This test consists of 10 problems. All work must be shown in order to get credit. Please write legibly and explain your logic by words whenever appropriate. If more space is needed, write on the back of the pages and/or ask for more paper. No calculators, notes, books etc. are allowed.

**Problem 1.** Find the dimensions of a cheapest box, if the box has the volume $3/2$ ft$^3$, the material for the bottom costs $.50/ft^2$, the material for the sides costs $.20/ft^2$, and the material for the lid costs $.10/ft^2$.

**Problem 2.** a) Find the absolute maximum and the absolute minimum values of the function $f(x, y) = x^2 + 6x + y^2$ on the curve $x^2 + 2y^2 = 1$.

b) Find the absolute maximum value of the function $f(x, y, z) = x^3 + y^3$ on the sphere $x^2 + y^2 + z^2 = 1$.

**Problem 3.** Evaluate the integral.

$$
\int_{0}^{3} \int_{\sqrt{\eta}}^{3} \sin(x^3) \, dx \, dy
$$

**Problem 4.** Find the center of mass of the lamina in the shape of the circle $x^2 + y^2 \leq 4$ and the density $\rho(x, y) = y + 2$.

**Problem 5.** Evaluate the triple integral of $f(x, y, z) = z^3 + 1$ over the solid bounded by the surfaces $y = 2x^2 + 2z^2$ and $y = x^2 + z^2 + 1$.

**Problem 6.** Find and classify all critical points of the function $f(x, y) = e^x \cdot y - e^y$.

**Problem 7.** Evaluate the integral of the function $f(x, y, z) = z - 1$ over the ball $x^2 + y^2 + z^2 \leq 4$.

**Problem 8.** Set up an iterated integral for the volume of the solid below the plane $x + y + z = 6$ and above the region in the $xy$–plane bounded by $y = 0$, $y = x - 2$, and $y = \sqrt{x}$.
Problem 9. Use the transformation \( x = v - u, \ y = v \) to change the variables in the integral
\[
\int_{R} \int f(x, y) \, dA,
\]
where \( R \) is the region bounded by the lines \( y = 0, \ y = 2, \ y = x, \ y = x + 2. \)

Problem 10. Find the absolute maximum and the absolute minimum values of the function \( f(x, y) = 2x^3 + 3x^2 - y^2 + 1 \) on the domain bounded by the triangle with the vertices \((-2, -3), \ (0, 1) \) and \((4, -3), \) and the points where the absolute maximum(s) and the absolute minimum(s) occur.