This test consists of 10 problems, each worth 10 points. All work must be shown in order to get credit. Please write legibly and explain your logic by words whenever appropriate. No calculators, notes, books etc. are allowed.

Problem 1. Consider the points $A(0, 0, 0)$, $B(1, 0, 1)$, $C(1, 2, 1)$. Find the area of the triangle $\triangle ABC$.

Problem 2. Find the equation of the plane tangent to the surface 
$$z^2 - 2x^2y + xyz + 5 = 0$$
at the point $(1, 2, -1)$.

Problem 3. Find the linearization $L(x, y)$ of the function
$$f(x, y) = 3\sin(\pi(x^2 + y^2)) + 4$$
at the point $(1, -2)$.

Problem 4. Find the maximum rate of change of $f(x, y) = x^3y^2 - 3x + 2$ at the point $(1, -2)$ and the direction in which it occurs.

Problem 5. Assume that the equation
$$xe^y + y \cdot \sin z + z e^x = 0$$
defines implicitly $z$ as a function of $x$ and $y$. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Problem 6. Given the function
$$f(x, y) = \frac{2x^2 - 3y^2}{4x^2 + 5y^2}.$$
Find
\[ \lim_{(x,y) \to (0,0)} f(x, y) \]
or show that the limit does not exist.

**Problem 7.** Find \( \frac{\partial f}{\partial t} \) for the function \( f(x, y) \) at the point where \( s = 3, t = -1 \) if
\[
\frac{\partial f}{\partial x} = 2x + 3xy, \quad \frac{\partial f}{\partial y} = xe^{y^2},
\]
x(s, t) = s^2 + 2t and y(s, t) = 3t^2 - s.

**Problem 8.** Assume that \( \vec{u} = <\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} > \) and \( \vec{v} = <\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} > \). Also assume that \( D_{\vec{u}} f(1,2) = 2\sqrt{2} \) and \( D_{\vec{v}} f(1,2) = \frac{7}{5}\sqrt{5} \). Find \( f_x(1,2) \) and \( f_y(1,2) \).

**Problem 9.** Find the length of the curve with vector equation,
\[
\vec{r}(t) = <e^t, e^t \sin t, e^t \cos t>
\]
for \( 0 \leq t \leq 2\pi \).

**Problem 10.** Consider the curve given by
\[
\vec{r}(t) = <1 - t, t^2 + 1, \frac{2t^3}{3} + 1 >.
\]
Find the curvature of this curve at the point \( t = 1 \).