Lecture 22: Chapter 9, Section 2
Inference for Categorical Variable: Hypothesis Tests

- 4 steps in Hypothesis Test: Posing Hypotheses
- Details of 4 Steps, Definitions and Notation
- 3 Forms of Alternative Hypothesis
- P-Value
- Example with “Greater Than” Alternative

Looking Back: Review

- 4 Stages of Statistics
  - Data Production (discussed in Lectures 1-4)
  - Displaying and Summarizing (Lectures 5-12)
  - Probability (discussed in Lectures 13-20)
  - Statistical Inference
    - 1 categorical: confidence intervals, hypothesis tests
    - 1 quantitative
    - categorical and quantitative
    - 2 categorical
    - 2 quantitative

Three Types of Inference Problem (Review)

In a sample of 446 students, 0.55 ate breakfast.
1. What is our best guess for the proportion of all students who eat breakfast?
   **Point Estimate**
2. What interval should contain the proportion of all students who eat breakfast?
   **Confidence Interval**
3. Do more than half (50%) of all students eat breakfast?
   **Hypothesis Test**

4 Steps in Hypothesis Test About $p$

(First pose question as choice between 2 opposing views about $p$.)

1. Check data production for bias.
2. We summarize with $\hat{p}$, standardize to $z$.
3. Find probability of $\hat{p}$ this extreme.

These correspond to 4 Processes of Statistics.
Example: *Posing Hypothesis Test Question*

- **Background:** In a sample of 446 students, 0.55 ate breakfast. Do more than half of all students at that university eat breakfast?
- **Question:** How can we pose above question as two opposing points of view about \( p \)?
- **Response:**

Example: *Considering Data Production*

- **Background:** In a sample of 446 college students, 0.55 ate breakfast. We want to draw conclusions about breakfast habits of all students at that university.
- **Question:** What data production issues should be considered?
- **Response:** (discussed with confidence intervals)
  - Sampling: ____________________________ (for claims about _____)
  - Study design: _________________________ (for claims about _____)

Also, (for claims about _____) is population \( \geq 10n \)?
And (for claims about _____) is \( n \) large enough?

4 Steps in Hypothesis Test About \( p \)

(First pose question as choice between 2 opposing views about \( p \).)

1. Check data production for bias.
2. We summarize with \( \hat{p} \), standardize to \( z \).
3. Find probability of \( \hat{p} \) this extreme.
4. Perform inference, drawing conclusions about population proportion \( p \).
Behavior of Sample Proportion (Review)

For random sample of size $n$ from population with $p$ in category of interest, sample proportion $\hat{p}$ has

- mean $p$
- standard deviation $\sqrt{\frac{p(1-p)}{n}}$

Hypothesis test: assume pop. proportion $p$ is proposed value $= 0.50$ for breakfast example.

Looking Back: For confidence intervals, we had to substitute sample proportion for unknown $p$.

Example: Summarizing and Standardizing

- **Background:** In a sample of 446 students, 0.55 ate breakfast. Do more than half of all students at that university eat breakfast?
- **Question:** How do we summarize the data?
- **Response:** Summarize with $\hat{p}$. Standardize to $z$.

Looking Back: For confidence intervals, we had to substitute sample proportion for unknown $p$.

So 0.55 is ______ standard deviations above 0.50: pretty unusual.

Example: Estimating Relevant Probability

- **Background:** In a sample of 446 students, 0.55 ate breakfast. Do more than half of all students at that university eat breakfast?
  
  We summarized with $\hat{p} = 0.55$ and $z = \frac{0.55 - 0.50}{\sqrt{0.50(1-0.50)}/446} = +2.11$.

- **Question:** If $p=0.50$, how unlikely is it to get $\hat{p}$ as high as 0.55 (that is, for $z$ to be $\geq +2.11$)?

- **Response:** 68-95-99.7 Rule $\rightarrow$ since $2.11 > 2$, $P(Z \geq +2.11)$ is ______________.

Such a probability can be considered to be __________.

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Illustration of Relevant Probability

Looking Ahead: The relevant probability for testing a hypothesis will be defined as the P-value.

4 Steps in Hypothesis Test About p

(First pose question as choice between 2 opposing views about p.)
1. Check data production for bias.
2. We summarize with \( \hat{p} \), standardize to \( z \).
3. Find probability of \( \hat{p} \) this extreme.
4. Perform inference, drawing conclusions about population proportion \( p \).

Example: Drawing Conclusions About p

- **Background**: In a sample of 446 students, 0.55 ate breakfast. Do more than half of all students eat breakfast? We summarized with \( \hat{p} = 0.55 \) and \( z = \frac{0.55 - 0.50}{\sqrt{0.50(1-0.50) \times \frac{1}{446}}} = +2.11 \)

The probability of \( z \) being +2.11 or higher is less than (1-0.95)+2= 0.025, (fairly unlikely).
- **Question**: What do we conclude about \( p \)?
- **Response**: 

Hypothesis Test About p (More Details)

First state 2 opposing views about \( p \), called null and alternative hypotheses \( H_0 \) and \( H_a \).
1. Consider sampling and study design as for C.I.
2. Summarize with \( \hat{p} \); does it tend in the suspected direction? Standardize to \( z \), assuming \( p = p_0 \) \( (p_0 \text{ is proposed value}) \); consider if \( z \) is “large”.
3. Find prob. of \( \hat{p} \) this high/low/different, called ‘P-value’ of the test; consider if it is “small”.
4. Draw conclusions about \( p \): choose between null and alternative hypotheses. (Statistical Inference)
Definitions

- **Null hypothesis** \( H_0 \) : claim that parameter equals proposed value.
- **Alternative hypothesis** \( H_a \) : claim that parameter differs in some way from proposed value.
- **P-value**: probability, assuming \( H_0 \) is true, of obtaining sample data at least as extreme as what has been observed.

**Looking Back**: We considered the probability, assuming \( p = 0.5 \) cards are red, of getting as few as 0 red cards in 4 or 5 picks.

Notation

- Proposed value of population proportion: \( p_0 \)

Null and alternative hypotheses in test about unknown population proportion:

\[
H_0 : p = p_0 \quad \text{vs.} \quad H_a : \begin{cases} 
  p > p_0 \\
  p < p_0 \\
  p \neq p_0
\end{cases}
\]

**Looking Ahead**: The form of the alternative hypothesis will affect Steps 2, 3, 4 of the test.

Example: What Are We Testing About?

- **Background**: Consider 3 problems:
  - \( \frac{30}{400} = 0.075 \) students picked #7 “at random” from 1 to 20. Is this evidence of bias for #7?
  - Do fewer than half of commuters walk? \( \frac{111}{230} \) of surveyed commuters at a university walked.
  - % disadvantaged in Florida community colleges is 43%. Is Florida Keys College unusual with 47.5% disadvantaged?
- **Question**: In each case, are we trying to draw conclusions about a sample proportion \( \hat{p} \) or a population proportion \( p \)?
- **Response**:___________________

**Looking Ahead**: We’ll refer to sample proportion later, to decide which of two claims to believe about the unknown population proportion.

Example: Three Forms of Alternative

- **Background**: Consider 3 problems:
  - \( \frac{30}{400} = 0.075 \) students picked #7 “at random” from 1 to 20. Is this evidence of bias for #7?
  - Do fewer than half of commuters walk? \( \frac{111}{230} \) of surveyed commuters at a university walked.
  - % disadvantaged in Florida community colleges is 43%. Is Florida Keys College unusual with 47.5% disadvantaged?
- **Question**: How do we write the hypotheses in each case?
- **Response**:___________________

**Looking Ahead**: We’ll refer to sample proportion later, to decide which of two claims to believe about the unknown population proportion.
Definitions

- **One-sided alternative hypothesis** refutes equality with $> \text{ or } <$ sign
- **Two-sided alternative hypothesis** features a not-equal sign

**Note:** For a one-sided alternative, sometimes the accompanying null hypothesis is written as a (not strict) inequality. Either way, the same conclusions will be reached.

Assessing Merit of Data in One-Sided Test

If sample proportion does not tend in the direction claimed by alternative hypothesis in a 1-sided test, there is no need to proceed further.

Example: *When Test Can Be Cut Short*

- **Background:** The moon has four phases: new moon, first quarter, full moon, and last quarter, each in effect for 25% of the time. A neurologist whose patients claimed their seizures tended to be triggered by a full moon found 20% of 470 seizures were at full moon.
- **Question:** Do we need to carry out all 4 steps in the test?
- **Response:**

How to Assess P-Value

**P-value:** probability, assuming $H_0$ is true, of obtaining sample data at least as extreme as what has been observed. How to find P-value depends on form of alternative hypothesis:

- **Right-tailed probability** for $H_a : p > p_0$
- **Left-tailed probability** for $H_a : p < p_0$
- **Two-tailed probability** for $H_a : p \neq p_0$


**P-Value for** $H_a : p > p_0$ **is Right-tailed Probability**

$H_0 : p = p_0$ vs. $H_a : p > p_0$

**P-Value for** $H_a : p < p_0$ **is Left-tailed Probability**

$H_0 : p = p_0$ vs. $H_a : p < p_0$

**P-Value for** $H_a : p \neq p_0$ **is Two-tailed Probability**

$H_0 : p = p_0$ vs. $H_a : p \neq p_0$

**Drawing Correct Conclusions**

Two possible conclusions:

- **P-value small** $\Rightarrow$ reject $H_0$ $\Rightarrow$ conclude $H_a$.  
State we have evidence in favor of $H_a$.  
(not same as proving $H_a$ true and $H_0$ false).
- **P-value not small** $\Rightarrow$ don’t reject $H_0$ $\Rightarrow$ conclude $H_0$ may be true.  
(not same as proving $H_0$ true and $H_a$ false)
**Example: Test with “Greater Than” Alternative**

- **Background**: 30/400 = 0.075 students picked #7 “at random” from 1 to 20.
- **Question**: In general, is \( p > 0.05 \) (evidence of bias?)
- **Response**: First write \( H_0: __________ vs. H_a: _________ \)
  1. Students are “typical” humans; bias is issue at hand.
  2. \( 0.075 > 0.05 \) so the sample did favor #7. If \( p = 0.05 \), \( \hat{p} \) standardizes to \( z = \)
  3. \( P\)-value = __________________
  4. Reject \( H_0 \)? _____ Conclude? __________________

**Lecture Summary**

*Inference for Proportions: Hypothesis Test*

- 4 steps in hypothesis test
  - Checking data production
  - Summarizing and standardizing
  - Finding a probability (\( P\)-value)
  - Conclusions as inference
- Posing null and alternative hypotheses
- Definitions and notation
- 3 forms of alternative hypothesis
- Assessing \( P\)-value
- Example with “greater than” alternative