Sample Final Problems

(Calculators NOT allowed!)

1. (15 points, 5 points each) Find the derivatives of the following functions. It is NOT necessary to simplify your answers.

   (a) $g(x) = \ln\left(\dfrac{(5x+1)^7(2x^4+3)^2}{(\arctan x)^{10}}\right)$

   (b) $y = (3x+2)^{4x}$

   (c) $f(x) = \displaystyle\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$

2. (10 points) Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16cm^2?

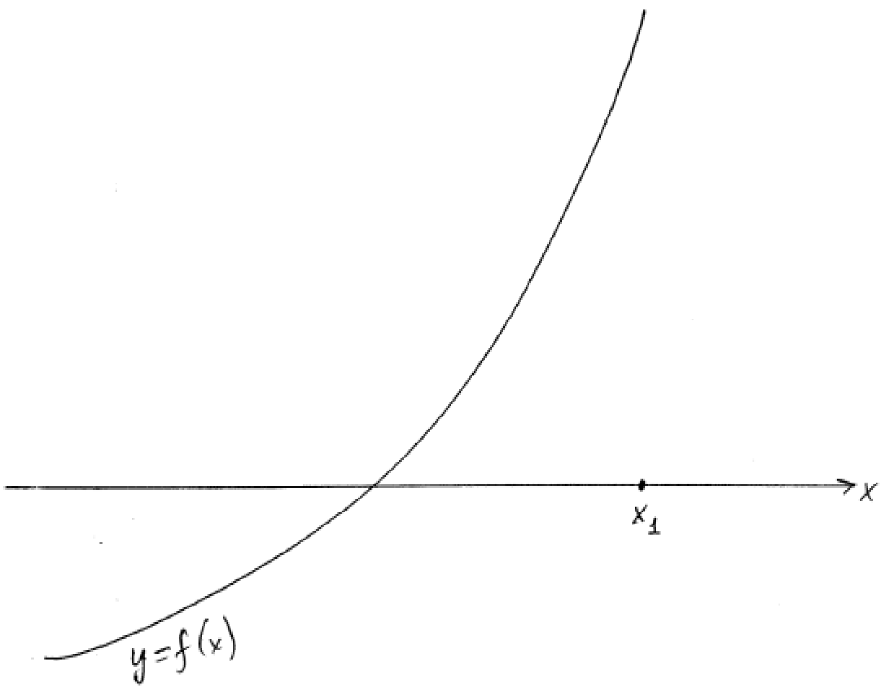
3. (10 points) If a snowball melts so that its surface area decreases at a rate of $1\text{cm}^2/\text{min}$, find the rate at which the diameter decreases when the diameter is 10 cm.

4. (20 points)

$$f(x) = x^3 + 3x^2 - 24x$$

(a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use the information from parts (a)-(c) to sketch the graph.

5. (10 points) We use Newton's Method to find the approximate value of the solution of the equation $f(x) = 0$. The graph of the function f and the position of x_1 is shown. Draw on the same picture the positions of x_2, x_3, and x_4.



6. (10 points) Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$.

7. (15 points, 5 points each) Evaluate the following limit. It is Not necessary to simplify your answer.

(a) $\lim_{x\to\infty} \dfrac{3x^2 - x - 2}{5x^2 + 4x + 1}$

(b) $\lim_{x\to 0} \dfrac{\sin(6x)}{\ln(x+1)}$

(c) $\lim_{x\to 0^+} (1 + 3x)^{1/x}$

8. (15 points, 5 points each) Evaluate the following integral. It is NOT necessary to simplify the answer.

(a) $\int_0^2 t\sqrt{4+t^2}\,dt$

(b) $\int_{-2}^2 \sqrt{4-x^2}\,dx$. (HINT: Use Geometry!)

(c) $\int \frac{\ln x}{x}\,dx$

9. (15 points, 5 points each) Evaluate the following integral. It is NOT necessary to simplify the answer.

(a) $\int_1^2 p \ln p \, dp$

(b) $\int x^3 e^x \, dx$

(c) $\int e^x \cos x \, dx$

10. (10 points) Find a function f and a number a such that

$$4 + \int_a^{2x} t f(t)\, dt = x^3 \quad \text{for all } x > 0$$

11. (10 points) If 1200 cm^2 of material is available to make a box with a square base and an open top. Find the largest possible volume of the box.

12. (10 points) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point (1,0).

13. (10 points) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.