

Sample for Midterm Exam

Notice: This sample set containing problems worth 140 points, while our Midterm Exam only containing problems worth 100 points. So problems in the Midterm will be less but worth similar weights. Plus: Calculators NOT allowed!

1. (30 points in total)

(a) (5 points) Find the logarithmic function

$$f(x) = \log_a(x - C) \quad (a > 0)$$

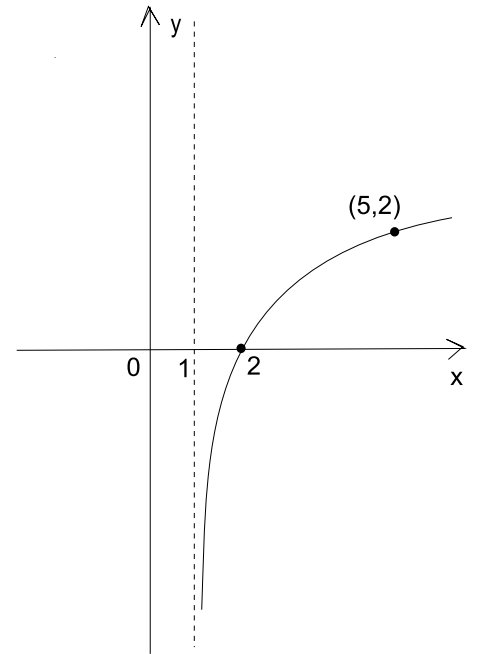
whose graph is given on the right.

(b) (5 points) Use derivative to determine if $f(x)$ is an increasing function or a decreasing function.

(c) (5 points) Use derivative to determine the concavity of $f(x)$.

(d) (10 points) Find the inverse function of the function you found above, denote it as $g(x)$ and sketch it on the same graph with $f(x)$.

(e) (5 points) Find the value for $(f \circ g)(78)$.



2. (15 points in total)

(a) (5 points) Find the exact value of the following expression

$$\log_2 64 + \frac{\ln 5}{\ln 2} - \log_2 10 + \log_7 \frac{1}{49}$$

(b) (10 points) Solve for x :

$$\ln(2x) + \ln(x - 5) = 2$$

3. (15 points in total)

$$f(x) = \frac{3x^2 + 8x + 5}{5x^2 + 4x - 1}$$

- (a) (5 points) Find the domain of the given function.
- (b) (10 points) Find the horizontal and vertical asymptotes of the curve.

4. (10 points in total, 5 points each) Differentiate the following function. NO NEED to simplify your answer.

(a) $f(t) = \frac{3x - 2}{\sqrt[4]{2x + 1}}$

(b) $g(x) = \ln \left(\frac{(5x^3 + 1)^2}{(\arcsin x)^4} \right) + e^{\sin 2x}$

5. (10 points) Find $\frac{dy}{dx}$ where

$$\begin{cases} x = 1 + \sqrt{t}, & 0 < t \leq 5 \\ y = t^2 - 4t \end{cases}$$

6. (20 points, 10 points each) Find $\frac{dy}{dx}$ where

(a) $e^{x/y} = \arctan(x) - y$

(b) $x^2y^2 + x \sin y = 4$

7. (10 points) Show that the two curves are orthogonal

$$\begin{cases} 2x^2 + y^2 = 3 \\ x = y^2 \end{cases}$$

8. (20 points, 10 each) Use definition of derivative (DO NOT use differentiation Laws) to find out $f'(x)$ when

(a) $f(x) = \frac{x+1}{x-3}$

(b) $f(x) = x + \sqrt{x}$

9. (10 points, 5 points each)

(a) Find the linearization $L(x)$ of the function

$$f(x) = x^4 + 3x^2 \quad \text{at } a = -1$$

(b) Find the differential of the function

$$y = e^{-u} \cos u$$