The Romer Model

David N. DeJong
University of Pittsburgh

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Overview of the Model

The model extends the Solow model by **endogenizing the characterization of technological progress**.

In Romer’s model, technological progress is represented as the generation of ideas, or blueprints, useful for producing **new intermediate goods** (goods used to produce final goods). The Solow model assumed a single intermediate good; the Romer model features $A_t$ intermediate goods, which evolves over time.

In both the Solow and Romer models, we assume a single final good (thus, we abstract from the innovation of consumption goods, and focus exclusively on the innovation of new intermediate goods).
The model features four sectors:

- Final-goods sector
- Intermediate-goods sector
- R&D sector
- Labor market
The model is represented in diagram form in the following slide. The arrows represent flows of resources between the sectors. These will be described in detail below.
R&D Sector

Produces blueprints for constructing intermediate goods

Workers

Supply labor to the R&D and Final-Goods sectors

Intermediate-Goods Sector

Group of monopoly producers of intermediate goods

Final-Goods Sector

Produces the single final good using labor and existing intermediate goods
In this sector, blueprints for producing new intermediate goods are generated by workers who spend their time exclusively towards this goal.

Physical capital is not required to produce blueprints; however, existing ideas are useful for generating new ideas.
Let the stock and flow of ideas be given by:

- Stock: $A_t$
- Flow: $\dot{A}_t$

Also, let $L_A = s_R L$ denote the proportion of the workforce devoted to the R&D sector.
With this notation, the law of motion for new ideas is given by

\[ \dot{A}_t = \delta L_A, \]

where

\[ \bar{\delta} = \delta A^\phi, \quad \delta > 0, \quad 0 < \phi < 1. \]

Note: individuals take \( \bar{\delta} \) as given, but their actions have an impact on \( \bar{\delta} \) (i.e., the search for new ideas generates an *externality*).
Combining equations, the law of motion for new ideas is given by

\[ \dot{A}_t = \delta A^\phi L_A, \quad \delta > 0, \quad 0 < \phi < 1. \]
When a new blueprint is generated, it is sold to a firm in the intermediate-goods sector at price \( P_A \).

Revenue generated by the sales of new blueprints are paid fully to workers. The wage rate in this sector is given by \( w_R \).

The flow of resources into and out of the R&D sector is depicted in the following diagram.
R&D Sector

- Produces blueprints for constructing intermediate goods
- \[ \dot{A}_t = \delta L_A \]

Workers

- Supply labor to the R&D and Final-Goods sectors

Intermediate-Goods Sector

- Group of monopoly producers of intermediate goods

Final-Goods Sector

- Produces the single final good using labor and existing intermediate goods
The Intermediate Goods Sector

This consists of a large number of firms, each of which has obtained the monopoly power to produce a unique intermediate good from the R&D sector at price $P_A$.

The $j^{th}$ firm produces the $j^{th}$ unique good $x_j$ at the marginal resource cost $r$, which represents the rental rate of physical capital.

The production of $x_j$ does not require a labor input, just the sacrifice of $x_j$ units of the final good (foregone consumption, as in the Solow model).
The demand curve for $x_j$ stems from the final-goods sector; it is given by

$$p_j = p_j(x_j), \quad p'_j(x_j) < 0.$$ 

The profit function of the $j^{th}$ firm is given by

$$\pi_j = p_j(x_j)x_j - rx_j.$$
The total quantity of intermediate goods supplied to the final-goods sector is the sum (or integral) of the quantities supplied by the individual monopolists:

$$\sum_{j=1}^{A_t} x_j, \quad \int_0^{A_t} x_j \, dj.$$ 

In turn, the total resources paid to the individual monopolists is given by

$$\sum_{j=1}^{A_t} p_j (x_j) x_j, \quad \int_0^{A_t} p_j (x_j) x_j \, dj.$$ 

The flow of resources into and out of this sector is indicated in the following diagram.
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\[ \sum_{j=1}^{A_I} p(x_j)x_j \]

\[ \sum_{j=1}^{A_I} x_j \]
A single consumption good is produced in a perfectly competitive industry. (I.E., prices are taken as given.)

Here, labor $L_Y = (1 - s_R) L$ and intermediate goods combine to produce $Y$, according to

$$Y = L_Y^{1-\alpha} \sum_{j=1}^{A_t} x_j^\alpha$$

or

$$Y = L_Y^{1-\alpha} \int_0^{A_t} x_j^\alpha \, dj.$$
The objective in this sector is once again to maximize profits:

$$\max_{L_Y, x_j} \int_0^{A_t} x_j^\alpha d\gamma - w_Y L_Y - \int_0^{A_t} p_j (x_j) x_j d\gamma.$$ 

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\[ \sum_{j=1}^{A_i} p(x_j) x_j \]

\[ \sum_{j=1}^{A_i} x_j \]
Workers are equally able and indifferent in supplying labor in the R&D and Final Goods sectors.

Therefore, they choose $s_R$, the fraction of labor devoted to the R&D sector, such that wages are equated across sectors:

$$w_Y = w_R = w.$$

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$L_R$

$w_R L_R$

$w_Y L_Y$

$L_Y$
Aggregating the Model

\[ Y_t = L_{Y_t}^{1-\alpha} \int_0^{A_t} x_j^\alpha \, dj \]

\[ K_t = \int_0^{A_t} x_j^\alpha \, dj \]

\[ \dot{K}_t = I_t - dK_t \]

\[ Y_t = C_t + I_t \]

\[ C_t = (1 - s) \cdot Y_t \]

\[ \dot{A}_t = \delta A_t^\phi L_{At} \]

\[ L_{At} = s_R L_t, \quad L_{Y_t} = (1 - s_R) L_t \]

\[ \frac{\dot{L}_t}{L_t} = n \]
Endogenous prices:

- $w_R, w_Y$: determined as the marginal product of labor in the R&D and final-goods sectors, constrained to be equal
- $p(x_j)$: determined as the marginal product of $x_j$ in the final-goods sector
- $P_A$: determined by the profits a new blueprint can generate for an intermediate-goods monopolist
- $r$: determined as the marginal product of $K$ in producing $Y$

Also endogenous:

$S_R$

The aggregated flow of resources is indicated in the following diagram.
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To solve the model, we must solve for \((w_R, w_Y, p(x_j), P_A, r, s_R)\), then using these solutions, draw inferences for aggregate behavior.

This approach is tedious and complicated, so we’ll work in reverse: we’ll take as granted these solutions, and move directly to the study of aggregate behavior. Time permitting, we’ll then return to the solution of \((w_R, w_Y, p(x_j), P_A, r, s_R)\).
Further model analysis will be pursued interactively during class.