Calculus 1; Preparation for the Final

Question 1
Simplify the following expressions:
- \(3 \log_3 3 + 2 \log_3 4 - \log_3 3\)
- \(e^{2 \ln 4 + 3 \ln 2}\)

Question 2
Find the inverse functions for the following functions:
- \(y = x^3 - 16\)
- \(y = \frac{2x + 1}{3x + 2}\)

Question 3
Eliminate the parameter \(t\) to find an equation relating \(x\) and \(y\) in each of the following:
- \(x = 1 - t^3, \ y = 1 + t + t^2\)
- \(x = 1 - 4t, \ y = 1 - 5t + 2t^2\)

Question 4
Find the equations of the tangent lines to the following curves at the given point:
- \(y = 1 - 2x^2 + x^4\) at the point \((2, 9)\).
- \(y = \frac{x + 2}{x - 1}\) at the point \((2, 4)\).
Question 5

Using the limit formula: \( f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \), find the following derivatives, from first principles, for each of the following functions:

- \( f'(2) \) where \( f(x) = \frac{x^2 - 4}{x^2 + 4} \).
- \( g'(3) \) where \( g(x) = (x - 3)3^x/3 + x^2 \).

Question 6

Using geometry (not FTC), find the following integrals:

- \( \int_{-2}^{2} (2 + 3t)dt \)
- \( \int_{-2}^{2} (\sqrt{4 - x^2} + 2x + 5)dx \)

Question 7

At time \( t \) a particle moving along a straight line has position \( s(t) \) and velocity \( v(t) \).

- Given that \( s(0) = 0 \) and \( v(t) = 40 - 32t \), find \( s(t) \) and \( s(4) \)
- Instead, given that \( s(0) = 0 \) and \( v(t) = \sin(t) \) find \( s(t) \) and \( s(\pi) \).
Question 8

Find the following limits:

- \( \lim_{x \to 3^-} \frac{|x - 3|}{x^2 - 2x - 3} \)
- \( \lim_{x \to 4^-} \frac{x^2 - 16}{|x - 4|} \)
- \( \lim_{x \to 0} \frac{2^x - 3^x}{x} \)
- \( \lim_{x \to 0} \frac{1 - e^{2x}}{4x} \)
- \( \lim_{x \to 0} \left( \frac{1}{x} - \frac{2}{\sin(2x)} \right) \)
- \( \lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{\tan(x)} \right) \)
- \( \lim_{x \to 0} (1 - x)^{\frac{1}{2}} \)
- \( \lim_{x \to 0} (1 + 2x)^{\frac{3}{2}} \)

Question 9

Let \( h(x) = f(g(x)) \) and \( k(x) = f(x)g(x) \).

Suppose that \( f(2) = 4, f(5) = 3, g(2) = 5, g(5) = 2, f'(2) = 4, f'(5) = -3, g'(2) = -1 \) and \( g'(5) = -3 \).

- Find \( h'(2) \) and \( k'(2) \).
- Find \( h'(5) \) and \( k'(5) \).

Question 10

Let \( y(x) \) be defined implicitly by \( x = 2y + y^3 \).

- Find \( y(3), y'(3) \) and \( y''(3) \).
- Find \( y(12), y'(12) \) and \( y''(12) \).
Question 11

Find the derivative of the following functions:

- \( f(x) = e^{2x} + \sin(x) - \cos(x) + \tan(x) + 5 \ln(x) - 3 \arctan(x) - 4 \arcsin(x) \)
- \( g(x) = (2x + 1)^{11} + \cos(3x - 1) - \sin(2x + 5) + \ln(5x) \)
- \( h(x) = x^{-2x} \)
- \( k(x) = (1 + x)^{2x} \)
- \( m(x) = \sin(\sqrt{1 - x^2}) \)
- \( n(x) = \cos(\sqrt[3]{1 - x^2}) \)
- \( p(x) = \int_0^{5x} \sin(t^3)\,dt \)
- \( q(x) = \int_0^{4x} e^{t^3}\,dt \)

Question 12

Find the following integrals:

- \( \int \left( x^3 - \frac{5}{x} + 4 \cos(x) - 4 \sin(x) + 2 \sec^2(x) \right) \,dx \)
- \( \int \left( x^4 + \frac{10}{x^2} + 3x - \frac{2}{\sqrt{1 - x^2}} + \frac{2}{1 + x^2} \right) \,dx \)
- \( \int_0^1 (3x^2 - 2)(x^3 - 2x + 1)^5 \,dx \)
- \( \int (4x + 3)(2x^2 + 3x - 1)^{11} \,dx \)
- \( \int x \sin(x) \,dx \)
- \( \int x \cos(2x) \,dx \)
• \[ \int \frac{2x + 1}{x^2 - x - 2} \, dx \]

• \[ \int \frac{2x - 1}{(3 - x)(x + 2)} \, dx \]

**Question 13**

A circular tin can of radius \( r \) and height \( h \) centimeters has surface area \( A = 2\pi r^2 + 2\pi rh \) square centimeters and volume \( V = \pi r^2 h \) cubic centimeters.

- If its surface area is \( 150\pi \) square centimeters, what is the maximum volume of the can?
- If instead its volume \( 2000\pi \) cubic centimeters, what is the minimum surface area of the can?

**Question 14**

A truck is traveling East at 120km/hr and a car is traveling north at 40km/hr. At a certain time, the truck is 30 kms East of an intersection and the car is 40 kms North of the intersection. What is the rate of departing between the car and truck at that time?

**Question 15**

A truck is traveling East at 80km/hr and a car is traveling north at 120km/hr. At a certain time, the truck is 50 kms West of an intersection and the car is 120 kms North of the intersection. What is the rate of departing between the car and truck at that time?

**Question 16**

Use Newton’s method with two iterations starting at \( x = 0 \) to estimate the root of the equation \( x^3 + x + 1 = 0 \).

**Question 17**

Use Newton’s method with two iterations starting at \( x = 5 \) to estimate the root of the equation \( x^3 - 25 = 0 \).
Question 18
Find the linear approximation \( L(x) \) for the function \( f(x) = x^{\frac{1}{12}} \) based at the point \( a = 1 \) and use it to estimate \( (1.2)^{\frac{1}{12}} \).

Question 19
Find the linear approximation \( L(x) \) for the function \( f(x) = e^x \) based at the point \( a = 0 \) and use it to estimate \( e^{0.3} \).

Question 20
Find the Riemann sum \( R_4 = \sum_{i=1}^{4} f(c_i)(x_i - x_{i-1}) \) for the integral \( \int_0^{16} x^3 \, dx \) with regular partition points \( x_i = 4i \) for \( i = 0, 1, 2, 3, 4 \) and the middle point rule: \( c_i = \frac{1}{2}(x_{i-1} + x_i) \).

Question 21
Find the Riemann sum \( R_4 = \sum_{i=1}^{4} f(c_i)(x_i - x_{i-1}) \) for the integral \( \int_0^{8} \sqrt{x} \, dx \) with regular partition points \( x_i = 2i \) for \( i = 0, 1, 2, 3, 4 \) and the middle point rule: \( c_i = \frac{1}{2}(x_{i-1} + x_i) \).

Question 22
Let \( f(x) = x^3 - 12x + 11 \), defined for \(-3 \leq x \leq 3\).
Find all local and global maxima and minima for \( f \) and all its inflection points and plot its graph.

Question 23
Let \( f(x) = xe^{-2x^2} \), defined for all \( x \).
Then by differentiating we find:
\[
f'(x) = -e^{-2x^2}(2x - 1)(2x + 1), \quad f''(x) = 4xe^{-2x^2}(4x^2 - 3).
\]
Find all local and global maxima and minima for \( f \) and all its inflection points and plot its graph. Also find the intervals where \( f \) is increasing the intervals where \( f \) is decreasing and the intervals where \( f \) is concave up and the intervals where \( f \) is concave down. Also describe the asymptotes of \( f \) if any.