1. [10 points] Find a vector equation and parametric equations for the line segment that joins \( P(-2, 4, 0) \) to \( Q(6, -1, 2) \).

Vector eq: \( \overrightarrow{r}(t) = (1-t)\overrightarrow{r}_0 + t\overrightarrow{r}_1 \), where \( 0 \leq t \leq 1 \), \( \overrightarrow{r}_0 = \langle -2, 4, 0 \rangle \), \( \overrightarrow{r}_1 = \langle 6, -1, 2 \rangle \)

\[ \overrightarrow{r}(t) = \langle -2+2t+6t, 4-4t-t, 0+2t \rangle \]

\[ \overrightarrow{r}(t) = \langle -2+8t, 4-5t, 2t \rangle, \ 0 \leq t \leq 1 \]

Parametric eq’s:

\[
\begin{align*}
x &= -2+8t \\
y &= 4-5t \\
z &= 2t \\
0 &\leq t \leq 1
\end{align*}
\]

(w/ou the condition \( 0 \leq t \leq 1 \) it is equation of a line, not a line segment)
2. [10 points] Explain why the function \( f(x, y) = x/y \) is differentiable at the point \((6, 3)\). Find the linearization \( L(x, y) \) of the function at that point.

\[
fx = \frac{1}{y}, \quad fy = -\frac{x}{y^2}
\]

\( fx \) and \( fy \) exist and are continuous near \((6, 3)\). Therefore \( f(x, y) \) is differentiable at \((6, 3)\).

\[
fx(6, 3) = \frac{1}{3}, \quad fy(6, 3) = -\frac{6}{9} = -\frac{2}{3}
\]

\[
f(6, 3) = \frac{6}{3} = 2
\]

\[
L(x, y) = f(6, 3) + fx(6, 3)(x-6) + fy(6, 3)(y-3)
\]

\[
L(x, y) = 2 + \frac{1}{3}(x-6) - \frac{2}{3}(y-3)
\]

\[
L(x, y) = \frac{1}{3}x - \frac{2}{3}y + 2
\]
3. (a) [10 points] Find the gradient of $f(x, y) = y \ln x$.

$$\nabla f(x, y) = \left< f_x, f_y \right> = \left< \frac{y}{x}, \ln x \right>$$

(b) [10 points] Evaluate the gradient at the point $P(1, -3)$.

$$\nabla f(1, -3) = \left< \frac{-3}{1}, \ln 1 \right> = \left< -3, 0 \right>$$
(c) [10 points] Find the rate of change of $f$ at $P$ in the direction of the vector $\mathbf{u} = \left(-\frac{4}{5}, \frac{3}{5}\right)$.

$$\nabla f(1, -3) \cdot \mathbf{u} = \langle -3, 0 \rangle \cdot \left\langle -\frac{4}{5}, \frac{3}{5} \right\rangle =$$

$$= (-3)(-\frac{4}{5}) + 0 = \frac{12}{5} = 2.4$$
4. [10 points] Find parametric equations and symmetric equations for the line of intersection of the planes \( x + y + z = 1 \) and \( x + z = 0 \).

Normal vectors to the planes are 
\[
\vec{n}_1 = \langle 1, 1, 1 \rangle, \quad \vec{n}_2 = \langle 1, 0, 1 \rangle
\]
The direction vector \( \vec{v} \) of the line is orthogonal to both \( \vec{n}_1 \) and \( \vec{n}_2 \):
\[
\vec{v} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
1 & 1 & 1 \\
0 & 1 & 0
\end{vmatrix} = \vec{i} - \vec{k} = \langle 1, 0, -1 \rangle
\]
The point \( P_0 (0, 1, 0) \) is on the line (it is on both planes, easy to see).

Vector equation:
\[
F(t) = \langle 0, 1, 0 \rangle + t \langle 1, 0, -1 \rangle = \langle t, 1, -t \rangle
\]
param. eq's: \[
\begin{align*}
X &= t, \\
Y &= 1, \\
Z &= -t
\end{align*}
\]
\[
t = X, \quad t = -Z
\]
Symmetric eq's: \[
\begin{align*}
X &= -Z, \\
Y &= 1
\end{align*}
\]
5. [20 points] Find the absolute maximum and minimum values of the function \( f(x, y) = 3 + xy - x - 2y \) on the set \( D \) if \( D \) is the closed triangular region with vertices \((1, 0), (5, 0), \) and \((1, 4)\).

CP's inside \( D \):
- \( f_x = y - 1 = 0 \Rightarrow y = 1 \)
- \( f_y = x - 2 = 0 \Rightarrow x = 2 \)
- CP: \((2,1)\), \( f(2,1) = 1 \)

On \( AB \) \( y = 0 \) \( 1 \leq x \leq 5 \)
\( g_1(x) = f(x, 0) = 3 - x \) is decreasing function
- \( g_1(x) \) attains max at \( x = 1 \) \( g(1) = 2 \)
- \( g_1(x) \) attains min at \( x = 5 \) \( g(5) = -2 \)

On \( CB \) \( y = -x + 5 \) \( 1 \leq x \leq 5 \)
- \( g_2(x) = f(x, -x + 5) = -x^2 + 6x - 7 \)
- \( g_2'(x) = -2x + 6 = 0 \), \( x = 3 \), \( g_2(1) = -2 \), \( g_2(3) = 2 \), \( g_2(5) = -2 \)

On \( AC \) \( x = 1 \), \( 0 \leq y \leq 4 \), \( g_3(y) = f(1, y) = 2 - y \) (decreasing
- \( g_3(0) = 2 \), \( g_3(4) = -2 \)

Abs max value: \( \boxed{2} \) Abs min value: \( \boxed{-2} \)
6. [10 points] Find and sketch the domain of the function

\[ f(x, y) = \sqrt{y - x} \ln(y + x) \]

\[ D = \{ (x, y) \mid y-x \geq 0, \ y+x > 0 \} \]

\[ = \{ (x, y) \mid y \geq x, \ y > -x \} \]
7. (10 points) Find the scalar and vector projections of \( \vec{b} \) onto \( \vec{a} \) and \( \vec{b} \) onto \( \vec{a} \) if 
\( \vec{a} = \langle 1, 1, 1 \rangle \), \( \vec{b} = \langle 1, -1, 1 \rangle \)

\[ |\vec{a}| = \sqrt{3}, \quad |\vec{b}| = \sqrt{3}, \quad \vec{a} \cdot \vec{b} = 1 \cdot 1 + 1 = 1 \]

\[ \text{comp}_\vec{a} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{1}{\sqrt{3}} \]

\[ \text{proj}_\vec{a} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a} = \frac{1}{3} \langle 1, 1, 1 \rangle = \langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle \]

\[ \text{comp}_\vec{b} \vec{a} = \frac{\vec{b} \cdot \vec{a}}{|\vec{b}|} = \frac{1}{\sqrt{3}} \]

\[ \text{proj}_\vec{b} \vec{a} = \frac{\vec{b} \cdot \vec{a}}{|\vec{b}|^2} \vec{b} = \frac{1}{3} \langle 1, -1, 1 \rangle = \langle \frac{1}{3}, -\frac{1}{3}, \frac{1}{3} \rangle \]
bonus problem [7 points extra] Reduce the equation \(-x^2 + 2y^2 + 3z^2 = 0\) to one of the standard forms, classify the surface and sketch it.

\[ x^2 = 2y^2 + 3z^2 \iff x^2 = \frac{y^2}{\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{z^2}{\left(\frac{1}{\sqrt{3}}\right)^2} \]

a cone along \(x\)-axis:

![Sketch of a cone along the x-axis](image)