Economic Development

To date, we’ve studied endogenous growth using a model designed to characterize growth at the *frontier* of technological development. Here, we will study growth in a model designed to characterize *developing* economies.

Recall the production function for a frontier economy:

\[ Y = L Y^{1-\alpha} \int_0^A x_j^\alpha \, dj \]

The production function to be adopted for our hypothetical developing economy is analogous:

\[ Y = L_Y^{1-\alpha} \int_0^h x_j^\alpha \, dj \]

where \( h \leq A \). The size of \( h \) relative to \( A \) indicates how close to the technological frontier the developing economy is.
As in the Romer model, $x_j = x$ for all $j$, thus

$$K = \int_0^h x_j \, dj = \int_0^h x \, dj = x \int_0^h dj = x \left[ j \right]_0^h = x(h - 0) = hx$$

So, $x = K/h$. Substituting for $x$ in the production function yields

$$Y = K^\alpha (YA_L)^{1-\alpha},$$

exactly as in the Romer model. Moreover, as usual,

$$\cdot K = I - dK,$$

so the law of motion of $K$ is standard.

Thus using familiar arguments, the model can be shown to yield balanced growth for per capita output $y$ and capital $k$, linked to the growth rate of technology $h$:

$$g_y = g_k = g_h$$

What remains to be specified is a law of motion for $h$. 
We will assume that the growth rate of $h$ depends on two variables: the amount of time spent studying, captured by the variable $u$ introduced in Ch. 3 (recall specifically equation 3.2, p. 48); and the distance of the developing economy away from the frontier, captured by the variable $A/h$.

Note: the closer $A/h$ is to 1, the closer the developing economy is to the frontier. The larger is $A/h$, the farther is the economy from the frontier.

The growth rate of $h$ is given by equation 6.5:

$$\frac{\dot{h}}{h} = \mu e^{\varphi u} \left( \frac{A}{h} \right)^\gamma$$

where $\mu > 0$, $\varphi > 0$, and $0 < \gamma < 1$.

So, $g_h$ is increasing in $u$ and $A/h$, but increases with $A/h$ at a decreasing rate.

In the steady state, $g_h$ is constant.

Tasks:

- Show $g_h = g_A$ in the steady state (note: here, $g_A$ is exogenous).
- Calculate the steady state value of $A/h$.
- Graph $g_h$ against $A/h$, use to illustrate transition dynamics.
- Show how changes in $u$ alter steady state values and transition dynamics.