MATH 0240 Midterm Examination II Sample 2

This test consists of 11 problems. All work must be shown in order to get credit. Please write legibly and explain your logic by words whenever appropriate. If more space is needed, write on the back of the pages and/or ask for more paper. No calculators, notes, books etc. are allowed.

Problem 1. a) Find all critical points of \( f(x, y) = x^2y^3 + xy^2 + 2xy. \)
   b) For each critical point, classify the point as a local maximum, a local minimum, or a saddle point.

Problem 2. Find the absolute maximum value of the function \( f(x, y) = x^2 - y \) on the circle \( x^2 + y^2 = 25. \)

Problem 3. Find the center of mass of the lamina that has a constant density and occupies the region \( D, \) bounded by the curves \( x = 1 - y^2 \) and \( y = x + 1. \)

Problem 4. Evaluate the integral.
\[
\int_0^2 \int_{2x}^4 \cos(y^2) \, dy \, dx
\]

Problem 5. Evaluate the integral.
\[
\iint_{x^2+y^2 \leq 4} (x^2 + 3xy) \, dxdy
\]

Problem 6. Find the shortest distance from the surface \( xyz = 1 \) to the origin.

Problem 7. Find the mass of the pyramid bounded by the coordinate planes and the plane \( x+3y+5z = 30, \) if its density is given by \( \rho(x, y, z) = z+2. \)
Problem 8. Evaluate the integral of the function \( f(x, y) = \frac{x}{y^2} - \frac{1}{x^2} \) over the rectangular region \( 1 \leq x \leq 2, \, 2 \leq y \leq 4 \)

Problem 9. Change the order of the integration in the iterated integral

\[
\int_0^1 \int_0^{(2-2x)/(3-3x-\frac{3}{2}y)} \int_0^{(x+5y+7z)} dz \, dy \, dx,
\]

so that the function is integrated first with respect to \( y \), then with respect to \( x \), and then with respect to \( z \).

Problem 10. Use the transformation \( x = \frac{1}{3}(u + v), \, y = \frac{1}{3}(u - 2v) \) to evaluate the integral \( \int_D (6x - 12y) \, dA \), where \( D \) is the triangular region with vertices \( (0, 0), \, (1, 1) \) and \( (-1, 2) \).

Problem 11. Evaluate the triple integral of \( f(x, y, z) = \sin(\sqrt{x^2 + y^2 + z^2}) \) over the top half of the sphere centered at \( (0, 0, 0) \) with the radius 1.