Slopes

The slope $m$ of the line passing through the points $(x_1, y_1)$ and $(x_2, y_2)$ is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Example: Find the slope of the line passing though the given points:

a) $(-1, 2)$ and $(3, 5)$
   
   \[
   m = \frac{5 - 2}{3 - (-1)} = \frac{3}{4}
   \]

b) $(0, 1)$ and $(2, 6)$
   
   \[
   m = \frac{6 - 1}{2 - 0} = \frac{5}{2}
   \]

c) $(5, 1)$ and $(1, 3)$
   
   \[
   m = \frac{3 - 1}{1 - 5} = \frac{2}{-4} = \frac{1}{2}
   \]

d) $(2, -3)$ and $(-1, -9)$
   
   \[
   m = \frac{-9 - (-3)}{-1 - 2} = \frac{-6}{-3} = 2
   \]

e) $(1, 3)$ and $(4, 6)$
   
   \[
   m = \frac{6 - 3}{4 - 1} = \frac{3}{3} = 1
   \]

f) $(3, 6)$ and $(1, 6)$
   
   \[
   m = \frac{6 - 6}{1 - 3} = \frac{0}{-2} = 0
   \]

g) $(-3, 2)$ and $(-3, 5)$
   
   \[
   m = \frac{5 - 2}{-3 - (-3)} = \frac{3}{0}
   \]
   slope undefined
Examples: Find the slope and the y–intercept of each of the following lines:

a) \(-2x + 3y = 6\)

\[
3y = 2x + 6 \\
y = \frac{2}{3}x + 2
\]

slope \(\frac{2}{3}\)

y–intercept 2

b) \(y = -3x + 2\)

slope –3

y–intercept 2

c) \(5x – 4y = 20\)

\[
-4y = -5x + 20 \\
y = \frac{5}{4}x - 5
\]

slope \(\frac{5}{4}\)

y–intercept –5

d) \(3y + 6 = 0\)

\[
3y = -6 \\
y = -2
\]

slope 0

y–intercept –2

e) \(y = 6\)

slope 0

y–intercept 6

f) \(x = 3\)

slope undefined

no y–intercept
Finding Equations Of Lines

To find the equation of any line, you always need two types of information: slope information and point information.

1. **Point–Slope** \[ y - y_1 = m(x - x_1) \]

2. **Slope–Intercept** \[ y = mx + b \]

3. **Horizontal line** \[ y = b \]

4. **Vertical line** \[ x = a \]

a. The slope of the line passing through two points is \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

b. Slopes of parallel lines are equal.

c. Slopes of perpendicular lines are negative reciprocals.
Examples: Find the equation of the line: (Put answers in slope–intercept form if possible)

a) through (2, 3) with slope \(-\frac{1}{2}\)

\[
y - (3) = -\frac{1}{2} (x - 2)
\]

\[
y - 3 = -\frac{1}{2} x + 1
\]

\[
y = -\frac{1}{2} x + 4
\]

b) through (–4, –1) with slope 2

\[
y - (-1) = 2(x - (-4))
\]

\[
y + 1 = 2(x + 4)
\]

\[
y + 1 = 2x + 8
\]

\[
y = 2x + 7
\]

c) through (–1, 2) and (4, –2)

\[
m = \frac{-2 - 2}{4 - (-1)} = -\frac{4}{5}
\]

\[
y - 2 = -\frac{4}{5} (x - (-1))
\]

\[
y - 2 = -\frac{4}{5} (x + 1)
\]

\[
y - 2 = -\frac{4}{5} x - \frac{4}{5}
\]

\[
y = - \frac{4}{5} x + \frac{6}{5}
\]

d) through (6, –2) and (2, 0)

\[
m = \frac{0 - (-2)}{2 - 6} = -\frac{1}{2}
\]

\[
y - 0 = -\frac{1}{2} (x - 2)
\]

\[
y = -\frac{1}{2} x + 1
\]

e) through (2, 3) and horizontal

\[
y = 3
\]

f) through (–2, 6) and vertical

\[
x = -2
\]
g) with slope 3 and y–intercept 5
\[ y = 3x + 5 \]

h) with slope 2 and x–intercept 7
\[ y - 0 = 2(x - 7) \]
\[ y = 2x - 14 \]

i) through (2, 3) parallel to
\[ 5x + 3y = 1 \]
find slope:  \[ 5x + 3y = 1 \]
\[ y = -\frac{5}{3}x + \frac{1}{3} \]
\[ m = -\frac{5}{3} \]
\[ y - 3 = -\frac{5}{3}(x - 2) \]
\[ y - 3 = -\frac{5}{3}x + \frac{10}{3} \]
\[ y = -\frac{5}{3}x + \frac{19}{3} \]

j) through (0, 0) perpendicular to
\[ 2x + y = 1 \]
find slope:  \[ 2x + y = 1 \]
\[ y = -2x + 1 \]
\[ m = \frac{1}{2} \]
\[ y - 0 = \frac{1}{2}(x - 0) \]
\[ y = \frac{1}{2}x \]

k) through (–1, –2) parallel to
\[ x + y = 6 \]
find slope:  \[ x + y = 6 \]
\[ y = -x + 6 \]
\[ m = -1 \]
\[ y - (-2) = -1(x - (-1)) \]
\[ y + 2 = -1(x + 1) \]
\[ y + 2 = -x - 1 \]
\[ y = -x - 3 \]

l) through (3, 0) perpendicular to
\[ x + 2y = 3 \]
find slope:  \[ x + 2y = 3 \]
\[ y = -\frac{1}{2}x + \frac{3}{2} \]
\[ m = 2 \]
\[ y - 0 = 2(x - 3) \]
\[ y = 2x - 6 \]
Graphing Linear Inequalities In Two Variables

1. Graph the boundary line (solid for ≤ and ≥, dotted for < and >).
2. Pick any point not on the line. Does it satisfies the inequality?
3. If yes, shade that side of the line. If not, shade the other side.

Example: Graph the following inequalities:

a) \(2x + y \leq 4\)  
b) \(y > 2x\)
c) \( x + y > 3 \)

d) \( y > 1 \)

e) \( x + 3y \leq 3 \)

f) \( x \leq 0 \)