Logic Cheat Sheet

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Definitions

Valid argument Reasoning in which a conclusion follows necessarily from the premises presented, so that the conclusion cannot be false if the premises are true.

Statements Either true or false, but not both. Represented by letters.

Not (negation)

$$\neg P$$

means "it is not the case that P"

And (conjunction)

$$P \wedge Q$$

means "both P and Q"

Or (disjunction)

$$P \vee Q$$

means "either P or Q (or both)"

Conditional connective

$$P \Rightarrow Q$$

means

- "P implies Q"
- "if P then Q,"
- "P is sufficient for Q"

- "Q is necessary for P"

Converse

$$Q \Rightarrow P \text{ is the converse of } P \Rightarrow Q$$

IMPORTANT! A conditional statement is NOT the same as its converse.

Contrapositive

$$\neg Q \Rightarrow \neg P \text{ is the contrapositive } of P \Rightarrow Q$$

A conditional statement IS EQUIVALENT to its contrapositive.

Biconditional connective

$$P \Leftrightarrow Q$$

means "P is necessary and sufficient for Q" or "P if and only if Q" (abbreviated iff)

Tautology A statement that is always true.

Contradiction A statement that is always false.

Logical equivalences

Double negation law

$$\neg\neg P \equiv P$$

Commutative laws

$$P \wedge Q \equiv Q \wedge P$$

$$P \vee Q \equiv Q \vee P$$

Associative laws

$$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$$

$$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$$

Idempotent laws

$$P \wedge P \equiv P$$

$$P \vee P \equiv P$$

Distributive laws

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

DeMorgan's laws

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

Conditional laws

$$P \Rightarrow Q \equiv \neg P \vee Q$$

$$P \Rightarrow Q \equiv \neg(P \wedge \neg Q)$$

Biconditional law

$$P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

Contrapositive law

$$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$$

Tautology laws

$$P \wedge (\text{a tautalogy}) \equiv P$$

$$P \vee (\text{a tautology}) \text{ is a tautology}$$

$$\neg(\text{a tautology}) \text{ is a contradiction}$$

Contradiction laws

$$P \wedge (\text{a contradiction}) \text{ is a contradiction}$$

$$P \vee (\text{a contradiction}) \equiv P$$

$$\neg(\text{a contradiction}) \text{ is a tautology}$$

Quantifier negation laws

$$\neg \exists x \text{ s.t. } P \equiv \forall x, \neg P$$

$$\neg \forall x, P \equiv \exists x \text{ s.t. } P$$