1. First order ODEs

Separation of variables:

* ODE of the form \( y' = f(x)g(y) \).
* Models: particle motion, exponential growth/decay; logistic model.

Linear equations:

* ODE of the form \( y' + p(x)y + q(x) = 0 \).
* Modeling: personal finance.

2. Second order ODEs

Modeling problems

* Spring vibration;
* LRC circuits.

Linear equations with constant coefficients:

* Homogeneous equations:
  (i) characteristic equations;
  (ii) harmonic oscillation, amplitude-phase formula.
* Inhomogeneous equations: \( y(t) = y_h(t) + y_p(t) \).
  (i) undetermined coefficients, resonance;
  (ii) variation of parameters;
  (iii) forced harmonic oscillation, transient state, steady state.

3. Laplace Transform

Table of Laplace transform for elementary functions (will be provided in the exam).

Basic properties

* Laplace transform of derivatives.
* The first translation theorem \( \mathcal{L}\{e^{ct}f(t)\} = F(s - c) \), partial fractions.
* Discontinuous/piecewise functions, Heaviside function representation, second translation theorem \( \mathcal{L}\{f(t - c)H(t - c)\} = e^{-cs}F(s) \).
* Delta functions.
* Using Laplace transform to solve ODEs, convolutions.

4. Euler’s method

\[ y_{k+1} = y_k + f(t_k, y_k)h. \]
5. **SYSTEM OF ODEs**

Converting higher order differential equations to a first order system.

**Planar linear systems with constant coefficients**

- Homogeneous systems.
  - (i) Eigenvalues and eigenvectors: simple real eigenvalue, complex eigenvalue, repeated real eigenvalue, fundamental set of solutions.
  - (ii) Phase portraits: saddle, nodal source/sink, spiral source/sink, center.

**Planar autonomous systems**

- Equilibria, nullclines.
- Linearization, Jacobian, characterization of the equilibrium points.
- Stability of the equilibrium points.

6. **FOURIER SERIES**

Computing Fourier coefficients on \([-\pi, \pi]\)

Even/odd functions, Fourier sine/cosine series on \([0, \pi]\)

7. **THE HOMOGENEOUS HEAT EQUATION**

Separation of variables

Solving the Dirichlet/Neumann boundary-value problems on \([0, \pi]\).