Practice Problems on Sections 13.6, 13.7

1. Find an equation of the tangent plane to the given parametric surfaces at the given point:
   a) \( \mathbf{r}(u, v) = (u + 2v)\mathbf{i} + (u^2)\mathbf{j} + (2v - u)\mathbf{k} \) at \( P = (-1, 1, -3) \)
   b) \( \mathbf{r}(u, v) = (\arctan(v))\mathbf{i} + (v^3 - u^2)\mathbf{j} + (\cos(u - 2v))\mathbf{k} \) at \( P = (\frac{\pi}{4}, -3, 1) \)

2. Set up the iterated integral representing the surface area for each of the following surfaces (do not evaluate):
   a) the surface is given by \( \mathbf{r}(u, v) = < u + v, uv, uv^2 >, 1 \leq u \leq 2, 3 \leq v \leq 4 \).
   b) the surface is the part of the sphere \( x^2 + y^2 + z^2 = 4 \) between two planes \( z = -1 \) and \( z = \sqrt{2} \).
   c) the surface is the part of the graph \( z = xy \), where \( x \geq 0, y \geq 0, x + y \leq 1 \).

3. Set up the iterated integral representing the given surface integral:
   a) \( \int \int_S (x + e^{yz}) \cdot dS \), where \( S \) is given by \( \mathbf{r}(u, v) = < u + v, uv, uv^2 >, 1 \leq u \leq 2, 3 \leq v \leq 4 \).
   b) \( \int \int_S \cos(xy^2z) \cdot dS \), where \( S \) is the part of the plane \( 2x + 3y + z = 12 \) in the first octant.
   c) \( \int \int_S (x + 2y + 3z) \cdot dS \), where \( S \) is the part of the surface \( z = x^2 + y^2 - 4 \) that lies below the \((x, y)\)–plane.
   d) \( \int \int_S \frac{x}{y^{1/2}} \cdot dS \), where \( S \) is the part of the surface \( z = x^y \) that lies above the region in \((x, y)\)–plane bounded by the curves \( x = 4 - (y - 2)^2 \) and \( y = x \).

4. Set up the iterated integral representing the given surface integral:
   a) \( \int \int_S \mathbf{F} \cdot dS \), if \( \mathbf{F}(x, y, z) = 2yi + 3xj - \sin(yz)k \), and \( S \) is given by \( \mathbf{r}(u, v) = < u + v, uv, uv^2 >, 1 \leq u \leq 2, 3 \leq v \leq 4 \).
   b) \( \int \int_S \mathbf{F} \cdot dS \), if \( \mathbf{F}(x, y, z) = yi - zj + xk \), and \( S \) is the part of the graph \( z = xy \), where \( x \geq 0, y \geq 0, x + y \leq 1 \).
   c) \( \int \int_S \mathbf{F} \cdot dS \), if \( \mathbf{F}(x, y, z) = 2xi - \sin(yz)k \), and \( S \) is the upper half of a unit sphere.

5. Evaluate \( \int \int_S \mathbf{F} \cdot dS \), where \( S \) is the surface of the part of the unit ball in the first octant with the normal pointing outward, if \( \mathbf{F}(x, y, z) = xyi + yzj + xzk \).