Problem 1: (a) Write down the linear programming problem corresponding to the max flow problem in the network below. (b) Also write down the linear programming problem corresponding to the minimum cut in this network. (i.e. the inequalities and equalities as well as the function to be maximized/minimized.) For linear programming of max flow and min cut you can consult the online notes posted in the webpage of the course.

Problem 2: Consider the following linear programming problem: Maximize $-2x_1 - x_2$ subject to the conditions:
\begin{align*}
x_2 - x_1 & \leq 1 \\
x_2 & \leq 2 \\
x_1 + x_2 & \leq 4 \\
(1/2)x_1 - x_2 & \leq 1/2 \\
-x_1 - x_2 & \leq -1
\end{align*}

and $x_1, x_2 \geq 0$. Draw the region defined by the inequalities. Solve this linear programming problem using the picture, i.e. find a solution $x = (x_1, x_2)$ for which $-2x_1 - x_2$ is maximum. Write down the dual linear programming problem. Find the minimum of the dual problem using Strong Duality Theorem.

**Problem 3:** Write down the linear programming problem corresponding to the shortest path problem in the directed graph in Problem 1. No need to solve it.