage/rental ratio in the shoe industry – another way of saying that the computer industry is assumed to be labor intensive.

The overall Lbar/Kbar ratio is shown by the vertical line at L/K = 40.

The solid red line indicates what the weighted average would be if capital were equally divided between the two industries – \( w/r = 75 - L/K \); the dashed red line would represent the situation after a rise in the price of shoes, leading to the migration of capital to the shoe industry. I assume that Kc/Kbar = 0.25 and Ks/Kbar = .75, to obtain the dashed line \( w/r = 60 - L/K \).

Clearly the result of the shift is that the wage/return ratio will fall (in this case from 35 to 20).

Note that a straight line relationship within industries is unrealistic – the curved lines in the text are more accurate – but it is easier to draw and attach specific numbers to.

Problem 6. Russia – importer of capital-intensive goods

In a simple Heckscher-Ohlin model, if Russia is an importer of capital-intensive goods it must be labor-abundant and have a comparative advantage in labor-intensive goods. (Actually, Russia is a major exporter of oil, and natural resource abundance is the key in the real world).

The Stolper-Samuelson Theorem suggests that the Russian real wage will rise as a result of the increase in the price of the labor-intensive goods it is exporting; the real rental rate of capital would fall in Russia.

Since capital would be the loser from trade, Russian capitalists would favor protectionism.