Problem 1. Let \( f(x, y) = xy \) be defined on the closed triangular region \( T \) with vertices \((-1, 0), (1, 0), \) and \((0, 1)\). Find the points on \( T \) where \( f \) has its absolute maximum and absolute minimum values, and find these values.

Problem 2. Let \( f(x, y, z) = ze^{x/y} \), and \( x = u + v, \ y = uv^2, \ z = v \).

a) Find the gradient of \( f \) at the point \((u, v) = (1, -1)\).

b) Calculate \( \frac{\partial f}{\partial v} \) and evaluate it at the point \((u, v) = (1, -1)\).

Problem 3. Let \( D \) be the region above the paraboloid \( z = 3 + x^2 + y^2 \) and below the paraboloid \( z = 11 - x^2 - y^2 \). Use the divergence theorem to find the surface integral \( \int \int_S \mathbf{F} \cdot \mathbf{n} \, dS \), where \( \mathbf{F}(x, y, z) = <2x, 2y, -3z> \), \( S \) is the boundary of \( D \), and \( \mathbf{n} \) is the outward unit normal vector to \( S \). Give a complete numerical answer.

Problem 4. a) A wire in the shape of the curve \( W \) described by the position vector \( r(t) = <t, t^3, 1> \), \( 0 \leq t \leq 1 \), has density \( 36yz \) at point \((x, y, z) \) on the curve. Find the mass of \( W \).

b) Evaluate the integral \( \int_W \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y, z) = <x, y - x^3, \sin(x + yz)> \).

Problem 5. Consider the scalar field \( f(x, y, z) = xy - z^2x \).

a) Write the equation of the tangent plane \( T \) to the surface \( f(x, y, z) = 0 \) at the point \( A = (2, 1, -1) \).

b) Calculate the derivative of \( f \) at the point \( A \) in the direction of any non-zero vector \( \mathbf{v} \) laying in the tangent plane \( T \). Justify your answer.

c) Calculate the derivative of \( f \) at the point \( A \) in the direction opposite to the gradient of \( f \) at \( A \).

Problem 6. Let \( D = \{(x, y) : x^2 + y^2 \leq 1\} \), and \( C \) be the boundary of \( D \). Let \( \mathbf{F} \) be the vector field \( \mathbf{F}(x, y) = <-y, x> \). Evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \).

Problem 7. Let \( S \) be the surface of a cylindrical glass described by equations \( x^2 + y^2 = r^2, \ z = 0, \ z = h \) with the bottom \( z = 0 \) being the part of \( S \), but the top \( z = h \) being open and thus not the part of \( S \). Use the divergence theorem to compute the surface integral \( \int \int_S \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F} \) is the vector field \( \mathbf{F}(x, y, z) = < y^2x, xy(x - y), 2xyz> \).

Hint: Cover up the top of the glass.
**Problem 8.** Let $D$ be the region above the paraboloid $z = 1 + x^2 + y^2$ and below the plane $z - 9 = 0$. Use the divergence theorem to find the surface integral $\int \int_{S} F \cdot \mathbf{n} \, dS$, where $F(x, y, z) = <2xy, -y^2, z>$, $S$ is the boundary of $D$, and $\mathbf{n}$ is the outward unit normal vector to $S$.

**Problem 9.** a) Determine whether or not $F = <y^2e^z, 2xye^z, xy^2e^z>$ is a gradient field.

   b) Evaluate $\int_{L} F \cdot d\mathbf{r}$, where $L$ is the straight line segment joining points $(-1, 1, 0)$ and $(1, 1, 0)$.

**Problem 10.** Let $F = <x - y, xz \ln(2 + y), z>$. Use Stokes’ theorem to evaluate the surface integral $\int \int_{S} \text{curl}(F) \cdot dS$, where $S$ is the boundary of the unit cube $\{(x, y, z) : 0 \leq x, y, z \leq 1\}$ with the face in the plane $z = 0$ removed, and $\mathbf{n}$ denotes the outward normal to $S$. 