POINT-OF-VIEW

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Functional Degrees of Freedom

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Mechanical degrees of freedom (DOF) are defined as the minimum number of independent coordinates needed to describe a system's position. The human musculoskeletal system has many mechanical DOF through which countless movements are accomplished. In the motor control field, one of the aspirations is to understand how the many DOF are organized for movement execution—the so-called DOF problem. Natural movements are characterized by the coordination of the DOF such that few vary independently. The concept of *functional degrees of freedom* (fDOF) is introduced to describe the very limited DOF of purposeful, coordinated movements. Deterministic (i.e., constraint satisfaction) and statistical (i.e., principal component analysis) approaches are used to determine fDOF. In contrast to DOF as a mechanical descriptor, fDOF emphasizes the mechanisms of human movements and corroborates our search for the solution to the DOF problem.

Key Words: constraint satisfaction problem, principal component analysis, human movement, motor control

The purpose of this article is to introduce the concept of *functional degrees of freedom* (fDOF), an idea built upon the well defined notion of *degrees of freedom* (DOF) in classical mechanics and the contemporary discoveries of human movement coordination. In this article, I will start with a brief description of the mechanical definition of DOF, and then present a mathematical formulation of the DOF problem. Next, I will introduce fDOF using deterministic and statistical approaches, and use an example of finger pinch movement to illustrate the calculation of fDOF. Finally, I will discuss the generalization of fDOF beyond the mechanical DOF to address the DOF problem in a broader perspective.

Degrees of Freedom and the Degrees of Freedom Problem

In classical mechanics, DOF is defined as the minimum number of independent coordinates used to describe a system's position. For example, a particle freely moving in space has three DOF, and can be described with three Cartesian coordinates, whereas a rigid body freely moving in space has six DOF: three translations and three rotations. A system composed of n rigid bodies has a maximum of $6n$

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DOF, but the actual DOF are usually much fewer due to structural constraints.

Kinematically, the human skeletal system is conveniently treated as a chain structure. It is estimated that the human skeletal system has 148 movable bones connected by various types of joints, resulting in 244 DOF (Zatsiorsky, 1998). While mobility and flexibility are rendered by the many DOF of the neuromusculoskeletal system, it is puzzling how these DOF are organized to accomplish countless movement tasks (Latash, 1996). This question was initially deliberated by Bernstein, who presented it as the DOF problem (Bernstein, 1967).

Human movements are accomplished under various integrated constraints that are mechanical, anatomical, physiological, neural, environmental, and/or task-related (Chiel & Beer, 1997; Zatsiorsky, 1998; Valero-Cuevas, 2005). In recent years, recognition of these constraints has shifted motor control theories from control-based central programs to constraint-based self-organization (Gelfand et al., 1971; Schoner & Kelso, 1988; Turvey, 1990; Thelen, 1995). The self-organizing behavior during performance of a task is thought to emerge from a combination of different types of constraints. It is as if constraint equations are written with the DOF variables so that movements are highly coordinated and self-regulated (Tuller et al., 1982).

Consonant with the mathematical approach to the description of movement structure by Gelfand et al. (1971), the DOF problem can be viewed mathematically as a type of *constraint satisfaction problem*, which has been studied in many fields. A constraint satisfaction problem is defined by: (1) a set of variables $\{x_1, x_2, .., x_n\}$ with their corresponding domains $\{D_1, D_2, ..., D_n\}$, and (2) a set of m constraint equations that specify the relationships among the variables:

$$f_k(x_1, x_2, \ldots, x_n) = 0 \; k = 1, 2, \ldots, m \quad (1)$$

A constraint satisfaction problem is solved by assigning values to the variables from their respective domains so that the constraint equations are satisfied. Depending on the relationship between m and n, the problem can be determinate ($m = n$), overdetermined ($m > n$), or indeterminate ($m < n$). If the problem is determinate, the problem is solvable and there is a unique solution that satisfies the constraint equations. If the problem is overdetermined, no exact solution exists but an approximate solution may be chosen to satisfy the constraint equations as closely as possible. When the problem is indeterminate, an infinite number of solutions exist to satisfy the constraint equations. In studying human movements, we are often confronted with indeterminate problems because either the variables are not completely defined by n constraint equations or all constraint equations are not known to us.

Functional Degrees of Freedom (fDOF)

Human movements would appear uncoordinated and purposeless if the DOF varied independently (Neilson & Neilson, 2004). Indeed, most movements are typified by spatiotemporal patterns that are repeatable and stereotypical, suggesting that the seemingly unsolvable DOF problem is somehow resolved as if the DOF were eliminated, a phenomenon pointed out by Bernstein (Bernstein, 1967). If this is so, then by what process are the DOF eliminated? One way is to freeze specific DOF, a situation that is observed at the early phase of motor learning (Newell, 1991). Another way of eliminating DOF is to use existing (or establish new) relationships among the DOF, a strategy commonly referred to as coordination, synergy, linkage, or coupling (Bernstein, 1967; Turvey, 1990; Rosenbaum, 1996). DOF reduction, whether by freezing, coordination, or other means, is equivalent to adding constraint equations. According to the constraint satisfaction hypothesis, Equation 1 can be generalized to movements by including the time series data of the DOF variables:

$$f_k(x_{1t}, x_{2t}, \ldots, x_{nt}) = 0 \quad k = 1, 2, \ldots, m \quad (2)$$

For a movement to occur, the DOF cannot be completely constrained, i.e., the number of constraint functions must be less than the DOF, $m < n$. The difference between n and m signifies the number of variables that can be independently varied, which is the genuine DOF during a movement. The term *functional degrees of freedom* (fDOF) (Sanguineti et al., 1997; Li & Tang, in press) or *control degrees of freedom* (Neilson & Neilson, 2005) has been suggested to describe the very limited DOF during specific movement tasks. Building upon the DOF concept, fDOF can be defined as the remaining DOF that are allowed to independently vary during a movement task due to the imposition of m task-specific constraints on the original n DOF:

$$fDOF = n - m \quad (3)$$

fDOF is consistent with the idea of the *uncontrolled manifold* (Schoner, 1995; Scholz & Schoner, 1999; Latash et al., 2002). This uncontrolled manifold defines a subspace formed by the variables in which changing the values of the variables does not influence performance. If the performance criteria are formulated as the set of the constraint equations, fDOF is the dimension of the subspace defined by the uncontrolled manifold. Figure 1 shows a hypothetical DOF problem in terms of fDOF and the uncontrolled manifold.

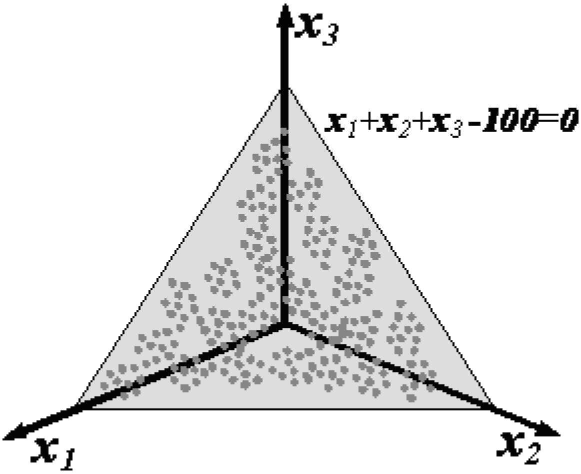

Figure 1—A set of hypothetical DOF variables (x_1, x_2, x_3) constrained by equation $f(x_1, x_2, x_3) = x_1 + x_2 + x_3 - 100 = 0$. fDOF is 3 – 1 = 2. In other words, the three variables can freely move in a 2D plane defined by the constraint equation, as shown by the dots. If the equation is viewed as a performance criterion, it defines an uncontrolled manifold, i.e., a 2D subspace.

Using Principal Component Analysis

Unfortunately, the relationships among the DOF as stated in Equation 2 are often unknown a priori; instead, they manifest in functional tasks (Kay, 1988). Analyses of the behavior of the DOF variables provide insight into their relationships. For example, movement coordination has been examined using various methods such as variable-variable plots, cross-correlation, continuous relative phase analysis (Sparrow et al., 1987), and vector coding (Tepavac & Field-Fote, 2001), but these methods are usually limited to a few DOF variables. Multivariate analysis techniques are needed to interpret relationships among the many DOF variables.

Principal component analysis (PCA) is one such multivariate analysis technique used to interpret high dimensional data in many fields of study, and recently has been applied to various relevant biomechanical problems (Daffertshofer et al., 2004). This technique transforms correlated variables into a reduced number of uncorrelated variables called principal components. They are sequentially derived based on their accountability of variance, starting with the first principal component to account for the greatest amount of variance. The inherent variability of human movement data allows us to derive as many principal components as the number of DOF variables. Geometrically, the spatiotemporal data can be viewed as a series of spatial coordinates in an n-dimensional space. The principal components derived by PCA define a new orthogonal coordinate frame so that the new coordinate axes (i.e., eigenvectors) are oriented to maximally account for the variances in a descending order. If the original data are expressed in the new coordinate frame, some of the axes corresponding to the small principal components can be removed with minimum loss to the original data. As such, the movement data can be represented adequately by a number of principal components smaller than the original number of variables. Indeed, previous studies have shown that the number of principal components needed for the reconstruction of many movements is rather small even though the movements are described by a large number of kinematic DOF variables (Soechting & Flanders, 1997; Santello et al., 2002; Braido & Zhang, 2004).

Unlike the definition of fDOF with known constraint equations, PCA provide a means to statistically define the small number of independent variables from the original DOF variables. By virtue of PCA, fDOF can be defined as the number of principal components, p, that adequately represent the original kinematic data:

$$fDOF = p \quad (4)$$

A number of methods may be used to determine p (Jackson, 1991). For example, principal components are retained if: (1) they are statistically significant, (2) their variances do not level off on a SCREE plot, or (3) they accumulatively account for most (e.g., 95%) of the total variance.

As a statistical method, PCA derives fDOF without defining the exact relationships among the DOF variables. This statistical approach is crucial for the study of human movements because (1) it is rather challenging to explicitly specify a set of hard-wired and task-dependent constraint functions in complex biological systems (Kay, 1988), and (2) variability is always superimposed on movement stereotypy (Newell et al., 1993; Todorov & Jordan, 2002). However, the standard PCA has a major limitation in that each derived principal component is a linear combination of the original variables. Recognizing this limitation, researchers in the mathematics community have been developing nonlinear multivariate analysis methods such as local linear embedding, neural network implementation, and isometric mapping. It is necessary to apply advanced techniques to analyze movement data resulting from nonlinear interactions among the DOF variables so that fDOF is not overestimated by the linear PCA. Another drawback of PCA is that the original kinematic meaning may not be directly interpretable via the principal components as the principal components are combinations of the original DOF variables. However, this drawback can be ameliorated by examining the contributions/loadings of the original DOF to the principal components (Alexandrov et al., 1998).

The Advantages of fDOF

In contrast to DOF as a mechanical descriptor, fDOF emphasizes the mechanisms of human movements and enhances our understanding of movement control. While a system may have a large number of kinematic DOF, its fDOF is usually very limited. The reduction of many DOF to few fDOF is considered a hallmark of a purposeful, coordinated movement. This simplification is compatible with the limited central resources (Neilson & Neilson, 2005), and is consistent with the idea that the commands necessary for movement realization cannot be complicated (Gelfand et al., 1971). The parsimonious nature of fDOF has been demonstrated in finger movement (Soechting & Flanders, 1997; Santello et al., 2002; Braido & Zhang, 2004), upper limb movement (Forner-Cordero et al., 2005), gait (Courtine & Schieppati, 2004), trunk bending (Alexandrov et al., 1998), squatting (St-Onge & Feldman, 2003), tongue movement (Sanguineti et al., 1997), and thumb opposition (Li and Tang, in press). These studies suggest that it is enough to specify a small number of independent variables out of many DOF for an execution of a coordinated movement (Gelfand et al., 1971).

Another important feature of fDOF is that it is invariant for a specific movement of a kinematic system, independent of various ways of describing the movement. Redundant kinematic variables must be accompanied by additional constraint equations, making the fDOF constant. An application of PCA to a redundant set of variables eliminates the unneeded variables. Lastly, fDOF captures the functional characteristics of the movement system. fDOF is movement-specific, that is, the same kinematic system that has constant kinematic DOF may have different fDOF when different movement tasks are performed. For example, the fingers may have more fDOF during piano playing than they have during grasping.

An Example of Finger Movement

An example of finger movement is presented herein to illustrate the fDOF concept. Consider the mechanical chain of the index finger in 2D (Figure 2). Points A, B, C, and D stand for the wrist, metacarpophalangeal, proximal interphalangeal, and distal interphalangeal joints, respectively. The lengths of the metacarpal, proximal phalanx, middle phalanx, and distal phalanx are assumed as 71 mm, 46 mm, 28 mm, and 18 mm, respectively. The kinematic model is further simplified by immobilizing the wrist joint (φ_A) at 30° extension, leaving the model with three DOF at the metacarpophalangeal (φ_B), promximal interphalangeal (φ_C), and distal interphalangeal (φ_D) joints.

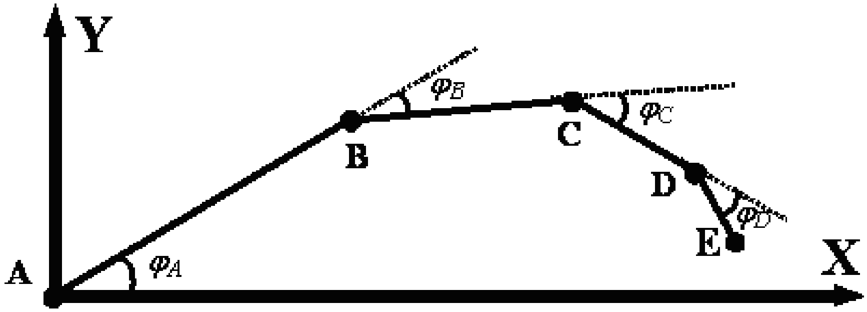

Figure 2—A 2D kinematic finger model represented by multiple links. If φ_A is stabilized, the kinematic configuration of the finger can be specified by three DOF, i.e., generalized coordinates (φ_B, φ_C, φ_D).

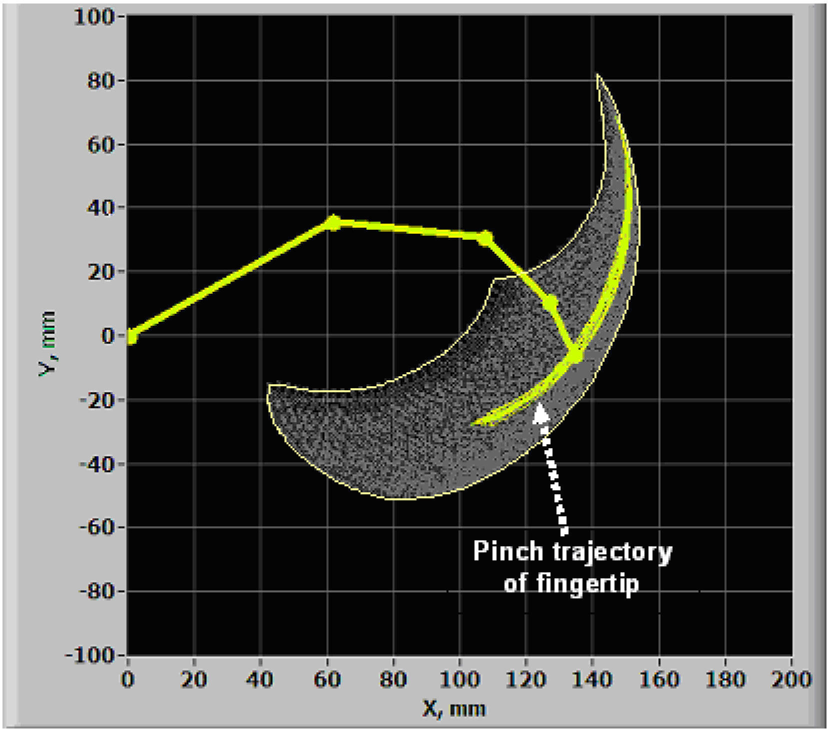

Figure 3—The trajectory of the fingertip during precision pinch movements within the crescent-shaped, theoretical workspace. The fingertip workspace is simulated with the wrist at 30° of extension, the metacarpophalangeal joint between 0° and 90° of flexion, the proximal interphalangeal joint between 0° and 90° of flexion, and the distal interphalangeal joint between 0° and 45° of flexion.

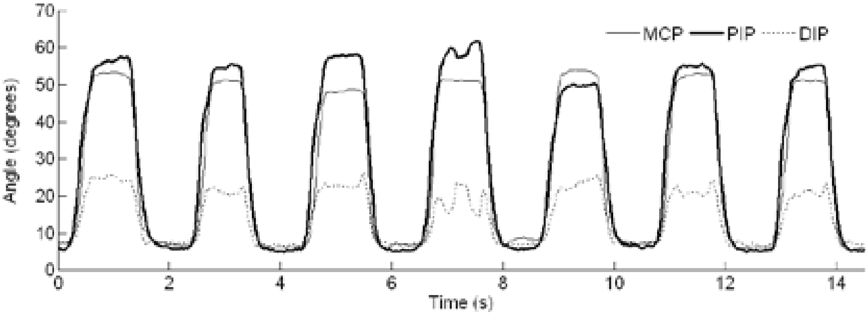

Figure 4—The time series data of the metacarpophalangeal (MCP), proximal interphalangeal (PIP), and distal interphalangeal (DIP) joints during seven cycles of precision pinch movements.

Based on the flexion/extension ranges of these joints, the fingertip attains considerable workspace (the crescent shaped area in Figure 3) if the three joints are allowed to move independently. However, the fingertip does not move arbitrarily in the attainable workspace during specific finger movements (Kamper et al., 2003). For example, our experimental data from precision pinch movements with the thumb and index finger demonstrate characteristic, repeatable fingertip trajectories (Figure 3). Furthermore, the time series data of the metacarpophalangeal and interphalangeal joints show strong coordination of the angular movements (Figure 4). Linear relationships exist among the three angles as:

$$\varphi_C = 1.07\,\varphi_B + 0.35\ (R^2 = 0.968)$$

$$\varphi_D = 0.33\,\varphi_B + 5.24\ (R^2 = 0.932)$$

According to the definition of fDOF in Equation 3, the two equations eliminate two of the original three DOF, leading to fDOF = 3 – 2 = 1.

The fDOF of the pinch movement can also be analyzed by PCA, with only one principal component needed to reconstruct the three angle time series data. The first principal component explains more than 98% of the total variance of the data, and therefore, there is still one fDOF.

Final Remarks

Thus far, I have purposely limited the discussion of fDOF using mechanical DOF, conforming to the original formulation of the DOF problem using kinematic coordinates (Bernstein, 1967). The essential mathematical feature of the DOF problem, however, is applicable to different motor entities, such as muscle forces (Prilutsky et al., 1994), electromyographic signals (Krishnamoorthy et al., 2003; Ting & Macpherson, 2005), finger forces (Li et al., 1998), and the neuronal population (Sporns & Edelman, 1993). Indeed, DOF have been used in motor control literature to describe many different elements including neurons, motor units, muscles, segments, forces, motion, electromyography, and visuomotor information. A system, whatever its elements, is made coherent and meaningful by ubiquitous constraints imposed on those elements. Consequently, fDOF can be generalized to any movement subsystem, allowing us to appreciate the low-dimensional structure hidden in the high-dimensional observations.

Indeed, a new language is needed in the motor control field (Gelfand & Latash, 1998). fDOF can be considered as a biologically relevant concept to aid our searching of principles governing human movements, no matter what level of the movement system one may choose to study. In the words of Bernstein, "[The movement system] is presented to us as given data, a given problem, and our task lies in the search for non-contradictory explanations of the mechanisms of multistaged functioning of this sort" [(Bernstein, 1967), p. 27]. In the quest for explanations, fDOF is effective in deciphering the large amount of data generated from human movements, and thus, helps reduce the difficulty of the DOF problem.

Acknowledgments

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