You are planning a sample survey of small businesses in your area. You will choose an SRS of businesses listed in the telephone book’s Yellow Pages. Experience shows that only about half the businesses you contact will respond.

a. If you contact 150 businesses, it is reasonable to use the Binomial \((150,0.5)\) distribution for the number \(X\) of businesses that respond. Explain why.

It is reasonable to use the binomial distribution because there are a fixed number of observations (150), all the observations are independent, and each observation falls into one of two categories: success or failure. Also the probability of a success is the same for each observation.

b. What is the mean number of businesses that respond to surveys like yours?

\[
\mu = np = 150 \times 0.5 = 75
\]

c. What is the probability that 70 or fewer will respond? (Use the Normal approximation.)

\[
S = \sqrt{np(1-p)} = \sqrt{150 \times 0.5 \times (1 - 0.5)} = 6.12
\]

\[
Z = \frac{70 - 75}{6.12} = -0.82
\]

From table A, \(P(X < 70) = 0.2061\)

d. How large a sample must you take to increase the mean number of respondents to 100?

\[
\mu = np = n \times 0.5 = 100
\]

\[
n = \frac{100}{0.5} = 200
\]

2. A sample of 1000 high school students gained an average of \(\bar{X} = 22\) points in their second attempt at the SAT mathematics exam. The change in score has a Normal distribution with standard deviation \(\sigma = 50\).

a. Give a 95% confidence interval for the mean score gain \(\mu\) in the population of all students.

\[
\bar{X} = 22, \ \sigma = 50 \text{ and } n = 1000
\]

\[
\bar{X} \pm Z \frac{\sigma}{\sqrt{n}} = 22 \pm 1.96 \times \frac{50}{\sqrt{1000}} = [18.9, 25.1]
\]

b. Suppose that a sample of 250 students had produced the sample mean \(x-bar=22\). Would the 95% confidence interval for the mean score gain \(\mu\), be wider or narrower than the one computed above? No computations are necessary.

Wider
c. How large a sample of high school students would be needed to estimate the mean score gain in SAT score $\mu$ to within 2 points with 95% confidence?

\[ m = Z \times \frac{\sigma}{\sqrt{n}} \]

\[
2 = 1.96 \times 50 \quad \Rightarrow \quad n = \left( \frac{1.96 \times 50}{2} \right)^2 = 2401
\]

3. The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures the motivation, attitude toward school, and study habits of students. Scores range from 0 to 200. The mean score for US college students is about 115, and the standard deviation is about 30. A teacher who suspects that older students have better attitudes toward school gives the SSHA to 36 students who are at least 30 years of age. Their mean score is 135. Assume that the standard deviation of the population of older students is 30, and that the 36 students in the teacher's class are a SRS.

a. Write down the hypotheses (in symbol) to test the teacher's statement.

\[ H_0: \mu = 115 \]
\[ H_a: \mu > 115 \]

b. What is the value of the test statistic?

\[ Z = \frac{135 - 115}{\frac{30}{\sqrt{36}}} = 4 \]

c. What is the P-value?

\[ P(Z > 4) \approx 0 \]

d. What is your conclusion?

There is strong evidence to reject the null hypothesis in favor of the alternative hypothesis, that is, older students seem to have a higher mean score.

4. The level of various substances in the blood of kidney dialysis patients is of concern because kidney failure and dialysis can lead to nutritional problems. A researcher performed blood tests on several dialysis patients on six consecutive clinic visits. One variable measured was the level of phosphate in the blood. Phosphate levels for an individual tend to vary normally over time. The data on one patient, in milligrams of phosphate per deciliter (mg/dl) of blood, are: 5.6, 5.1, 4.6, 4.8, 5.7, 6.4.
a. Calculate the sample mean and its standard error. (Hint: the sum of the 6 values given above is 32.2 and its sample variance is 0.4427.)

\[
X = \frac{32.2}{6} = 5.37
\]

\[
\text{Std. Err} = \frac{S}{\sqrt{n}} = \frac{\sqrt{0.4427}}{\sqrt{6}} = 0.2716
\]

b. Give a 90% confidence interval for this patient’s mean phosphate level.

From Table C with d.f=5,

\[
t_{0.05} = 2.015
\]

\[
X \pm t_{0.05,5} \frac{S}{\sqrt{n}} = 5.37 \pm 2.015 \times 0.2716 = [4.819, 5.914]
\]