Review Problems (III)

1. Evaluate
\[ \iiint_B e^{(x^2+y^2+z^2)/2} \, dV, \]
where \( B \) is the unit ball \( B = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1\} \).

2. Let \( f \) be a scalar-valued function and \( F \) a vector field on \( \mathbb{R}^3 \). Determine if the given expression is meaningful.

   (i) \( \text{curl(curl \, F)} \) \hspace{1cm} YES \hspace{1cm} NO

   (ii) \( \text{curl(div \, F)} \) \hspace{1cm} YES \hspace{1cm} NO

   (iii) \( \text{grad(curl \, F)} \) \hspace{1cm} YES \hspace{1cm} NO

   (iv) \( (\text{grad} \, f) \times (\text{curl} \, F) \) \hspace{1cm} YES \hspace{1cm} NO

   (v) \( \text{grad}[(\text{grad} \, f) \cdot F] \) \hspace{1cm} YES \hspace{1cm} NO

3. Let \( C \) denote the path from \((0, 4)\) to \((2, 0)\) along the parabola \( y = 4 - x^2 \). Evaluate the line integral
\[ \int_C 2x \sin y \, dx + (x^2 \cos y - 3y^2) \, dy. \]

4. Evaluate
\[ \int_C (2x^2y + e^x) \, dx + (x^3 + xy^2) \, dy, \]
where \( C \) consists of the line segment from \((-2, 0)\) to \((2, 0)\) and the top half of the circle \( x^2 + y^2 = 4 \).

5. Let \( \mathbf{F}(x, y, z) = xi + yj + zk \). Evaluate the surface integral
\[ \iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma, \]
where \( S \) is the boundary surface of the cylinder \( \{(x, y, z) : x^2 + y^2 = 4, 0 \leq z \leq 4\} \).