Selling Authority

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Abstract

Conflict of interests between an uninformed principal and an informed agent prevents the principal from fully using the agent's information when making a decision. This informational loss often results in a socially undesirable outcome. In this paper, we show that inefficiency caused by the informational loss can be resolved via bargaining with monetary transfers to efficiently reallocate decision-making authority. We consider bargaining over decision-making authority in which an informed but self-interested agent makes a price offer to buy decision-making authority to an uninformed principal who then decides either to accept or to reject the offer. No matter how large the difference between parties' preferences, there exists a continuum of perfect Bayesian equilibria, each of which yields an *ex-post* efficient action for any realization of the state. Furthermore, any equilibrium outcome is *ex-ante* Pareto superior to several dispute-resolution schemes studied in the framework of Crawford and Sobel (1982, Econometrica) and Holmström (1977) when the parties' preferences are substantially misaligned.

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1 Introduction

In many economic situations, a party, such as a government or firm (principal), initially has full authority to make a decision, but lacks information about the task or project at hand. There is often another, better-informed party (agent) but it lacks the authority to make the decision. Examples of this principal-agent relationship include an international manufacturer who is less informed about specific national market conditions than a domestic distributer, a patentee who is less-experienced in commercializing than a manufacturing company, a policy-maker who knows less about potential impact of a fiscal policy than an economist, a central office who is less informed about a local division than a lower level division manager, and a private investor who is less informed about stock market conditions than a fund manager.

Sometimes the informed agent's interest is so different from the uninformed principal's interest that the informed agent does not want to share useful information with the principal. Crawford and Sobel [9] show that if costless communication, cheap-talk, is allowed between parties whose preferences diverge, then information is transmitted in a strategic way. That is, the informed agent has no incentive to fully reveal his information. Furthermore, when the parties' interests diverge substantially information cannot be transmitted through cheap-talk at all. Although more complicated communication protocols such as communication via a neutral trustworthy mediator (Goltsman, Hörner, Pavlov and Squintani [14]), a biased mediator (Ivanov [23]), an extensive communication (Krishna and Morgan [26]), and communication including noise (Blume, Board, and Kawamura [5]) could facilitate communication between parties, meaningful information is not transmitted when the degree of conflict is sufficiently large.

Organizational theory suggests that delegation of authority might resolve this problem. For example, Milgrom and Roberts [29] point out that comprehensive decision making in a large organization must involve considerable delegation of authority to lower levels of the organization. However, the principal is not always willing to delegate authority because the agent has his own agenda: Dessein [10] shows that the uninformed principal does not prefer to delegate his decision-making authority to the informed agent when the preferences diverge substantially. Although the principal can optimally delegate authority with the restricted set of actions that the agent can take, it is optimal for the principal not to delegate authority when the degree of conflict is large enough (Alonso and Matouschek [3], Holmström [21], Kováč and Mylovanov [24] and Melamud and Shibano [28]).

In this paper, we show that the inefficiency caused by the informational loss can be resolved by efficient reallocation of authority via bargaining with monetary transfers over authority to make a decision, no matter how large the difference between parties' preferences is. This idea is inspired by Coase [8] who asserts that if property rights are well-defined, *voluntary bargaining* between parties results in an efficient outcome under complete information. Since we can think of the authority to make decisions as a well-defined property right, it is natural to investigate how bargaining over the decision-making authority affects the social outcome. One might also expect a socially efficient outcome in our framework. However, the result is not certain in our case because of the presence of an informational asymmetry between the parties. For example, Farrell [11] shows that in the presence of two-sided private information, voluntary negotiation does not lead to the first-best outcome that maximizes joint surplus.

We consider a bargaining game in which an informed but self-interested agent makes a price offer for decision-making authority to an uninformed principal who then decides either to accept or to reject the offer. We show that there exists a continuum of equilibria in which an efficient action is taken for any realization of the state. An agreement between parties is always reached in this kind of equilibrium and, thus, delegation takes place with probability one and no information is transmitted in equilibrium. If we allow the principal to randomize between accepting and rejecting price offers, however, there always exists an equilibrium with informative price offers. We construct an infinite sequence of informative equilibria with n number of partition intervals in which the agent types in the same interval makes a common price offer to the principal. We show that this sequence of equilibria converges to the ex-post efficient outcome as n goes to infinity.

Bargaining over authority to make a decision is common in the corporate world between two separately owned companies. When an international manufacturer enters a particular national market, it typically lacks relevant information about local market conditions and has difficulties making decisions on pricing, marketing, advertising, distribution and so on. As a result, it sells an exclusive distributorship to a domestic company who is better-informed but lacks authority to make such decisions. If a license agreement is reached through bargaining, the domestic company pays royalties in return for the exclusive right to make decisions about pricing, marketing, advertising, distribution, and so on in the domestic market.[1] For example, I.B.M., the world's largest computer maker in the 1990's, agreed to allow Mitsubishi to sell an I.B.M. mainframe computer under its own name in Japan in April, 1991.[2] More recently, tobacco industry leader Philip Morris International announced an agreement with Chinese National Tobacco under which Chinese National Tobacco will manufacture Marlboro cigarettes

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[1] This license agreement is different from contracting in the adverse selection literature: the international manufacturer does not make a contracting offer which is contingent on every possible decisions or actions. Instead, it usally sells "the right" to make a decision.

[2] Andrew Pollack, "IBM Model to Be Sold By Mitsubishi," *The New York Times* (April 29, 1991), 17.

for marketing in China.[3] If international manufacturers cannot find any partners to make a license agreement, then they are able to found their own corporation in the national market and to start their businesses by themselves. For instance, in the mid-1990s, dozens of foreign beer brewers such as Annheuser-Busch, Heineken, South African Breweries (SAB), Carlsberg, Interbrew, San Miguel, Kirin, Lion Nathan and Foster's entered the Chinese market without making any license agreement.[4]

The situation considered arises frequently in the pharmaceutical industry, especially between an R&D firm with a patent who is less-experienced in commercializing and an experienced manufacturing company. For instance, Animas Corporation, an insulin infusion pump manufacturing company, set up license and development agreements with the Swiss R&D company, Debiotech for intellectual property related to next-generation insulin pumps and micro-needles. In return for the exclusive worldwide license to make, use and sell products utilizing the intellectual property portfolio that includes over 70 issued patents, Animas paid \$12 million in cash and issued 400,000 restricted share of Animas common stock for the right.[5]

The efficiently reallocated authority via bargaining allows parties to make a full use of the decision-relevant information and as a result, leads to a Pareto improvement. We compare the equilibrium outcomes of the model with several dispute resolution schemes studied in the literature such as communication (Crawford and Sobel [9]), optimal mediation (Goltsman, Hörner, Pavlov and Squintani [14]), optimal delegation (Alonso and Matouschek [3], Holmström [21], Kováč and Mylovanov [24] and Melamud and Shibano [28]) and optimal compensation contract (Krishna and Morgan [27]). We show that any equilibrium outcomes of this model are Pareto superior to outcomes of those other schemes when the parties' preferences diverge to a substantial degree.

The rest of the paper is organized as follows. In the next section, we discuss the related literature. In section 3, we describe the model. Focusing on the principal's binary decision between accepting and rejecting a price offer, Section 4 provides the full characterization of the set of perfect Bayesian equilibrium outcomes of the model and its properties are analyzed. In section 5, we show that if we extend the principal's strategy space by allowing randomization between accepting and rejecting price offers, there exists an equilibrium with informative price offers that always yields almost efficient outcomes *ex-post*. Section 6 is devoted to analyzing

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[3] Nicholas Zamiska and Juliet Ye, "Chinese Cigarettes to Go Global" *The Wall Street Journal*, (January 30, 2008) B4.

[4] After years of failing to break into the market, many of them have recently been cutting back, even selling their new state-of-the-art production facilities to local brewers. See Heracleous [19] for more details.

[5] Rick Baron, "Animas acquires technology for disposable insulin micro-pumps and micro-neddles," http://www.bioalps.org/Bioalps/en/Internet/Documents/1996.pdf

welfare of the model. In section 7, we adopt a stronger equilibrium concept called sequential perfect equilibrium and show that this refinement gives us the unique outcome of the game, which satisfies *ex-post* efficiency. In section 8, we investigate the robustness of the existence of *ex-post* efficient equilibrium against state-dependent biases. We conclude in section 9.

2 Related Literature

We may divide literature on strategic interactions between an uninformed principal and a self-interested but informed agent into two strands: one on the reallocation of decision rights and the other on the strategic transmission of information.

Holmström [20], [21] initiates works on the reallocation of authority or delegation problem: the uninformed principal's choice from a set of admissible actions from which the agent can take an action. Focusing on the uniform distribution of the state space and particular preferences, Melumad and Shibano [28] provide full characterization of equilibria. With more general distributions of the state space and preferences, Alonso and Matouschek [3] show that the optimal set of admissible actions takes the form of a single interval if the informed party's preferences are sufficiently similar to the uninformed party's. While most papers have restricted attention to deterministic mechanisms, Kováč and Mylovanov [24] study relative performance of both stochastic and deterministic mechanisms and show that stochastic mechanisms perform strictly better than deterministic ones under some circumstances.

Another strand of the literature investigates the strategic information transmission or simply cheap talk between an informed but self-interested agent and an uninformed principal. Crawford and Sobel [9] (hereafter CS) develop a model of strategic communication in which a better-informed agent sends a possibly noisy signal to a principal, who then takes an action that determines the welfare of both. They show that all equilibria in their model have the form of partition equilibria in which there is only a *finite number of actions* chosen in equilibrium and each action is associated with an interval of states. This means that the final outcome of communication may still be inefficient, even though it can improve social welfare by helping parties to transmit information. An important question arises here concerning how to facilitate communication between parties or when the information transmission can be improved. Several papers answer this question by modifying the CS model to allow more extensive communication (Krishna and Morgan [26]) or to consider the possibility of error in communication (Blume, Board and Kawamura [5].) Recently, Goltsman, Hörner, Pavlov and Squintani [14] allow the parties to use any communication protocol including ones that call for a neutral trustworthy mediator and show that information transmission is improved under the optimal mediation rule. Ivanov [23] demonstrates that for any bias in the parties' preferences, there exists a biased mediator that provides the highest expected payoff to the principal as if the players communicated through a neutral mediator. Although there is no explicit communication in our setting, information transmission is an important issue because the informed agent's price offer can be used as a signalling device. We show that meaningful information can be transmitted through bargaining in which the parties are allowed to use monetary transfers, no matter how widely the parties' preferences diverge.

Some recent papers either allow parties to reallocate both information and authority, or consider the principal's choice between communication and delegation. In a setting with a single decision and a single agent, Dessein [10] studies the optimal allocation of decision-making authority when the uninformed party only can commit to an *ex-ante* allocation of decision rights. He assumes that cheap talk takes place whenever the uninformed principal retains some decision rights and shows that complete delegation dominates communication if the conflict of interests is not serious. The same result is obtained in settings with a multi-divisional organization in which there are two agents who have independent private information (Alonso, Dessein and Matouschek [4].) By exploring a setting with multiple, interdependent decisions, Alonso [2] shows that if activities are complementary the uninformed principal can always improve the informativeness of communication by sharing control with the informed agent.

Although we also consider the reallocation of decision-making authority, our paper is significantly different from others in the following ways. First, we allow explicit monetary transfers for parties which are not possible in other papers mentioned above. Second, contrary to most papers on the delegation problem, the informed agent has commitment power in our model so that he makes a price offer for authority to make decisions. This assumption makes the strategic information transmission, which is not an issue in other papers on the delegation problem, play an important role in our paper. Interestingly in our paper, the aspects of both information transmission and delegation appear at the same time, even though we neither allow the parties to communicate via cheap talk nor consider the principal's choice between communication and delegation. This is because the principal may be able to get meaningful information from the price offer for decision-making authority, which is a main determinant for the reallocation of authority.

There are several papers that consider contracting with monetary transfers in the framework of CS. Krishna and Morgan [27] and Bester and Strausz [7] consider the imperfect commitment model in which the uninformed principal is able to commit on the schedule of transfer payment but not on his action rule. Contrary to our model, the uninformed principal always retains decision-making authority in these models since commitment is only on the transfer but not on the allocation of decision-making authority. Under an optimal contract in Krishna and Morgan [27], the principal should never induce the agent to fully reveal what he knows even though this is feasible. Moreover, the principal never pay the agent for imprecise information. Krähmer [25] considers message-contingent delegation in which the principal can commit the allocation of decision rights after observing cheap talk messages from the informed agent and shows that it creates incentives for information revelation. Bester [6] also studies the contracting problem in the setting with monetary transfer when only decision rights are contractible *ex-ante* and focuses on the question of whether a direct and truthful mechanism can implement the same allocation of decision rights as under perfect information. Contrary to all these papers, the informed agent has a bargaining power in our setting so that he makes an offer to the principal.

Our paper shows that reallocation of decision-making authority is important to attain *ex-post* efficient outcomes, whereas most papers mentioned above focus their attention on either informativeness of equilibrium or the principal's welfare. In this sense, our paper is related to the *incomplete contracting* literature. Grossman and Hart [16] and Hart and Moore [18] develop a theory of property rights based on incentives by considering *incomplete contracting* between principal and agent. They assume that parties are not able to make a complete contract that encompasses all contingencies that might arise and argue that it can lead to *inefficient outcomes*. They suggest an optimal allocation of decision-making authority that minimizes the *ex-post* inefficiency. In this framework, Aghion and Tirole [1] explore the determinants of control rights in an alliance between a research unit and a customer firm to develop new technologies when the lack of financial resources makes the research unit eager to form the alliance with a customer.[6] They consider two different cases: when the research unit has bargaining power and when the customer does. When the research unit has the bargaining power, the ownership of the research output will be efficiently allocated. However, when the customer has the bargaining power, an inefficient allocation of the property rights might occur. Instead of assuming the lack of financial resources, however, this paper focuses on the lack of information that the principal faces and shows that voluntary bargaining over decision-making authority yields efficient outcomes *ex-post*.

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[6] In Aghion and Tirole [1], the research unit initially has decision-making authority. Thus, it is natural to think of the research unit as a principal in our framework.

3 The Model

3.1 Environment

There are two parties, a principal (P) and an agent (A). The principal who initially has decision-making authority has little information about the state of the world $\theta \in \Theta \equiv [0,1]$. She has her prior distribution F over $[0,1]$ which has an absolutely continuous density function $f > 0$. The agent who has different interests from the principal knows the true state of the world θ but does not have decision-making authority. The payoffs for a given allocation of authority depend on an action y taken by the party who has decision-making authority and the state of the world θ. The payoff functions of the parties for a given action being taken are of the form $U^P(y,\theta) = -l(|y-\theta|)$ for the principal and $U^A(y,\theta,b) = -l(|y-(\theta+b)|)$ for the agent.[7] We refer to l as the loss function and assume that $l''(\cdot) > 0$, $l'(0) = 0$ and $l(0) = 0$. This means that the ideal action of the principal is $\overline{y}^P(\theta) = \theta$ and the ideal action of the agent is $\overline{y}^A(\theta,b) = \theta + b$ where $b > 0$ is a parameter that measures how nearly the agent's interest coincides with that of the principal. All of these are common knowledge between parties.

3.2 Bargaining Game

Consider bargaining over the decision-making authority between the informed agent and the uninformed principal. The timing of the game is as follows:

1. The agent privately observes the state of the world $\theta \in \Theta \equiv [0,1]$.
2. The agent offers to pay a price $p \in \mathbb{R}$ for the authority to take an action.[8]
3. The principal decides whether to accept or reject the offer.
4. If the principal accepts the offer then the agent pays the price offered by himself and takes an action, denoted by y^A. In this case, payoffs become $U^P(y^A,\theta) + p$ and $U^A(y^A,\theta,b) - p$ for the principal and the agent respectively. If the principal rejects the offer, however, she takes an action, denoted by y^P, without transferring the decision-making authority. Then payoffs are $U^P(y^P,\theta)$ and $U^A(y^P,\theta,b)$ for the principal and the agent, respectively.

The solution concept we use here is perfect Bayesian equilibrium. For the agent, a strategy consists of a price offer and an action rule. The price offer $\mu^A : \Theta \to \Delta(\mathbb{R})$ specifies the agent's choice of p when the state is θ. The agent's action rule $y^A : \Theta \times \mathbb{R} \to \mathbb{R}$ specifies the agent's choice of action after the principal's acceptance of p, i.e. $y^A(\theta,p)$ is the action taken by the agent type θ whose offer of p is accepted. For the principal, a strategy consists of a decision

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[7] A special case is a quadratic utility ($U^P(y,\theta) = -(y-\theta)^2$ and $U^A(y,\theta,b) = -(y-\theta-b)^2$) which we are assumed in most examples and applications.

[8] We allow p to be negative, which means that the principal pays to the agent.

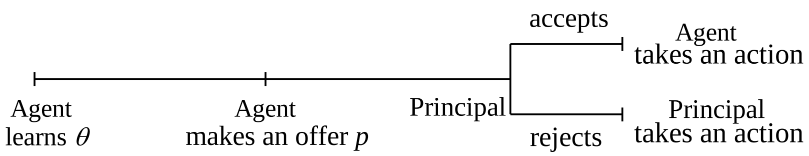

Figure 1: Timing of the game

rule and an action rule. The decision rule, denoted by $d^P : \mathbb{R} \to \{0, 1\}$, specifies the principal's binary decision between accepting and rejecting for each price offer $p \in \mathbb{R}$ that the principal might receive. That is, $d^P(p) = 0$ implies that the principal accepts the offer p while $d^P(p) = 1$ implies that she rejects the offer. It is important to note that we don't allow the principal to use mixed-strategies in her decision rule and we are focusing on the principal's pure strategy for a while. In section 5, we will investigate equilibria in which the principal is mixing in her decision rule. The action rule $y^P : \mathbb{R} \to \mathbb{R}$ specifies the principal's choice of action for each price offer that the principal might receive. The strategy profile $\{\mu^A, y^A, d^P, y^P\}$ and the principal's posterior belief ρ form a perfect Bayesian equilibrium if:

(**BA1**) for each $\theta \in [0, 1]$, $\int_{\mathbb{R}} \mu^A(p|\theta)dp = 1$ and if $p^* \in \mathbb{R}$ is in the support of $\mu^A(\cdot|\theta)$ then p^* solves

$$\max_{p\in\mathbb{R}} d^P(p)U^A(y^P(p), \theta, b) + (1 - d^P(p))(U^A(y^A(\theta, p), \theta, b) - p)$$

(**BA2**) for each $p \in \mathbb{R}$, $d^P(p)$ solves

$$\max_{d^P\in\{0,1\}} (1 - d^P)(p + \int_0^1 U^P(y^A(\theta, p), \theta)\rho(\theta|p)d\theta) + d^P \int_0^1 U^P(y^P(p), \theta)\rho(\theta|p)d\theta$$

(**BA3**) for each $\theta \in [0, 1]$ and each $p \in \mathbb{R}$, $y^A(\theta, p)$ solves

$$\max_{y\in\mathbb{R}} U^A(y, \theta, b)$$

(**BA4**) for each $p \in \mathbb{R}$, $y^P(p)$ solves

$$\max_{y\in\mathbb{R}} \int_0^1 U^P(y, \theta)\rho(\theta|p)d\theta$$

where $\rho(\theta|p)$ is given by Bayes' rule whenever possible.

Notice that the optimal behavior of the informed agent type θ is to take an action $\overline{y}^A(\theta, b) = \theta + b$. After substituting this into (**BA1**), (**BA2**), and (**BA3**), we have the following conditions for the equilibrium.

(**BA1′**) for each $\theta \in [0,1]$, $\int_{\mathbb{R}} \mu^A(p|\theta)dp = 1$ and if $p^* \in \mathbb{R}$ is in the support of $\mu^A(\cdot|\theta)$ then p^* solves

$$\max_{p\in\mathbb{R}} d^P(p)U^A(y^P(p),\theta,b) - p(1-d^P(p))$$

(**BA2′**) for each $p \in \mathbb{R}$, $d^P(p)$ solves

$$\max_{d^P\in\{0,1\}}(1-d^P)(p-l(b)) + d^P\int_0^1 U^P(y^P(p),\theta)\rho(\theta|p)d\theta$$

(**BA3′**) for each $\theta \in [0,1]$ and each $p \in \mathbb{R}$, $y^A(\theta,p) = \theta + b$.

4 Analysis

In this section, we identify the set of equilibria of the model. In what follows, we will first categorize equilibrium candidates into several subcategories according to their properties. Then we will show that it is impossible for some of those candidates to be equilibria of the model. Notice that price offers could be used as signaling devices which convey some information to the principal since the informed agent makes an offer in our model. Thus, a price offer could be either *informative* or *uninformative*. Formally, a price offer on the equilibrium path is informative if and only if the posterior belief after observing the price offer is not the same as its prior belief so that the expected value of θ conditional on p is different from $E_P(\theta)$, the prior expectation of θ.

Definition 1. *A price offer $p \in \mathbb{R}$ on the equilibrium path is informative if $E_P(\theta|p) \neq E_P(\theta)$ where $E_P(\theta|p) = \int_\Theta \theta \cdot \rho(\theta|p)d\theta$.*

Moreover, the agent's price offer could be either a *costless* or a *costly* message. Whether it is costless or costly is determined endogenously in equilibrium. To see this explicitly, let us define the set of acceptable prices in equilibrium as

$$\mathcal{P}^\alpha := \{p \in \mathbb{R}|d^P(p) = 0\}$$

and the set of prices offered in equilibrium as

$$\mathcal{P}^o := \{p \in \mathbb{R}|\exists\theta \in [0,1] \quad s.t. \quad \mu^A(p|\theta) > 0\}.$$

A price offer p is *acceptable* if $p \in \mathcal{P}^\alpha$. A price offer p is *unacceptable* if $p \notin \mathcal{P}^\alpha$. Given $\mathcal{P}^\alpha$, if an agent type θ makes an offer $p \in \mathcal{P}^\alpha$ then the principal accepts the offer and the authority to make a decision is transferred to the agent. Then the principal and agent's payoffs are

$$U^P(y^A,\theta) + p \quad and \quad U^A(y^A,\theta,b) - p, \tag{1}$$

respectively. In this case, the agent's price offer is a costly message.[9]

On the other hand, if the agent type θ makes an offer $p \notin \mathcal{P}^{\alpha}$ then the principal rejects the offer and she retains the authority to make a decision. Then the principal's payoff and the agent's payoff are

$$U^P(y^P, \theta) \quad \text{and} \quad U^A(y^P, \theta, b), \tag{2}$$

respectively. In this case, the agent's price offer is a costless message because it is not included in the payoffs above.

Therefore, a price offer could be one of the followings: *informative* and *acceptable* (costly) price offer, *informative* but *unacceptable* (costless) price offer, *uninformative* but *acceptable* (costly) price offer, and *uninformative* and *unacceptable* (costless) price offer.

4.1 Equilibrium

This section provides the full characterization of equilibrium outcomes of the model. We first focus on the simplest kind of equilibria, in which all agent types make a common price offer which is acceptable. The next proposition shows that there exists a continuum of such equilibria in this model. Let $\sigma = -\int_0^1 U^P(\overline{y}^P, \theta) f(\theta) d\theta > 0$ where $\overline{y}^P = argmax_y \int_0^1 U^P(y, \theta) f(\theta) d\theta$. That is, $-\sigma$ is the principal's expected utility from her ex-ante optimal action $\overline{y}^P$.

Proposition 1. *For any $p^* \in [l(b) - \sigma, l(b)]$, the following strategies and belief form a perfect Bayesian equilibrium.*
i) For all $\theta \in [0, 1]$, the agent makes a price offer p^* with probability 1.
ii) $d^P(p) = \begin{cases} 0 & \text{if } p \geq p^*, \\ 1 & \text{otherwise.} \end{cases}$
iii) $y^P(p) = \begin{cases} \overline{y}^P & \text{if } p \geq p^*, \\ 0 & \text{otherwise.} \end{cases}$
iv) For any $\theta \in [0, 1]$ and any $p \in \mathbb{R}$, $y^A(\theta, p) = \theta + b$.
v) For any $p < p^*$, $\rho(\theta|p) = \begin{cases} 0 & \forall \theta \in (0, 1], \\ 1 & \text{if } \theta = 0, \end{cases}$ *and for any $p \geq p^*$, $\rho(\theta|p) = f(\theta)$ $\forall \theta \in [0, 1]$.*

Proof. See the appendix. □

Figure 2 illustrates the equilibria described in Proposition 1. The horizontal axis measures the state of the world θ. The vertical axis above zero stands for the action taken by players in different states, whereas the vertical axis below zero stands for the price offers made in different states. The dotted line in the upper quadrant represents the agent's action. It is

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[9] This is true only if the agent's price offer is not equal to zero. The price offer $p = 0$ is always costless no matter whether the principal accepts or not.

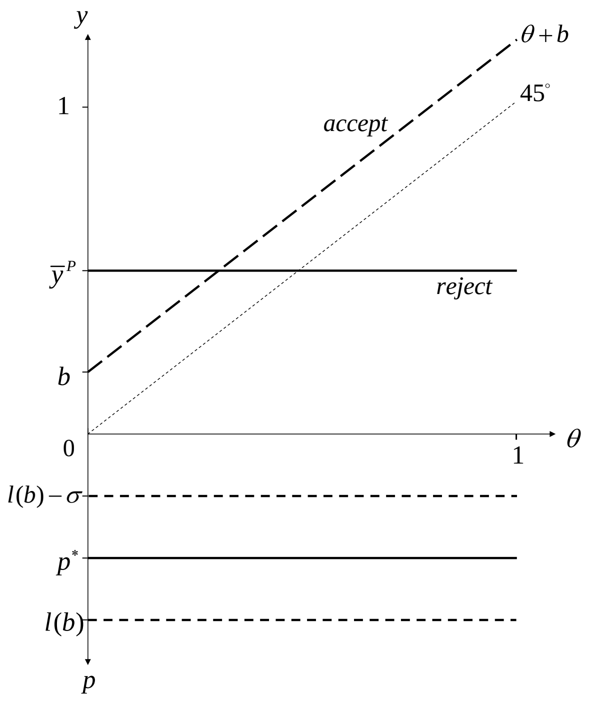

Figure 2: Equilibrium

clear that the equilibrium action should be $\theta + b$, which is the ideal action for the agent in each state. The solid line in the upper quadrant represents the principal's action whenever she has the authority. Since the equilibrium price offer is uninformative, the principal's action should be the same as her ex-ante optimal one, $\overline{y}^P$. The solid line in the lower quadrant depicts the price offers made in different states. It is constant in θ because otherwise the price offer conveys some information to the principal. Lastly, the figure shows that the equilibrium price offer must be in-between $l(b) - \sigma$ and $l(b)$.

Observe that in the equilibrium with $p^* = l(b)$ demonstrated in Proposition 1 the agent type 0 is indifferent between making a price offer p^* and making any unacceptable price offer $p < p^*$. Therefore, there exists an equilibrium where agent type 0 reveals itself by making an unacceptable price offer, which results in the principal's action 0, and all the other types make a price offer $l(b)$, which is accepted.

Proposition 2. *The following strategies and belief form a perfect Bayesian equilibrium.*
i) For all $\theta \in (0, 1]$*, the agent makes a price offer* $l(b)$ *with probability 1.*
i) When making a price offer, the agent type $\theta = 0$ *randomizes over* $(-\infty, l(b)]$.
iii) $d^P(p) = \begin{cases} 0 & \text{if } p \geq l(b), \\ 1 & \text{otherwise.} \end{cases}$

iv) $y^P(p) = \begin{cases} \overline{y}^P & \text{if } p \geq l(b), \\ 0 & \text{otherwise.} \end{cases}$

v) For any $\theta \in [0,1]$ *and any* $p \in \mathbb{R}$, $y^A(\theta, p) = \theta + b$.

vi) For any $p < l(b)$, $\rho(\theta|p) = \begin{cases} 0 & \forall\theta \in (0,1], \\ 1 & \text{if } \theta = 0, \end{cases}$ *and for any* $p \geq l(b)$, $\rho(\theta|p) = f(\theta)$ $\forall\theta \in [0,1]$.

Proof. Since the proof is obvious, I skip it. □

We say that an equilibrium price offer is non-degenerately informative if there is another informative price offer on the equilibrium path. In the equilibrium above, there is an informative price offer but not non-degenerately informative price offer. In the remainder of the paper, we simply say that an equilibrium is informative if and only if there exists a non-degenerately informative price offer on the equilibrium path. Is there any equilibrium with a non-degenerately informative price offer? It is interesting to notice that if all agent types make unacceptable offers in equilibrium, then price offers play exactly the same role as cheap talk messages so that outcome should be the same as one of the equilibria of CS model.[10] That is, unacceptable price offers could be informative non-degenerately. However, not only is there no such equilibrium in this model but also an unacceptable price offer made by a positive measure of agent types can never be part of an equilibrium.

Lemma 1. *There is no equilibrium in which a positive measure of agent types makes an unacceptable offer.*

Proof. See the appendix. □

Why does a positive measure of agent types not make an unacceptable offer in equilibrium? Notice that an unacceptable price offer in our model plays exactly the same role as a costless message in CS model[11] so that if there is an unacceptable price offer on the equilibrium path then the set of agent types who make the offer should be an interval. As a result, the unacceptable price offer could not convey precise information without noise to the principal. However, as shown in Figure 3, some agent types in the interval strictly prefer to take an action by themselves after obtaining decision-making authority rather than allowing the principal to make a decision based on imprecise information. In this figure, the solid curve depicts the agent's expected payoff from making the unacceptable offer p that induces the principal's

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[10] In this case, the message space is $\mathbb{R} \backslash \mathcal{P}^\alpha$.

[11] In CS, every equilibrium should be an interval partitional, i.e. for every message on the equilibrium path, the set of sender types who send the message is a convex interval. This implies that a cheap talk message sent by the informed sender conveys imprecise information to the receiver in equilibrium.

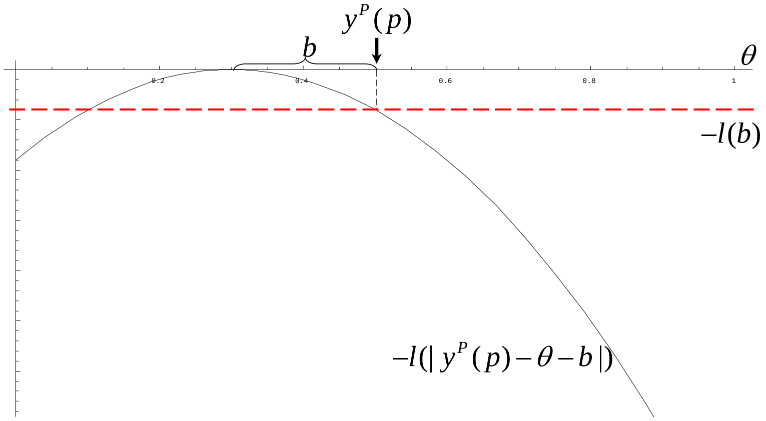

Figure 3: Nonexistence of equilibria with a non-degenerately informative offer

action $y^P(p)$, whereas the dotted line depicts the agent's expected payoff from deviating to the acceptable offer $l(b)$. Notice that the principal accepts any price offer greater than $l(b)$ because accepting the offer always gives her nonnegative expected payoff, while rejecting it could give her at most zero expected payoff. This implies that each agent type in the interval should get at least $-l(b)$ from making the unacceptable offer and does not have an incentive to make an acceptable offer. This is impossible because in any given interval, $y^P(p)$ is in the interior of this interval so that the agent types on the right of $y^P(p)$ are strictly better off by deviating to making the acceptable offer $l(b)$.

This result tells us that there remain only two possibilities: a price offer can be either an *informative and acceptable* offer or an *uninformative but acceptable* offer. The next lemma tells even more about the equilibrium price offer and allows us to eliminate the possibility for having an informative price offer made by a positive measure of agent types.

Lemma 2. *There is no equilibrium in which a positive measure of agent types makes an informative price offer.*

Proof. See the appendix. □

Where does the nonexistence of equilibrium with a non-degenerately informative price offer come from? In our model two different price offers on the equilibrium path cannot be accepted at the same time, because otherwise the agent type who makes a higher price offer has an incentive to make the lower offer. Thus, if an equilibrium is informative, there should be at least one unacceptable price offer on the equilibrium path, which leads to a contradiction.

Lemma 1 and 2 result in the following proposition.

Proposition 3. *All equilibria are outcome equivalent to equilibria demonstrated in Proposition 1 and 2.*

Proof. See the appendix. □

4.2 *Ex-post* Efficiency

In this section, we demonstrate ex-post efficiency of the equilibrium outcome in this model. Although the model of informed agent and uninformed principal has been extensively studied in the literature on communication (Crawford and Sobel [9]) and optimal delegation (Holmström [20]), most of the schemes considered do not completely resolve *ex-post* inefficiency caused by the tension between access to information and authority to make a decision. We demonstrate *ex-post* inefficiency of these schemes in the following example and show that our model leads to a socially desirable outcome.

Example: Suppose that F is uniform and utilities are quadratic. Let $b = 1/5$ and the realized state of the world $\theta = 7/8$. As you can see in Figure 4, if an action y is not in $[\theta, \theta + b] = [7/8, 43/40]$, then there exists another action y' such that both parties strictly prefer y' to y. Suppose that parties communicate via cheap talk. In the most informative equilibrium, only two actions $y_1 = \frac{1}{20}$ and $y_2 = \frac{11}{20}$ are induced.[12] Since $y_1 < \theta$ and $y_2 < \theta$, both actions are inefficient *ex-post*. Alternatively, suppose that the principal optimally proposes the set of admissible actions that the agent can take. In the optimal delegation, the proposed set is $[0, 1 - b] = [0, 4/5]$.[13] As the result, the agent cannot take any action $y \in [7/8, 43/40]$.

Our focus is on whether an ex-post efficient action $y \in [\theta, \theta + b]$ can be taken for any realization of the state of the world θ in an equilibrium of our model. From the previous analysis, we know that parties come to an agreement for (almost) all realization of the state of the world so that the informed agent takes his ex-post ideal action $\theta + b$. Therefore, a socially efficient outcome is always attained in our model. To be more formal, define *ex-post* efficiency as follows. An action is said to be *efficient ex-post* if and only if there is no other feasible action that makes some individual better off without making other individuals worse off after the true state of the world θ is publicly known.

Definition 2. *An action $y \in \mathbb{R}$ is efficient ex-post at θ if there is no other action $z \in \mathbb{R}$ such that*

$$U^P(z, \theta) \geq U^P(y, \theta) \quad \textit{and} \quad U^A(z, \theta, b) \geq U^A(y, \theta, b) \tag{3}$$

---

[12] See the leading example of Crawford and Sobel [9].

[13] See Holmström [20][21], Melumad and Shibano [28], Alonso and Matouschek [3] and Kováč and Mylovanov [24].

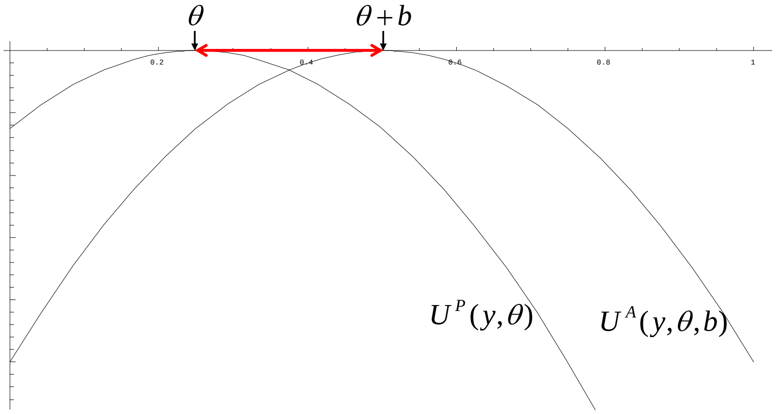

Figure 4: Ex-post Efficiency

with at least one strict inequality.

Now, we are ready to state the main result of this section. It is important to notice that in any perfect Bayesian equilibrium of this model the informed agent makes a decision after buying authority from the principal. The agent is able to use her private information fully so that a socially desirable outcome is attained. The following proposition summarizes this result.

Proposition 4. *In any perfect Bayesian equilibrium of the model an ex-post efficient action is taken for all* $\theta \in [0,1]$.

Proof. Since an action $y = (\theta + b)$ is a unique maximizer of $U^A(y, \theta, b)$, any other actions make the agent worse off. Similarly, an action $y = \theta$ is a unique maximizer of $U^P(y, \theta)$, any other actions make the principal worse off. Notice that the action taken by the party in control in any equilibrium is either θ or $\theta + b$. This completes the proof. $\square$

One might be tempted to argue that the concept of *ex-post* efficiency should be defined over the space of actions and transfers. However, in this paper, it is assumed that decision-right is transferable and contractible but not the decision or action itself. The principal can commit only on the *ex-ante* allocation of decision rights and corresponding monetary transfers. Once the *ex-ante* allocation of decision rights is determined and monetary transfers are made according to the agreement, the monetary transfers have no effect on the incentives of a decision-maker at all and what action can be taken is only an informational issue. Therefore, it is reasonable to discuss efficiency of actions separately from monetary transfers, especially when actions are not contractible.

5 Informative Equilibria

Several recent papers provide theoretical evidence showing that uncertainty could improve the informativeness of communication (Blume, Board and Kawamura [5], Krishna and Morgan [26], and Goltsman, Hörner, Pavlov and Squintani [14].) How then does uncertainty affect the informativeness of equilibria in our model? Instead of introducing an exogenous source of uncertainty, we consider the strategic uncertainty resulting from the players' randomization by allowing the principal to use mixed-strategy in her decision rule and investigate the effect of the strategic uncertainty on the informativeness of equilibria. In what follows, we first investigate some general properties of informative equilibria and show that there exist both monotonic[14] and non-monotonic equilibria with informative price offers. Next, we study the *ex-post* efficiency of these equilibria and see that there exists an informative equilibrium that yields an outcome arbitrarily close to the *ex-post* efficient one.

Let $d^P : \mathbb{R} \to [0,1]$ be the principal's decision rule. Precisely, $d^P(\cdot)$ specifies the *rejection probability* for each price offer $p \in \mathbb{R}$ that the principal might receive. The first question that arises is whether there is an informative equilibrium in this extended strategy space. The following proposition tells us that there is no fully separating equilibrium, even though we allow mixing in the principal's decision rule. Intuitively, if the agent type θ makes an informative price offer that fully reveals her private information, then the principal has a strong incentive to take her optimal action θ by herself after rejecting the offer. Therefore, it is profitable for the agent type $\theta - b$ to imitate the agent type θ.

Proposition 5. *There is no fully separating equilibrium.*

Proof. See the appendix. □

The nonexistence of a fully separating equilibrium does not imply that there is no equilibrium with informative price offers. There indeed exist equilibria with informative price offers in this model. Before we start looking at some examples of informative equilibria in the next section, let us explain why we suddenly have such equilibria once we remove the restriction on the principal's decision rule. Recall that two different acceptable price offers cannot be made by agent types in equilibrium if we prohibit the principal from mixing her strategies because otherwise, making the lower price offer is profitable for the agent types who are making higher but still acceptable offers. This prevents us from having an equilibrium with informative price offers. It turns out that two (or even more) price offers can be accepted *with positive probability* in equilibrium of the extended model. To see details, suppose that there are two (high and

---

[14] The equilibrium action taken by the principal is a nondecreasing function of the state.

low) price offers that will be accepted by the principal with some positive probabilities. It is clear that all agent types get higher payoff from making the low-price offer if it is accepted. However, some agent types might still prefer to make the high-price offer because they infer that an action induced by the offer is much more attractive to them in case of rejection that takes place with positive probability. We will discuss more on this by looking at some examples of informative equilibria in the next session.

5.1 Properties and Examples

What do equilibria with informative price offers look like? Recall that, by Lemma 2, there is no (non-degenerately) informative equilibrium when the principal uses pure strategies. This implies that for equilibria to be informative it is necessary for the principal to use a mixed-strategy in her decision rule. Thus, on the equilibrium path, agent types make some offers that the principal is indifferent between accepting and rejecting.

Lemma 3. *In any (non-degenerately) informative equilibrium, agent types make price offers that the principal is indifferent between accepting and rejecting.*

To get a clear picture of the informative equilibria, we first focus on the monotonic equilibrium in which the equilibrium action taken by the principal is an increasing function of the state, which is reported by the cheap talk literature. Let $\Theta(N) \equiv (\Theta_0(N), ..., \Theta_N(N))$ denote a partition of $[0,1]$ with N steps and dividing points between steps $\theta_0(N)$,...,$\theta_N(N)$, where $0 = \theta_0(N) < \theta_1(N) < \cdots < \theta_{N+1}(N) = 1$. Whenever it can be done without loss of clarity in what follows, we shall write θ or θ_n instead of $\theta(N)$ or $\theta_n(N)$. Define, for all $\underline{\theta}, \overline{\theta} \in [0,1]$ with $\underline{\theta} \leq \overline{\theta}$,

$$y(\underline{\theta}, \overline{\theta}) = \begin{cases} argmax \int_{\underline{\theta}}^{\overline{\theta}} U^P(y,\theta) f(\theta) & \text{if } \underline{\theta} < \overline{\theta}, \\ \overline{\theta} & \text{if } \underline{\theta} = \overline{\theta} \end{cases} \tag{4}$$

Then in any monotonic informative equilibria the following indifference conditions are necessary.

Proposition 6. *In any N-step monotonic informative equilibria,*

$$p_n - l(b) = \int_{\theta_{n-1}}^{\theta_n} U^P\left(y(\theta_{n-1}, \theta_n), \theta\right) \rho(\theta|p_n) d\theta, \qquad (ID-P)$$

$$U^A\left(y(\theta_{n-1}, \theta_n), \theta_n, b\right) d_n - p_n(1-d_n) = U^A\left(y(\theta_n, \theta_{n+1}), \theta_n, b\right) d_{n+1} - p_{n+1}(1-d_{n+1}), \quad (ID-A)$$

and

$$U^A\left(y(\theta_{n-1}, \theta_n), \theta_n, b\right) d_n - p_n(1-d_n) \geq -l(b), \quad \forall \theta \in \Theta_n,$$

for all $n = 1, 2, ...N-1$.

$(ID-P)$ is the indifference condition of the principal for equilibrium price offers and $(ID-A)$ is the indifference condition of the critical agent types. The third condition tells us that equilibrium payoffs of agent types are at least $-l(b)$ so that there are no agent types who have incentives to deviate to off-the-equilibrium-path price offers. The following example gives us one of the simplest two-step equilibrium. In this example, we take a uniform distribution and quadratic utilities for simplicity.

Example 1 (Monotonic equilibrium)**.** *Consider the following strategy profile.*

- *The agent types in* $[0, \theta_1]$ *makes a price offer* $p_1 = b^2 - \frac{\theta_1^2}{12}$.
- *The agent types in* $(\theta_1, 1]$ *makes a price offer* $p_2 = b^2 - \frac{(1-\theta_1)^2}{12}$.
- *The principal rejects the price offers* p_1 *with probability* $d^P(p_1) \in (0,1)$ *but accepts* p_2 *with probability 1.*
- *The principal accepts any price offer* $p \geq b^2$ *with probability 1 but rejects any* $p < b^2$ *which is different from* p_1 *and* p_2 *with probability 1.*
- *The principal takes an action* $y = 0$ *whenever she rejects a price offer* $p < b^2$ *which is different from* p_1 *or* p_2. *The principal takes an action* $y = \frac{1}{2}$ *whenever she rejects a price offer* $p \geq b^2$.
- *The principal takes an action* $y_1 = \frac{\theta_1}{2}$ *if she rejects* p_1 *and takes an action* $y_2 = \frac{1+\theta_1}{2}$ *if she rejects* p_2.
- *The agent type* θ *takes an action* $\theta + b$ *whenever she has decision-making authority.*

It is interesting to see that, for any $b > 0$, we can find proper beliefs and $d^P(p_1) \in (0,1)$ that form a perfect Bayesian equilibrium together with the strategy profile specified above. The strategy profile satisfies all three necessary conditions in Proposition 6. Figure 5 illustrates this equilibrium. All missing calculations can be found in the appendix.

In CS model, there exist indifference conditions similar to $(ID-A)$ for critical agent types which are both necessary and sufficient. Unlike CS, however, the above conditions are not sufficient for equilibria in our model because the agent's interim utility function does not satisfy the single-crossing property, the sorting condition crucially used by CS to get a full characterization of equilibria. This may give us another type of equilibria called non-monotonic equilibria in which equilibrium action taken by the principal is not an increasing function of the state. In a setting with a multi-stage communication, Krishna and Morgan [26] report the existence of such non-monotonic equilibria. Similarly, there are non-monotonic equilibria in our model. Again, for simplicity, we take a uniform distribution and quadratic utilities in the following example.

Example 2 (Non-monotonic equilibrium)**.** *Consider the following strategy profile as a simple candidate for the (three-step) non-monotonic informative equilibrium. Specifically, suppose*

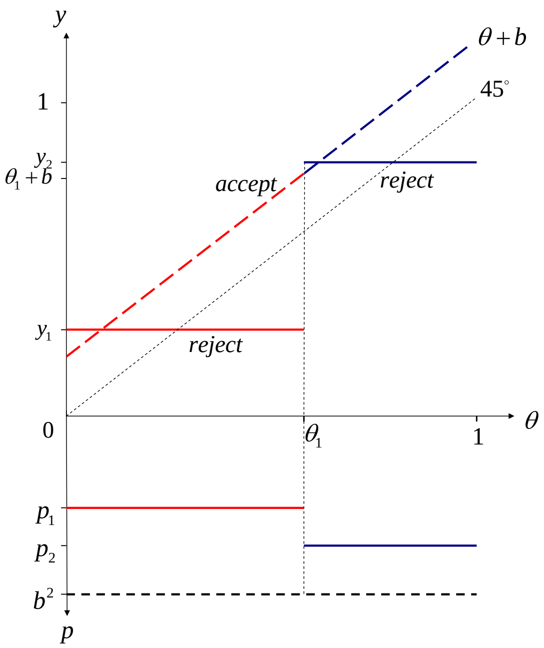

Figure 5: 2-step Monotonic Equilibrium

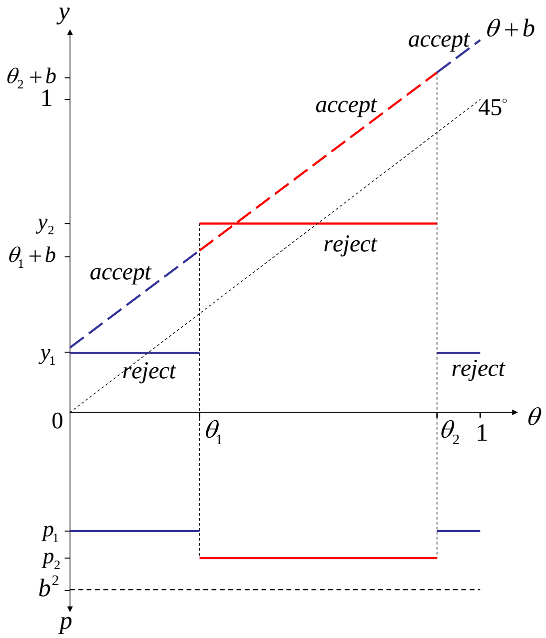

Figure 6: 3-step Non-monotonic Equilibrium

that $b = 1$, $\theta_1 = \frac{49}{100}$, *and* $\theta_2 = \frac{99}{100}$.

- *The agent types in* $[0, \theta_1] \cup [\theta_2, 1]$ *make a price offer* p_1 *that the principal is indifferent between accepting and rejecting.*
- *The agent types in* (θ_1, θ_2) *make a price offer* p_2 *that the principal is indifferent between accepting and rejecting.*
- *The principal rejects the price offers* p_1 *and* p_2 *with probability* $d^P(p_1)$ *and* $d^P(p_2)$ *respectively.*
- *The principal accepts any price offer* $p \geq b^2$ *with probability 1 but rejects any* $p < b^2$ *which is different from* p_1 *and* p_2 *with probability 1.*
- *The principal takes an action* $y = 0$ *whenever she rejects a price offer* $p < b^2$ *which is different from* p_1 *or* p_2. *The principal takes an action* $y = \frac{1}{2}$ *whenever she rejects a price offer* $p \geq b^2$.
- *The principal takes an action* $y_1 = \frac{1+\theta_1^2-\theta_2^2}{2(1+\theta_1-\theta_2)}$ *if she rejects* p_1 *and takes an action* $y_2 = \frac{\theta_1+\theta_2}{2}$ *if she rejects* p_2.
- *The agent type* θ *takes an action* $\theta + b$ *whenever she has decision-making authority.*

Figure 6 illustrates this equilibrium. Again, all missing calculations can be found in the appendix.

5.2 No Upper Bound in N: Uniform Quadratic Example

What is the full characterization of the equilibrium with informative price offers in this model? While we cannot offer a complete answer, this section demonstrates the existence of the infinite sequence of informative equilibria with N partition elements that converges to an *ex-post* efficient equilibrium as N tends to infinity, by focusing on uniform prior and quadratic utilities.

Let us construct a monotonic equilibrium with N interval partitions. Notice that a price offer in equilibrium depends solely on the conditional variance which is determined by the length of the interval in the case of a uniform distribution due to the indifference condition $(ID - P)$. Thus, no two interval partitions can have the same length. Finally, we have the following necessary conditions for monotonic informative equilibria:

1) The lengths of intervals are different from each other.

2) The agent types in each interval make a price offer which the principal is indifferent between accepting and rejecting.

3) The principal randomizes in her decision rule for any price offer on the equilibrium path.

4) The critical agent type θ_n is indifferent between making price offer p_n and p_{n+1}, for $n = 1, 2, ...N - 1$.

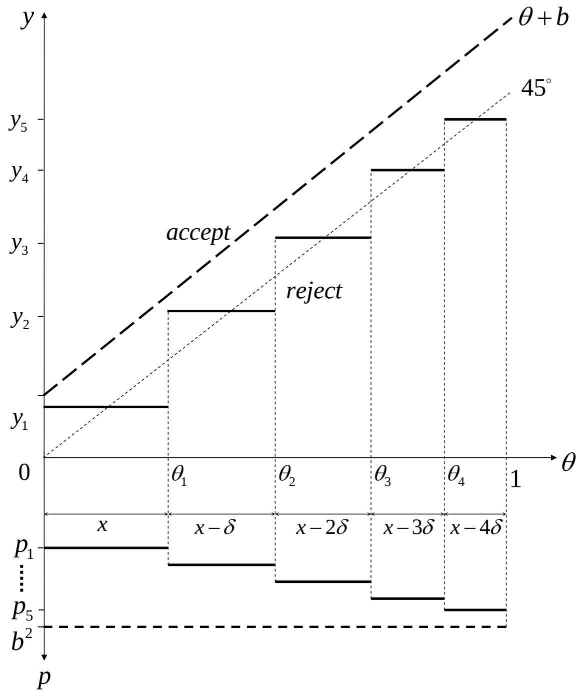

Figure 7: Construction of 5-step Equilibrium

Figure 7 illustrates this monotonic informative equilibrium in which the lengths of intervals are strictly decreasing in n for $N = 5$. Let x and δ denote the length of the first interval and the difference between the lengths of any two adjacent intervals, respectively. Then by the principal's indifference condition, the price offer p_n made by the agent types in the n_{th} interval is strictly increasing in n. Notice that any p_n is less than b^2. By sequential rationality, an agent type takes an action $\theta + b$ whenever she has the authority. If the principal rejects an offer p_n, she then takes the action which is a mid-point of the n_{th} interval. The principal randomizes between accepting and rejecting for any p_n. The following proposition demonstrates the existence of such informative equilibria for any positive integer N and any positive b.

Proposition 7. *Suppose that F is uniform and utilities are quadratic. For any $b > 0$ and any positive integer N, there exist $\delta \in (0, 1)$ and $x \in (0, 1)$ such that the following strategy profile forms a perfect Bayesian equilibrium;*

i) For all $\theta \in [\theta_{n-1}, \theta_n]$ the agent makes a price offer p_n with probability 1, for all $n = 1, ..., N$.

ii) $d^P(p) = \begin{cases} 0 & \textit{if } p \geq b^2, \\ d^* = \frac{\delta}{4(3b+\delta)} & \textit{if } p = p_n \textit{ for all } n = 1, ..., N, \\ 1 & \textit{if } p < b^2 \textit{ and } p \neq p_n. \end{cases}$

iii) $y^P(p) = \begin{cases} \frac{1}{2} & \textit{if } p \geq b^2, \\ y_n & \textit{if } p = p_n \textit{ for all } n = 1, ..., N, \\ 0 & \textit{if } p < b^2 \textit{ and } p \neq p_n. \end{cases}$

iv) For any $\theta \in [0, 1]$ and any $p \in \mathbb{R}$, $y^A(\theta, p) = \theta + b$.

v) $\rho(\theta|p) = \begin{cases} 0 & \textit{if } \theta \in (0, 1], \\ 1 & \textit{if } \theta = 0. \end{cases}$ *for any $p < b^2$ and $p \neq p_n$,* $\rho(\theta|p_n) = \begin{cases} \frac{1}{\theta_n - \theta_{n-1}} & \textit{if } \theta \in (\theta_{n-1}, \theta_n], \\ 0 & \textit{otherwise.} \end{cases}$

for all $n = 1, ..., N$, and $\rho(\theta|p) = 1$ for any $p \geq b^2$, where

(a) $\theta_n = nx - \frac{n(n-1)\delta}{2}$ for all $n = 1, ..., N$,

(b) $\theta_0 = 0$ and $\theta_N = 1$

(c) $p_n = b^2 - \frac{(x-(n-1)\delta)^2}{12}$ for all $n = 1, ..., N$,

(d) $y_n = \frac{(2n-1)x}{2} - \frac{(n-1)^2\delta}{2}$ for all $n = 1, ..., N$.

Proof. See the appendix. □

It is important to note that for any $b > 0$ there is no upper bound for N in this proposition. This means that meaningful information can be transmitted without limit, even though the agent's bias is so great that communication would not be possible via cheap talk. Surprisingly, this remains true even if the divergence of interests is sufficiently large that no meaningful communication occurs in an equilibrium that allows parties to use any communication protocol including the neutral trustworthy mediator studied by Goltsman, Hörner, Pavlov and Squintani [14]. Our model, however, allows the parties to use monetary transfers excluded in

Goltsman, Hörner, Pavlov and Squintani [14]. Krishna and Morgan [27] show that if parties can use monetary transfers, then full revelation of agent's private information can be induced by some contract, no matter how largely the parties' preferences diverge. Therefore, our result is consistent with that of contracting with monetary transfers.

Observe that these equilibria do not need to yield an *ex-post* efficient outcome. Any equilibrium with informative price offers, including the equilibria demonstrated in Proposition 7, contains the principal's mixing in her decision rule. Since there is no fully separating equilibrium, an action taken by the principal after the rejection of any equilibrium price offer is based on imprecise information. As a result, it has to yield an *ex-post* inefficient outcome in some states. However, it is always possible to achieve an outcome *arbitrarily close to* the *ex-post* efficient one in some equilibria. By construction of the equilibrium in Proposition 7, the length of each interval decreases as the number of partitions, N, increases. This implies that increasing N gives more precise information to the principal and allows her to choose an action very close to her (*ex-post*) optimal one, even though *ex-post* optimality is not guaranteed.

6 Welfare

In this section, we assume that utilities are quadratic in order to get not only more precise welfare analysis of our model but also clear welfare comparisons to other models that studied quadratic utility functions. That is,

$$U^P(y,\theta) = -(y-\theta)^2 \quad \textit{and} \quad U^A(y,\theta,b) = -(y-\theta-b)^2. \tag{5}$$

6.1 No *Ex-Ante* Pareto Ranking

Facing multiplicity of perfect Bayesian equilibria, what criterion can we use to get a unique prediction of the game? CS use *ex-ante* Pareto ranking to select the most informative equilibrium. We might also use the same argument to select one equilibrium if there is such a Pareto ranking among equilibria in this model. However, there is no Pareto ranking in our model.

Let $p(\theta)$ be an equilibrium price offer made by the agent type θ. Define the *ex-ante* expected payoff of the principal as

$$EU^P = \int_0^1 \{-(y^P(p(\theta))-\theta)^2 d^P(p(\theta)) + (1-d^P(p(\theta)))(p(\theta)-b^2)\} f(\theta)d\theta, \tag{6}$$

and the *ex-ante* expected payoff of the agent as

$$
\begin{aligned}
EU^A &= \int_0^1 \{-(y^P(p(\theta)) - \theta - b)^2 d^P(p(\theta)) - (1 - d^P(p(\theta)))p(\theta)\} f(\theta) d\theta \\
&= \int_0^1 \{-(y^P(p(\theta)) - \theta)^2 d^P(p(\theta)) - (1 - d^P(p(\theta)))p(\theta)\} f(\theta) d\theta - b^2 \int_0^1 d(p(\theta)) f(\theta) d\theta \\
&= EU^P - 2\int_0^1 (1 - d^P(p(\theta)))p(\theta) f(\theta) d\theta + b^2 \int_0^1 (1 - 2d^P(p(\theta))) f(\theta) d\theta. \qquad (7)
\end{aligned}
$$

While equilibrium actions taken by the principal and the bias between parties are the only determinant of parties' expected payoffs in CS, price offers and corresponding rejection probabilities play an additional important role to determine parties' payoffs in (7) so that we cannot have any Pareto ranking between equilibria in this model.

By $(ID - P)$, the following holds in any informative equilibrium:

$$
\int_0^1 -(y^P(p(\theta)) - \theta)^2 d^P(p(\theta)) f(\theta) d\theta = \int_0^1 (p(\theta) - b^2) d^P(p(\theta)) f(\theta) d\theta. \qquad (8)
$$

After substituting this into (6) and (7), we have

$$
EU^P = \int_0^1 p(\theta) f(\theta) d\theta - b^2 \;\; and \quad EU^A = -\int_0^1 \{(1 - 2d^P(p(\theta)))p(\theta) + 2d^P(p(\theta)) b^2\} f(\theta) d\theta. \qquad (9)
$$

Notice that by $(ID - P)$, $p(\theta) \in (b^2 - \sigma, b^2)$ for all $\theta \in [0, 1]$. This gives the following result.

Proposition 8. *Suppose that utility functions satisfy (5). In any informative equilibria,* $EU^P \in (-\sigma, 0)$ *and* $EU^A \in (-b^2, \sigma - b^2)$.

Proof. See the appendix. □

This result tells us that although there is no general *ex-ante* Pareto ranking among equilibria, both uninformative and informative equilibria should be in between two extreme pure-strategy equilibria- $E1$ (the perfect Bayesian equilibrium with $p^* = b^2 - \sigma$) and $E2$ (the perfect Bayesian equilibrium with $p^* = b^2$) in terms of ex-ante payoffs. This means that $E1(E2)$ is the best(worst) equilibrium for the agent and at the same time the worst(best) equilibrium for the principal, in terms of the *ex-ante* payoffs. We use this result to get a clear welfare comparison between our model and other models studied in the literature on Crawford and Sobel [9] and Holmström [20][21].

6.2 Benefit from Trading Decision-making Authority

In this section, we demonstrate the benefit from trade of decision-making authority by comparing the equilibrium outcomes of our model to those of several dispute resolution processes studied in the same framework: communication (Crawford and Sobel [9]), optimal mediation (Goltsman, Hörner, Pavlov, and Squintani [14]), optimal delegation (Holmström [20][21], Melumad and Shibano [28], Alonso and Matouschek [3] and Kováč and Mylovanov [24]) and optimal compensation contract (Krishna and Morgan [27]). To get more clear comparative results we focus on the uniform distribution in this section. It will be shown that there exist perfect Bayesian equilibria of this model Pareto superior to the equilibrium outcomes of all of these schemes especially when the parties' preferences are misaligned to a substantially large degree.

CS consider a situation in which the principal has no commitment power at all and sends cheap-talk messages to the agent. It is shown that all equilibria in their model are interval partitional so that there is only a finite number of actions chosen in equilibrium, each associated with an interval of states. With uniform quadratic assumption, they show that the number of distinct equilibrium outcome, denoted by $N_{CS}(b)$, is

$$N_{CS}(b) = \Big\langle -\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2}{b}}\Big\rangle \tag{10}$$

where $\langle z \rangle$ denotes the smallest integer greater than or equal to z. Moreover, there is a Pareto ranking among $N_{CS}(b)$ equilibria so that, for any $b > 0$, the number of elements of the partition associated with the *Pareto dominant* equilibrium, which we will call the *best* equilibrium, is $N_{CS}(b)$. The expected payoff of the principal in this *best* equilibrium is

$$EU^P_{CS}(b) = -\frac{1}{12N_{CS}(b)^2} - \frac{b^2(N_{CS}(b)^2-1)}{3} \tag{11}$$

while the *ex-ante* expected payoff for the informed agent is

$$EU^A_{CS}(b) = EU^P_{CS}(b) - b^2. \tag{12}$$

Recently, Goltsman, Hörner, Pavlov, and Squintani [14] allow the parties to use any communication protocol, including the ones that call for a neutral trustworthy mediator. According to the optimal mediation rule, the parties' expected payoffs are

$$EU^P_{mediation}(b) = -\frac{b(1-b)}{3} \quad \textit{and} \quad EU^A_{mediation}(b) = EU^P_{mediation}(b) - b^2. \tag{13}$$

Holmström [20][21], Melumad and Shibano [28], Alonso and Matouschek [3] and Kováč and Mylovanov [24] study the principal's optimal choice of the set of admissible actions that the agent can take and show that under the optimal delegation scheme, the principal restricts project choices of the agent to be from 0 up to a maximum of $1-b$. Under this scheme, the parties' expected payoffs are

$$EU^P_{delegation}(b) = -\frac{b^2(3-4b)}{3} \quad \textit{and} \quad EU^A_{delegation}(b) = -\frac{8b^3}{3}. \tag{14}$$

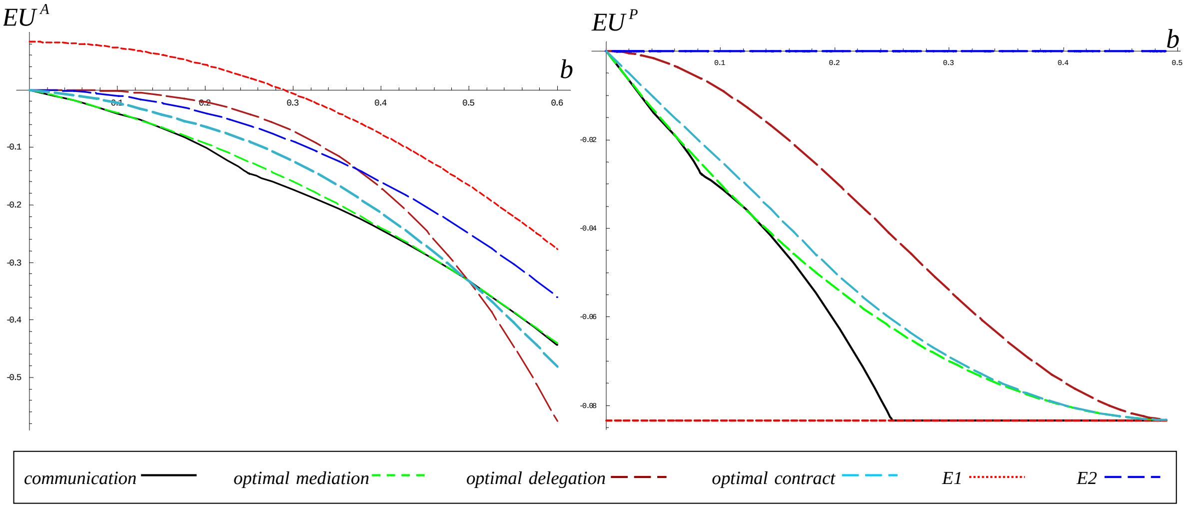

Figure 8: (a) Agent's expected payoff (b) Principal's expected payoff

In these papers, the principal also has imperfect commitment power so that she can only commit on the *ex-ante* allocation of decision rights. Moreover, the monetary transfer is impossible.

Krishna and Morgan [27] consider the situation in which the principal can commit to pay the agent for his advice but retains decision-making authority. They fully characterize the optimal compensation contract: the optimal compensation contract involves separation in low states and a finite number of pooling intervals in high state and the principal never pays for imprecise information. In this optimal compensation contract, the expected payoffs for the principal and the agent are

$$EU^P_{contract}(b) = -\int_0^{a_0} (2b(a_0-\theta)+t_0)d\theta - \frac{1}{12}\sum_{i=1}^{K}\left(\frac{1}{K}-\frac{a_0}{K}-2b(K-2i+1)\right)^3 \tag{15}$$

and

$$EU^A_{contract}(b) = EU^P_{contract}(b) - b^2 + 2\int_0^{a_0} (2b(a_0-\theta)+t_0)d\theta \tag{16}$$

where

$$K = \left\langle -\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{3}{2b}}\right\rangle,$$

$$a_0 = \frac{3}{4}-\frac{1}{4}\sqrt{4+\frac{1}{3}(3-8bK(K-1))(8bK(K+1)-3)} \quad \textit{and}$$

$$t_0 = \frac{(1-a_0-2K(K-1)b)(2bK(K+1)-(1-a_0))}{4K^2}.$$

In our model, the principal has stronger commitment power so that she can commit on the allocation of decision rights correspond to the agent's monetary transfers. In equilibrium, the agent takes an action $\theta + b$ after paying the price $p^* \in [b^2 - \frac{1}{12}, b^2]$ of authority to the principal.[15] As a result, the equilibrium payoffs are

$$EU^A(p^*, b) = \int_0^1 -((\theta + b) - \theta - b)^2 f(\theta)d\theta - p^* = -p^* \tag{17}$$

$$EU^P(p^*, b) = \int_0^1 -((\theta + b) - \theta)^2 f(\theta)d\theta + p^* = p^* - b^2 \tag{18}$$

for the agent and the principal, respectively.

The (*ex-ante*) payoff comparison between equilibrium outcomes of these schemes and equilibria of our model is shown in Figure 8. $E1$ and $E2$ represent the perfect Bayesian equilibria with $p^* = b^2 - \frac{1}{12}$ and $p^* = b^2$, respectively. For any $b > 0$ and any $p^* \in [b^2 - \frac{1}{12}, b^2]$, the informed agent's expected payoff in our model is strictly greater then the equilibrium payoff of CS and optimal mediation. Although optimal delegation yields higher expected payoff than some equilibrium of this model for the agent for small b, there always exists a continuum of equilibria that gives strictly higher payoff to the agent than all other schemes compared. Furthermore, for any $b > 0$, the set of equilibria that give the principal strictly higher payoff than the equilibrium of any other schemes is nonempty. It is interesting to note that this set becomes larger as b increases. When $b > \frac{1}{2}$, all equilibria in our model are Pareto superior to all other schemes considered. This result is summarized in the following proposition. Detailed proofs are omitted since Figure 8 demonstrates a clear welfare comparison.

Proposition 9. *Suppose that F is uniform and utilities are quadratic. For any $b > 0$, there exists a perfect Bayesian equilibrium in this model which is ex-ante Pareto superior to equilibria of several other dispute resolution schemes such as communication, optimal mediation, optimal delegation and optimal contract. Moreover, for sufficiently large b, all equilibria in this model are ex-ante Pareto superior to equilibria of these schemes.*

This welfare result does not implies that bargaining mechanism is superior to all other schemes considered in the literature. The higher ex-ante utilities comes from different assumptions on the principal's commitment power. Unlike the most papers in the literature, this model assumes not only the principal can commit on the ex-ante allocation of decision right but also monetary transfer is available. The welfare comparison shows the benefit of using monetary transfer to trade decision rights in our environment though.

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[15] Note that $\frac{1}{12}$ is a variance of the random variable distributed uniformly over $[0, 1]$.

7 Refinement

From the previous sections, we know that there is a continuum of pure strategy equilibria, each of which has a different price offer on the equilibrium path and (at least) infinitely many mixed-strategy equilibria. Therefore, it seems necessary to reduce the set of equilibrium outcomes to understand the model completely. In this section, we apply a stronger equilibrium concept called *perfect sequential equilibrium* developed by Grossman and Perry [17] to refine equilibria. We will first show that there is a unique pure-strategy perfect sequential equilibrium for any $b > 0$ in our model. Next, we will extensively apply the refinement to the mixed-strategy equilibria and show that there is no mixed-strategy perfect sequential equilibrium when $b < l^{-1}(\sigma)$.

Where does the multiplicity of equilibria come from? In any equilibrium of this game, there exists a continuum of out-of-equilibrium path price offers. If the price offer is one to which the equilibrium assigns positive probability, a posterior distribution of an agent's type can be computed using Bayes' rule. However, Bayes' rule does not determine the posterior distribution over type after observing the price offer to which the equilibrium assigns probability 0. Notice that the principal's choice of action after rejection of out-of-equilibrium price offer depends solely on the principal's posterior belief that we are completely free to choose. Therefore, without additional restriction on beliefs off the equilibrium path, any action can be induced as a best response to some beliefs off the equilibrium path and this leads to multiplicity of perfect Bayesian equilibrium.

To see this more precisely, compare the following extreme perfect Bayesian equilibria from proposition 1.

E1: the perfect Bayesian equilibrium with $p^* = l(b) - \sigma$,
E2: the perfect Bayesian equilibrium with $p^* = l(b)$

Notice that the price offer in *E1* is the least costly and most attractive to the agent. Thus, it seems reasonable that the agent makes this price offer to minimize the loss from the payment. This is also true in any equilibrium including *E2*. However, this could not happen in *E2* (and also any other equilibria except *E1*) because the price offer $l(b) - \sigma$ would be rejected in the equilibrium. Interestingly, the reason why the principal rejects the offer is that she certainly believes the state of the world is $\theta = 0$ if she accidently observes the offer. This belief gives the principal higher expected payoff from the rejection than from the acceptance, even though the price offer is the least costly. However, there is no reason why the principal has such an extreme belief off the equilibrium path. Thus, *E2*, as well as any other equilibria except *E1*, might not be supported by some different belief.

How could we eliminate some perfect Bayesian equilibria which contain unreasonable behaviors? In other words, how can we find a reasonable restriction on the belief off the equilibrium path? We adopt a stronger refinement called *perfect sequential equilibria* introduced by Grossman and Perry [17]. This refinement is closely related to the concept of *neologism-proof equilibria* developed by Farrell [12].[16] The refinement involves a *consistent interpretation* of a deviation from a perfect Bayesian equilibrium.[17] Formally, an interpretation of a deviation is a hypothesized (nonempty) subset of the type space by the principal who observes the deviation that members of the specified subset are responsible for the deviation. For a given interpretation, the principal's posterior belief is her prior belief renormalized over the interpretation. Then, sequential rationality determines whether the principal accepts or rejects the deviation. Each agent type θ in $[0, 1]$ can compute its payoff from offering the deviation and can compare this with its payoff in the perfect Bayesian equilibrium. A *consistent interpretation* is a fixed point of the map described above.

Definition 3 (Gertner, Gibbons and Scharfstein). *An interpretation of a deviation is consistent if the set of agent types who strictly prefer its payoff from offering the deviation to its equilibrium payoff is equivalent to the interpretation.*

It seems reasonable to take the associated posterior belief for the principal if a deviation has a consistent interpretation. By construction, this belief destroys the equilibrium by motivating agent types in the interpretation to deviate. This argument gives us the following additional restriction on the perfect Bayesian equilibrium.

(**BA5**) There is no deviation with a consistent interpretation.

An equilibrium that satisfies condition (**BA1**)$\sim$(**BA5**) is called as a *perfect sequential equilibrium.*

Let us go back to the previous comparison to see if there is a perfect sequential equilibrium. Does the deviation $\widehat{p} = p^* - \varepsilon > l(b) - \sigma$ (with arbitrarily small $\varepsilon > 0$) have a consistent interpretation in equilibrium $E2$? Observe that this deviation might be beneficial for all agent types in $[0, 1]$ if they believe that the deviation is acceptable. So, assume that the principal who observes the deviation interprets the deviation as an offer from all agent types in $[0, 1]$. The principal's posterior belief is the same as her prior so that her optimal action is μ once

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[16] However, *neologism-proof* differs from *perfect sequential equilibria* because it imposes stronger requirement to an equilibrium. Precisely, an equilibrium is *neologism-proof* only if it is supported by all, rather than one, credible updating rules for interpreting deviations off the equilibrium path.

[17] A *consistent interpretation* of a deviation is used first by Gertner *et al* [13]. It is equivalent to the *credibility of neologism* in Farrell [12] and *consistent belief* derived by credible updating rule in Grossman and Perry [17].

the offer is rejected. Then the principal expects to get $-\sigma$ from rejecting the offer. However, accepting the offer gives the principal $\widehat{p} - l(b) > -\sigma$ so that it is optimal for the principal to accept the offer. Under the deviation being accepted, all agent types become strictly better off by deviating to the offer. As the result, the deviation $\widehat{p}$ in equilibrium $E2$ has a consistent interpretation $[0, 1]$. Notice that this is true for any perfect Bayesian equilibria except $E1$. Therefore, any perfect Bayesian equilibrium with the price offer $p^* \in (l(b) - \sigma, l(b)]$ does not satisfy the condition (**BA5**). It remains to show that the set of perfect sequential equilibria is nonempty.[18] The following proposition shows that the perfect Bayesian equilibrium with the price offer $p^* = l(b) - \sigma$ demonstrated in proposition 1 is indeed a perfect sequential equilibrium.

Proposition 10. *There is a unique pure-strategy perfect sequential equilibrium in this model. In the perfect sequential equilibrium, all agent types make a common price offer $l(b) - \sigma$ and the principal accepts the offer.*

Proof. See the appendix. $\square$

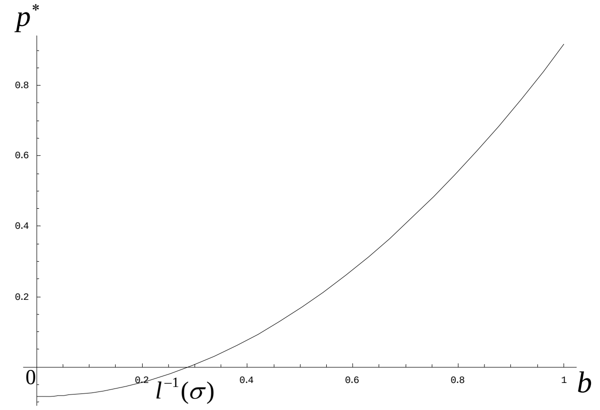

Figure 9: Equilibrium payment

Remark: The above result points to an interesting relationship between the equilibrium price offer and the parameter b, which measures the divergence of preferences. Notice that if $b > l^{-1}(\sigma)$, then the equilibrium price offer is positive. (See Figure 9.) This means that the agent pays some amount of money to the principal for the authority. In reality, such payment is often called a *royalty*. Intuitively, when b is large, using the agent's private information by means of selling authority becomes less valuable or sometimes even harmful for the principal

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[18] It is possible that the set of perfect sequential equilibria is empty. See Grossman and Perry [17]. Neologism-proof equilibria are also suffering from the same problem. See Farrell [12].

because it allows the agent to choose her optimal action, which is distant from the principal's optimal one. Thus, the agent has to give the principal monetary compensation in order to get authority. This explains the positive price offer when b is large. When $b < l^{-1}(\sigma)$, however, the equilibrium price offer is negative. This means that the principal pays the agent, as we frequently observe in interfirm relationships. Examples include *consulting fees* in projects conducted through the outside consultation by other firms. Intuitively, small b increases the principal's relative value of information against the authority because the agent's private information allows her to choose an action similar to the principal's optimal action. Therefore, the principal is willing to pay some amount of money to the informed agent for giving her the authority to make a decision that fully reflects the agent's private information.

We can apply the same logic to refine outcomes of informative perfect Bayesian equilibria. Take an arbitrary informative equilibrium of the model and consider the deviation $\widehat{p} = l(b) - \sigma$. Notice that an agent type's interim payoff is a convex combination of the price he would pay and some negative utility driven by some action taken by the principal. Since the price offer made by the principal in any equilibrium should be less than $l(b) - \sigma$, any agent type in $[0, 1]$ prefers to make the price offer $\widehat{p}$ if $\widehat{p} > 0$. Together with the fact that $\widehat{p}$ is one of the acceptable common price offer of uninformative equilibria, this results in the following proposition.

Proposition 11. *There is no informative perfect sequential equilibrium if* $b < l^{-1}(\sigma)$.

Proof. See the appendix. □

8 State-dependent Biases

In this section, we investigate the robustness of the existence of the equilibrium in which an *ex-post* efficient action is taken for any realization of the state under state-dependent biases: we find the necessary and sufficient condition for such equilibria to exist. The state-dependent biases are studied by several papers in the literature, for example, Alonso and Matouschek [3], Melumad and Shibano [28] and Gordon [15].

Let $b(\theta)$ be the state-dependent bias, a continuous function from type space to real number. Since the type space is compact and the function is continuous, $b(\theta)$ has its maximum and minimum. Recall that if there exists any *ex-post* efficient equilibrium then it should be pooling equilibria. This allows us to start with the following strategy of the agent: all agent types make a common price offer p^*. Since the equilibrium has to be efficient *ex-post*, the principal should accept the price offer with probability 1. This gives the following necessary condition:

$$-\sigma \leq p^* - \int_0^1 l(|b(\theta)|) \cdot f(\theta) d\theta = p^* - \overline{b}, \tag{19}$$

where $\bar{b} = \int_0^1 l(|b(\theta)|) \cdot f(\theta) d\theta$.

Consider the principal's strategy as conservative as possible that includes the worst action for agent types who deviate from the equilibrium behavior: the principal accepts any price offer $p \geq p^*$ and rejects any price offer $p < p^*$. After rejecting $p < p^*$, she takes an action y^* where

$$y^* = arg \min_{y \in [0,1]} \left(\max_{\theta \in [0,1]} U^A(y, \theta, b(\theta)) \right). \tag{20}$$

Notice that such a y^* exists since the set of state is compact and utilities are continuous. Under the principal's strategy specified above, the following is sufficient for any agent type in $[0, 1]$ not to have an incentive to deviate to any price offer $p < p^*$:

$$\min_{y \in [0,1]} \left(\max_{\theta \in [0,1]} U^A(y, \theta, b(\theta)) \right) \leq -p^*. \tag{21}$$

An *ex-post* efficient equilibrium exists if and only if there exists p^* satisfying (19) and (21).

Proposition 12. *There exists an equilibrium in which an ex-post efficient action is taken for any realization of the state if and only if*

$$\min_{y \in [0,1]} \left(\max_{\theta \in [0,1]} U^A(y, \theta, b(\theta)) \right) \leq \sigma - \bar{b}. \tag{22}$$

Observe that the left-hand side of equation (22) is at most 0. This means that if $\bar{b}$ is small enough then there always exists an equilibrium in which an *ex-post* efficient action is taken for any realization of the state.

9 Conclusion

This paper considers bargaining over the authority to make a decision between an uninformed principal and a privately informed but self-interested agent. In spite of private information, bargaining between parties results in *ex-post* efficient outcomes. This explains why we frequently observe delegation in both interfirm and intrafirm relationships.

Meaningful information can always be transmitted in equilibrium, however, if we allow mixing in the principal's decision rule. Although equilibria with informative price offers may not yield *ex-post* efficient outcomes, there exists an informative equilibrium that yields an outcome arbitrarily close to the *ex-post* efficient one.

Some extensions of this model might be interesting. We may investigate bargaining models with different procedures such as, multi-stage bargaining with alternative offers. Moreover, it seems natural that bargainers can communicate through cheap-talk in bargaining procedures. So, it might be worthwhile to consider bargaining either before or after communication. More general contracting offers by informed agents in the same environment is worth investigating. We leave them for future research.

A Appendix. Remaining Proofs and Calculations

Proof of Proposition 1. First, consider the principal's incentives on the equilibrium path. After observing p^*, the principal updates her belief by Bayes' rule, i.e.

$$\rho(\theta|p^*) = \frac{\mu^A(p^*|\theta)f(\theta)}{\int_0^1 \mu^A(p^*|\theta')f(\theta')d\theta'} = f(\theta), \quad \forall\theta \in [0,1]$$

Under this belief, the principal's optimal action after observing p^* is

$$y^P(p^*) = arg\max_y \int_0^1 U^P(y,\theta)f(\theta)d\theta = \overline{y}^P$$

Then the principal would get the expected payoff $\int_0^1 U^P(y^P(p^*),\theta)d\theta = -\sigma$ from rejecting p^* whereas she would get $p^* - l(b)$ from accepting p^*. Since $p^* \geq l(b) - \sigma$, the principal has no incentive to reject p^*.

Second, consider the principal's incentives off the equilibrium path. Notice that we are completely free to take any out-of-equilibrium path belief. Let us take the following out-of-equilibrium-path belief for any $p < p^*$:

$$\rho(\theta|p) = \begin{cases} 0 & \forall\theta \in (0,1], \\ 1 & \textit{if } \theta = 0. \end{cases}$$

In words, this implies that the principal is so certain and confident that she believes the true state of the world is $\theta = 0$ for sure whenever she observes any $p < p^*$. Under the belief we take, $y^P(p) = 0$ for any $p < p^*$. Then the principal would get 0 in expectation from rejecting any $p < p^*$ while she would get $p - l(b)$ from accepting it in expectation. Since $p < p^* \leq l(b)$, the principal rejects the offer p.

Let us take the following out-of-equilibrium-path belief for any $p > p^*$:

$$\rho(\theta|p) = f(\theta)$$

Under the belief we take, $y^P(p) = \mu$ for any $p > p^*$. Then the principal would get $-\sigma$ in expectation from rejecting any $p > p^*$ while she would get $p - l(b)$ from accepting it in expectation. Since $p^* \geq l(b) - \sigma$, the principal accepts the offer p.

Third, consider the agent's incentives. Given the principal's strategy and belief above, if the agent type θ makes an acceptable offer $p > p^*$ then her expected utility is $-p < -p^*$. If the agent type θ makes an unacceptable offer $p < p^*$, then she gets $-l(|\theta + b|) < -l(b) \leq -p^*$. If the agent type θ offers p^* then she gets $-p^*$ because the principal would accept this offer and the agent chooses her optimal action $y^A(\theta, p^*) = \theta + b$. Notice that $p^* \leq l(b)$. Since any agent type in $[0,1]$ would get a payoff less than $-p^*$ by offering any $p \neq p^*$, offering p^* with probability 1 is optimal for all agent types in $[0, 1]$. This completes the proof.

Proof of Lemma 1. The proof of Lemma 1 consists of three claims. Let $\overline{P} \equiv \{p | d^P(p) = 1 \ \ \textit{and} \ \ y^P(p) = y\}$. We say that principal's action y is induced by an agent type θ if $\int_{\overline{P}} \mu(p|\theta)dp > 0$.

Claim 1. *In any equilibrium, for every action y, the set of agent types who induce principal's action y is an interval. If this interval has a nonempty interior, then all types in the interior induce only action y.*

Proof. To get a contradiction, suppose that agent types θ_1 and θ_2 with $\theta_1 < \theta_2$ both induce the principal's action y, but an agent type $\theta_3 \in (\theta_1, \theta_2)$ induces the principal's action $y_1 > y$. Then since $U^A_{12}(y, \theta, b) > 0$, agent type θ_2 strictly prefer y_1 to y. Thus, types $\theta_3 \in (\theta_1, \theta_2)$ never induce an action $y_1 > y$. Similarly, types $\theta_3 \in (\theta_1, \theta_2)$ never induce an action $y_2 < y$, because otherwise, by single crossing property agent type θ_1 strictly prefer y_2 to y. Hence agent types in the interval (θ_1, θ_2) have no incentive to induce actions different from y.

Next, let us show that the agent type θ_3 has no incentive to make an acceptable offer. To see this, suppose that the agent type $\theta_3 \in (\theta_1, \theta_2)$ makes an acceptable price offer, denoted by ω. Then revealed preference yields $U^A(y, \theta_1, b) \geq -\omega$ and $U^A(y, \theta_2, b) \geq -\omega$. However, by the concavity of U^A in θ, $U^A(y, \theta_3, b) > -\omega$. That is, the agent type θ_3 strictly prefers to induce the action y rather than make the acceptable price offer ω. This is a contradiction. Thus, agent types in the interval (θ_1, θ_2) have no incentive to make an acceptable offer. This completes the proof. □

Claim 2. *In any equilibrium, for any principal's action y, y is in the interior of the set of agent types who induce the action.*

Proof. Let y denote the action taken by the principal after rejecting p. By Lemma 1, the set of agent types who induces the action y is an interval. $f > 0$ and the concavity of U^P in y complete the proof. □

Claim 3. *There is no equilibrium in which a positive measure of agent types makes an unacceptable offer.*

Proof. To get a contradiction, suppose that an unacceptable price offer p is made by a positive measure of agent types in equilibrium. Let y denote the action taken by the principal after rejecting p. By Claim 1 and 2, the set of agent types who induces the action y can be denoted by $[x, z]$ where $0 \leq x < y < z \leq 1$. By the concavity of U^A in θ, we have

$$U^A(y, \theta, b) < U^A(y, y, b) = -l(b), \quad \forall \theta \in (y, z] \tag{23}$$

However, by the sequential rationality (**BA2$'$**) the principal should accept any price offer $p > l(b)$ because she expects to get at most 0 from rejecting it while she expects to get $p - l(b) > 0$ from allowing the agent to take an action by accepting it. This implies that we are always able to find $\varepsilon > 0$ such that the acceptable price offer $l(b) + \varepsilon$ is profitable for these agent types. This leads to a contradiction. □

Proof of Lemma 2. I claim that $\mathcal{P}^\alpha \cap \mathcal{P}^o$ is a singleton.

Proof. To get a contradiction, suppose that $p', p'' \in \mathcal{P}^\alpha \cap \mathcal{P}^o$ where $p' > p''$. Then there exist $\theta', \theta'' \in [0, 1]$ (possibly $\theta' = \theta''$) such that $\mu^A(p'|\theta') > 0$ and $\mu^A(p''|\theta'') > 0$. Since $p', p'' \in \mathcal{P}^\alpha$, $d^P(p') = 0$ and $d^P(p'') = 0$. Then from **(BA1$'$)**, the agent type θ' gets $-p'$ if she makes the offer p' while she gets $-p''$ if she makes the offer p''. Since $p' > p''$, making the offer p'' is profitable for the agent type θ'. This is a contradiction. This implies that two different prices cannot be accepted in equilibrium. □

Proof of Proposition 3. From Lemma 1 and 2, the set of agent types making an unacceptable price offer is either a singleton or an empty set. Suppose that it is a singleton. Let $\widehat{\theta}$ denote the agent type making the unacceptable price offer. Let p^* denote an acceptable price offer in the equilibrium. Since the agent type $\widehat{\theta}$ reveals a weak preference for making the unacceptable price offer over making the acceptable price offer, we have

$$p^* \geq l(b). \tag{24}$$

Moreover, any agent types except $\widehat{\theta}$ reveal a weak preference for making the acceptable price offer over making the unacceptable price offer, we have

$$-l(\widehat{\theta} - \theta - b) \leq -p^*, \quad \forall \theta \in \Theta \setminus \{\widehat{\theta}\}. \tag{25}$$

By (24) and (25), we have

$$\widehat{\theta} = 0 \quad \textit{and} \quad l(b) = p^*.$$

This implies that any equilibria with informative price offer have to be outcome equivalent to the equilibrium demonstrated in Proposition 2.

Now, suppose that the set of agent types making an unacceptable price offer is empty. Suppose that all agent types in $[0, 1]$ make an acceptable price offer $p < l(b) - \sigma$. Notice that the principal would get $p - l(b) < -\sigma$ from accepting the offer. However, rejecting the offer is profitable for the principal because she can get the expected payoff $-\sigma$ by choosing $y^P(p) = \overline{y}^P$. This is a contradiction. Suppose that all agent types in $[0, 1]$ make an acceptable price offer $p > l(b)$. Suppose that the principal would take an action $y \in [0, 1]$ if the equilibrium price offer p is rejected. Since all agent types in $[0,1]$ reveal a weak preference for making an acceptable price offer over making an unacceptable price offer, we have $\forall y \in [0,1]$,

$$-l(y-\theta-b) \leq -p, \quad \forall \theta \in [0,1].$$

This implies that $p \leq l(b)$ which leads to a contradiction. Therefore, any equilibria with an uninformative price offer have to be outcome equivalent to one of the equilibrium demonstrated in Proposition 1.

Proof of Proposition 5. Suppose that there is a separating equilibrium. Then the agent type θ induces either the principal's action $y^P = \theta$ after the rejection, the agent's action $y^A = \theta + b$ after the acceptance, or both with positive probability. Since two different price offers cannot be accepted with probability 1 in equilibrium, there is at most one agent type who makes an acceptable price offer. Moreover, the principal randomizes accepting and rejecting only if she is indifferent between them, i.e. $p - l(b) = 0$. Hence, there is at most one agent type who makes the price offer of which the principal randomizes accepting and rejecting. This implies that there are at most two agent types who make the offer accepted with positive probability. Thus, almost all agent types in $[0,1]$ make a price offer rejected with probability 1 in this separating equilibrium. Therefore, we can choose one agent type, denoted by θ_1, who makes an offer rejected with probability 1 such that there exists an agent type $\theta_2 \in (\theta_1, \theta_1 + b]$ who also makes an offer rejected with probability 1 for any $b > 0$. Since $-l(|\theta_1 - \theta_1 - b|) = -l(b) < -l(|\theta_2 - \theta_1 - b|)$, the agent type θ_1 has an incentive to pretend to be an agent type θ_2. This is a contradiction.

Calculations for Example 1. In order to verify that the strategy forms a perfect Bayesian equilibrium with some belief, let us consider the principal's incentive on the equilibrium path. After observing p_1, the principal updates her belief by Bayes' rule, i.e.

$$\rho(\theta|p_1) = \frac{\mu^A(p_1|\theta)}{\int_0^1 \mu^A(p_1|\theta')d\theta'} = \begin{cases} \frac{1}{\theta_1} & \textit{if } \theta \in (0, \theta_1], \\ 0 & \textit{otherwise.} \end{cases}$$

Under this belief, the principal's optimal action after observing p_1 is

$$y_1 = arg \max_y \int_0^1 -(y-\theta)^2 \cdot \rho(\theta|p_1)d\theta = \frac{\theta_1}{2}$$

Notice that the expected payoff from accepting p_1 is exactly the same as the expected payoff from rejecting p_1 because

$$\int_0^1 -(y_1-\theta)^2 \cdot \rho(\theta|p_1)d\theta = -\frac{(\theta_1)^2}{12} = p_1 - b^2.$$

Hence, the principal is indifferent between accepting and rejecting p_1. Therefore, any $d^P(p_1) \in [0,1]$ is sequentially rational. Applying the same logic to p_2 allows us to conclude that the principal is indifferent between accepting and rejecting p_2 so that rejecting p_2 with probability 1 is also sequentially rational.

Second, let us look at the principal's incentive off the equilibrium under the following beliefs;

$$\rho(\theta|p) = \begin{cases} 0 & \textit{if } \theta \in (0,1], \\ 1 & \textit{if } \theta = 0 \end{cases} \qquad \textit{for any } p < b^2 \textit{ and } p \neq p_1, p_2,$$

and

$$\rho(\theta|p) = 1 \qquad \textit{for any } p \geq b^2.$$

Suppose that the out-of-equilibrium price offer is $p < b^2$. Then the principal's action, which is sequentially rational under the belief we take, is $y^P(p) = 0$. Then the principal's expected payoff from rejecting any $p < b^2$ is 0 while she would expect to get $p - b^2 < 0$ from accepting the offer. Therefore, the principal rejects the offer whenever $p < b^2$. On the other hand, suppose that the out-of-equilibrium price offer is $p \geq b^2$. Then the principal's action, which is sequentially rational under the belief we take, is $\frac{1}{2}$. Then the principal would expect to get $-\frac{1}{12}$ from rejecting any $p \geq b^2$ while her expected payoff from accepting the offers is $p - b^2 \geq 0$. Thus, the principal accepts the price offer whenever $p \geq b^2$.

Third, consider the agent's incentives. Let us define the agent type θ's expected utility from making a price offer p_i as follows;

$$EU(\theta, p_i) = -(y_i - \theta - b)^2 \cdot d^P(p_i) - (1 - d^P(p_i)) \cdot p_i. \tag{26}$$

We will show that any agent of type $\theta \in (\theta_{n-1}, \theta_n)$ has no incentive to make an offer p_m where $m \neq n$. By the strict concavity of $EU(\theta, p_1)$ on θ and the linearity of $EU(\theta, p_2)$ on θ, it is sufficient to show that the boundary type θ_1 is indifferent between making the offers p_1 and p_2. Formally, we need to find $\theta_1 \in (0,1)$ such that

$$EU(\theta_1, p_1) = EU(\theta_1, p_2). \tag{27}$$

Plugging equation (26) into (27) gives the following;

$$\left(\frac{\theta_1}{2} + b\right)^2 \cdot d^P(p_1) + \left(b^2 - \frac{\theta_1^2}{12}\right) \cdot (1 - d^P(p_1)) = b^2 - \frac{(1-\theta_1)^2}{12}. \tag{28}$$

After some rearrangement, we have

$$4d^P(p_1)\theta_1^2 + 2(6bd^P(p_1) - 1)\theta_1 + 1 = 0. \tag{29}$$

Since $d^P(p_1) \neq 0$, the above equation can be treated as a polynomial in θ_1. Therefore,

$$\theta_1 = \frac{(1 - 6bd^P(p_1)) \pm \sqrt{(1 - 6bd^P(p_1))^2 - 4d^P(p_1)}}{4d^P(p_1)}. \tag{30}$$

To complete our discussion on the existence of monotonic informative equilibrium, it is necessary to show that there exist $d^P(p_1) \in (0, 1)$ and $\theta_1 \in (0, 1)$ that satisfy the equation (30). It is easy to verify that equation (30) is satisfied by the following parameters:

$$\theta_1 = 0.6 \quad and \quad d^P(p_1) = \frac{5}{36(1 + 5b)}.$$

Since $0 < \frac{5}{36(1+5b)} < 1$ for all $b > 0$, we conclude that for any $b > 0$, there exists a (two-step) monotonic perfect Bayesian equilibrium with informative price offers.

Calculations for Example 2. Let us look at the principal's incentive. Under the strategy profile above, if the principal observes the price offer p_1 she then updates her belief by Bayes' rule, i.e.

$$\rho(\theta|p_1) = \begin{cases} \frac{1}{1+\theta_1-\theta_2} & \text{if } \theta \in [0, \theta_1] \cup [\theta_2, 1], \\ 0 & \text{if } \theta \in (\theta_1, \theta_2). \end{cases}$$

The principal's optimal action y_1 after rejecting p_1 is the following;

$$y_1 = arg \max_y \int_0^1 -(y - \theta)^2 \rho(\theta|p_1) d\theta = \frac{1 + \theta_1^2 - \theta_2^2}{2(1 + \theta_1 - \theta_2)} = \frac{13}{50} \tag{31}$$

If the principal observes the price offer p_2 then she again updates her belief by Bayes' rule, i.e.

$$\rho(\theta|p_2) = \begin{cases} \frac{1}{\theta_2-\theta_1} & \text{if } \theta \in (\theta_1, \theta_2), \\ 0 & \text{otherwise.} \end{cases}$$

The principal's optimal action y_2 after rejecting p_2 is the following;

$$y_2 = arg \max_y \int_0^1 -(y - \theta)^2 \rho(\theta|p_2) d\theta = \frac{\theta_1 + \theta_2}{2} = \frac{37}{50} \tag{32}$$

Thus, the principal's actions induced by the price offers on the equilibrium path are sequentially rational under the posterior belief derived from Bayes' rule. It is easy to see that the principal's out-of-equilibrium strategy is sequentially rational if we have the same belief as in the previous example.

Since the principal should be indifferent between accepting and rejecting p_1, we have

$$p_1 = b^2 + \int_0^1 - (y_1 - \theta)^2 \rho(\theta|p_1) d\theta = b^2 + \frac{(y_1 - \theta_1)^3 - y_1^3 + (y_1 - 1)^3 - (y_1 - \theta_2)^3}{3(1 + \theta_1 - \theta_2)} = \frac{29081}{30000} \tag{33}$$

Similarly, we have

$$p_2 = b^2 + \int_0^1 -(y_2 - \theta)^2 \rho(\theta|p_2) d\theta = b^2 - \frac{(\theta_1 - \theta_2)^2}{12} = \frac{47}{48} \tag{34}$$

To see that the agent's price offers are incentive compatible, it is sufficient to show that the boundary types θ_1 and θ_2 are indifferent between making the offer p_1 and p_2. From these arbitrage conditions, we get

$$EU(\theta_1, p_1) = EU(\theta_1, p_2) \tag{35}$$

and

$$EU(\theta_2, p_1) = EU(\theta_2, p_2), \tag{36}$$

where the agent type θ's expected payoff from making the offer p_i is

$$EU(\theta, p_i) = -(y_i - \theta - b)^2 d^P(p_i) - (1 - d^P(p_i))p_i.$$

From (31), (32), (33), (34), (35), and (36), we get

$$d^P(p_1) = 0.00844682 \quad and \quad d^P(p_2) = 0.0125013.$$

It remains to show that no agent type has an incentive to deviate to offers off the equilibrium path. This is equivalent to showing that all agent types who make the offer p_i get higher expected payoff than $-b^2$ for all $i = 1, 2$. Formally, we need to show that

$$EU(\theta, p_1) > -b^2, \quad \forall\theta \in [0, \theta_1] \quad and \quad EU(\theta, p_2) > -b^2, \quad \forall\theta \in (\theta_1, 1] \tag{37}$$

Notice that two indifference conditions and the convexity of utility function on θ guarantee that for any $\theta \in [0, \theta_1] \cap [\theta_2, 1]$, $EU(\theta, p_1) > EU(1, p_1)$ and for any $\theta \in [\theta_1, \theta_2]$, $EU(\theta, p_2) > EU(1, p_1)$. Since we have $EU(1, p_1) = -0.986752 > -1 = -b^2$, there is no agent type who wants to deviate to any other price offers. This completes our discussion on the existence of non-monotonic equilibria.

Proof of Proposition 7. First, consider the principal's incentives on the equilibrium path. After observing p_n, the principal updates her belief using Bayes rule, i.e.

$$\rho(\theta|p_n) = \frac{\mu^A(p_n|\theta)}{\int_0^1 \mu^A(p_n|\theta')d\theta'} = \begin{cases} \frac{1}{\theta_n - \theta_{n-1}} & if\ \theta \in (\theta_{n-1}, \theta_n], \\ 0 & otherwise. \end{cases}$$

Under this belief, the principal's optimal action after observing p_n is

$$y^P(p_n) = arg \max_y \int_0^1 -(y - \theta)^2 \cdot \rho(\theta|p_n) d\theta = \frac{\theta_{n-1} + \theta n}{2}$$

Rewriting this by using (a) gives us the following:

$$y^P(p_n) = \frac{(2n-1)x}{2} - \frac{(n-1)^2\delta}{2}.$$

Notice that the expected payoff from accepting p_n is exactly the same as the expected payoff from rejecting p_n because

$$\int_0^1 -(y^P(p_n) - \theta)^2 \cdot \rho(\theta|p_n)d\theta = -\frac{(x-(n-1)\delta)^2}{12} = p_n - b^2$$

Hence, the principal is indifferent between accepting and rejecting p_n for any $n = 1, ...N$.

Second, consider the principal's incentives off the equilibrium path. Suppose that the out-of-equilibrium price offer $p < b^2$. Then the principal's action which is sequentially rational under the belief we take is $y^P(p) = 0$. Then the principal would get 0 in expectation from rejecting any p while she would get $p - b^2 < 0$ from accepting it in expectation. Therefore, the principal rejects the offer whenever $p < b^2$. On the other hand, suppose that the out-of-equilibrium price offer $p > b^2$. Then the principal's action which is sequentially rational under the belief we take is $y^P(p) = \frac{1}{2}$. Then the principal would get $-\frac{1}{12}$ in expectation from rejecting any $p > b^2$ while she would get $p - b^2$ from accepting it in expectation. Thus, the principal accepts the price offer whenever $p > b^2$.

Third, consider the agent's incentives. We will show that we can choose $\delta \in (0, 1)$ and $x \in (0, 1)$ such that the agent's price offers are incentive compatible. As a first step, we will show that any agent type $\theta \in (\theta_{n-1}, \theta_n)$ has no incentive to make an offer p_m where $m \neq n$. As a second step, we will show that any agent type $\theta \in [0, 1]$ has no incentive to make an offer off the equilibrium path.

Given y_n, p_n, d^* and θ_n, let us define the agent type θ's expected utility from making a price offer p_n as follows;

$$EU(\theta, p_n) = -(y_n - \theta - b)^2 \cdot d^* - (1 - d^*) \cdot p_n. \tag{38}$$

Then, it is easy to see that

$$EU(\theta_n, p_n) = \frac{b(-12b^2 + x^2 + n(n-1)\delta^2 + (x(1-2n) - 4b)\delta)}{4(3b+\delta)} = EU(\theta_n, p_{n+1}) \tag{39}$$

Therefore, the following arbitrage condition hold for all $n = 1, ..., N-1$.

$$EU(\theta_n, p_n) = EU(\theta_n, p_{n+1}). \tag{40}$$

We need to show that for any $\theta \in (\theta_{n-1}, \theta_n)$,

$$EU(\theta, p_n) \geq EU(\theta, p_m), \quad \forall m \neq n.$$

By the arbitrage condition and the strict concavity of $EU(\theta, p)$ on θ, this is the same as showing that

$$EU(\theta_n, p_n) - EU(\theta_n, p_m) \geq 0, \quad \forall m \neq n. \tag{41}$$

From equation (38), we have

$$\begin{aligned} EU(\theta_n, p_n) - EU(\theta_n, p_m) &= -(y_n - \theta_n - b)^2 d^* - (1-d^*)p_n + (y_m - \theta_n - b)^2 d^* + (1-d^*)p_m \\ &= \frac{\delta}{16(3b+\delta)}(m-n)(m-n-1)(-2x+(m+n-2)\delta)(-2x+(m+n-1)\delta). \end{aligned} \tag{42}$$

Notice that the length of each interval should be positive, i.e.

$$x - (n-1)\delta > 0. \tag{43}$$

Then,

$$-2x + (m+n-2)\delta < (m-n)\delta, \ \textit{and} \ -2x + (m+n-1)\delta < (m-n+1)\delta. \tag{44}$$

Thus, if $m \leq n-1$ then

$$EU(\theta_n, p_n) - EU(\theta_n, p_m) > \frac{\delta^3}{16(3b+\delta)}(m-n)^2((m-n)^2-1) \geq 0. \tag{45}$$

Suppose that $m \geq n+1$. Then the choice of $\delta \in (0, \frac{2x}{m+n-1})$ gives us

$$(-2x + (m+n-2)\delta) < 0 \ \textit{and} \ (-2x + (m+n-1)\delta) < 0, \tag{46}$$

so that condition (41) holds. Choose $\delta = \delta_1^* \in (0, \frac{x}{N-1})$. Then, for all $n = 1, ...N$,

$$EU(\theta_n, p_n) \geq EU(\theta_n, p_m), \quad \textit{for all } m \neq n \tag{47}$$

It remains to show that for all $n = 1, ..., N$, $EU(\theta, p_n) > -b^2$ for any agent type $\theta \in (\theta_{n-1}, \theta_n)$. Since $EU(\theta_n, p_n) < EU(\theta, p_n)$ for all $\theta \in (\theta_{n-1}, \theta_n)$, it is sufficient to show that $EU(\theta_n, p_n) > -b^2$. From equation (38), we have

$$EU(\theta_n, p_n) + b^2 = \frac{b(x^2 + n(n-1)\delta^2 + n\delta(1-2n))}{4(3b+\delta)}. \tag{48}$$

Therefore, if we choose $\delta_n \in (0, \frac{x^2}{n(2n-1)})$, then $EU(\theta_n, p_n) + b^2 > 0$. Since N is finite, we can choose $\delta_2^* = \min\{\delta_1, \delta_2, ..., \delta_N\}$. Take $\delta = \min\{\delta_1^*, \delta_2^*\}$.

Notice that from the condition (a) and (b) we have $\delta = \frac{2x}{N-1} - \frac{2}{(N-1)N}$. To complete this proof, we need to show that there exists $x \in (0,1)$ such that $\delta \in (0, \min\{\delta_1^*, \delta_2^*\}] = (0, \min\{\frac{x}{N-1}, \frac{x^2}{N(2N-1)}\}]$. Take $x = \frac{1}{N} + \varepsilon$ with arbitrarily small $\varepsilon > 0$. Then $\delta = \frac{\varepsilon}{N-1}$. Since both δ_1^* and δ_2^* are strictly increasing in x, $\min\{\delta_1^*, \delta_2^*\} > \min\{\frac{1}{N(N-1)}, \frac{1}{N^3(2N-1)}\} = \frac{1}{N^3(2N-1)}$.

It is straight forward to see that with an arbitrarily small $\varepsilon > 0$ (or more precisely when $0 < \varepsilon < \frac{N-1}{N^3(2N-1)}$), $\delta = \frac{\varepsilon}{N-1} < \frac{1}{N^3(2N-1)}$ so that $\delta \in (0, \min\{\delta_1^*, \delta_2^*\}]$. This completes the proof.

Proof of Proposition 8. The first result is directly from the previous discussion. It remains to show that $EU^A \in (-b^2, \sigma - b^2)$. Before I show this, I will first show that $d^P(p(\theta)) \in [0, \frac{1}{2}]$ for all $\theta \in [0,1]$ and use this to prove the result. Take an arbitrary price offer p_0 on the equilibrium path and let Θ_0 be the set of all agent types who make the offer p_0. Let σ_0 denote the conditional variance of θ, i.e. $\sigma_0 = Var(\theta|\theta \in \Theta_0)$. Then by $(ID-P)$, $p_0 = b^2 - \sigma_0$. Moreover, in order for any agent types in Θ_0 not to have an incentive to deviate to off the equilibrium price offers, it is necessary that

$$-d^P(p_0) \cdot (y^P(p_0) - \theta - b)^2 - (1 - d^P(p_0))(b^2 - \sigma_0) \geq -b^2. \tag{49}$$

Take integral with respect to θ in both sides. After some rearrangements, we get

$$d^P(p_0)(-2\sigma_0^2) + \sigma_0^2 \geq 0 \ \ or \ \ \ d^P(p_0) \leq \frac{1}{2} \tag{50}$$

This is true for any equilibrium price offers so that we have $d^P(p(\theta)) \in [0, \frac{1}{2}]$ as was to be shown.

Next, to get the upper bound of EU^A, let us rearrange (9).

$$EU^A = -\int_0^1 p(\theta) f(\theta) d\theta + 2\int_0^1 d^P(p(\theta))\{p(\theta) - b^2\} f(\theta) d\theta.$$

Since $p(\theta) - b^2 \in (-\sigma, 0)$, $p(\theta) \in (b^2 - \sigma, b^2)$, and $d^P(p(\theta)) \in [0, \frac{1}{2}]$ for all $\theta \in [0,1]$, we have

$$-\sigma - b^2 < EU^A < \sigma - b^2. \tag{51}$$

In order for the agent type not to have an incentive to deviate to off-the-equilibrium-path price offers,

$$-(y^P(p(\theta)) - \theta - b)^2 d^P(p(\theta)) - (1 - d^P(p(\theta)))p(\theta) \geq -b^2, \quad \forall \theta \in [0,1].$$

The non-existence of fully separating equilibrium gives

$$-(y^P(p(\theta)) - \theta - b)^2 d^P(p(\theta)) - (1 - d^P(p(\theta)))p(\theta) > -b^2, \quad for\ some\ \theta \in [0,1].$$

Taking integral with respect to θ in both sides gives

$$EU^A > -b^2. \tag{52}$$

From (51) and (52), we have $EU^A \in (-b^2, \sigma - b^2)$. This completes our proof.

Proof of Proposition 10. First, we claim that the perfect Bayesian equilibrium with $p^* = l(b) - \sigma$ demonstrated in proposition 1 satisfies the condition (**BA5**). To see this, suppose that the principal observes the deviation $p < p^*$ and hypothesizes that a subset Θ' of Θ is responsible for the deviation. Let $y^P(\Theta')$ be the principal's optimal action under the posterior belief that is the prior belief renormalized over Θ'. Then the principal's expected payoff from rejecting the deviation is $\int_{\Theta'} U^P(y^P(\Theta'), \theta) \cdot \rho(\theta|p) d\theta \geq -\sigma$. Since the principal's expected payoff from accepting the deviation is $p - l(b) < -\sigma$, the principal rejects the deviation. Then the agent type θ gets $U^A(y^P(\Theta'), \theta, b)$ from offering the deviation while she gets $-p^* = \sigma - l(b)$ in equilibrium. In order for Θ' to be a consistent interpretation for the deviation p, we need the following condition:

$$\sigma - l(b) < U^A(y^P(\Theta'), \theta, b), \quad \forall \theta \in \Theta'. \tag{53}$$

After rearranging the righthand side of the equation , we get

$$\sigma < l(b) - l(|\theta - y^P(\Theta') + b|), \quad \forall \theta \in \Theta'. \tag{54}$$

Since $y^P(\Theta') \in C(\Theta')$, where $C(\Theta')$ is the convex hull of Θ', there exists $\theta \in \Theta'$ such that $\theta > y^P(\Theta')$. Then at $\theta > y^P(\Theta')$, $l(b) < l(|\theta - y^P(\Theta') + b|)$ so that we have

$$\sigma < 0,$$

which leads to a contradiction. Therefore, any deviation $p < p^*$ cannot have a consistent interpretation.

Similarly, suppose that the principal observes the deviation $p > p^*$ and hypothesize that a subset Θ' of Θ is responsible for the deviation. If the sequential rationality determines that the principal accepts the deviation, then the agent type θ gets $-p < -p^*$ from offering the deviation. Therefore, there is no agent type who wants to offer the deviation. If the sequential rationality determines that the principal rejects the deviation, then the agent type θ gets $-l(|y^P(\Theta') - \theta - b|)$ from offering the deviation while she gets $-p^* = \sigma - l(b)$ in equilibrium. Then, we get the condition (53) again for Θ' to be a consistent interpretation for the deviation p. This is a contradiction. Therefore, any deviation $p > p^*$ of the perfect Bayesian equilibrium cannot have a consistent deviation.

Second, I claim that in any perfect Bayesian equilibrium with $p^* \neq l(b) - \sigma$, a deviation $\widehat{p} = p^* - \varepsilon$ (with arbitrarily small $\varepsilon > 0$ so that $\widehat{p} > l(b) - \sigma$) has a consistent interpretation $\Theta = [0, 1]$. To prove this, suppose that the principal observes the deviation $\widehat{p}$ and hypothesizes that all agents types are responsible for the deviation. Then the principal's posterior belief is the same as the prior belief. Given this belief, the principal optimally chooses an action μ if he rejects the deviation and his expected payoff is $-\sigma$. If he accepts the deviation, then he also gets the expected payoff $(p^* - \varepsilon) - l(b) > -\sigma$. Thus, it is optimal for the principal to accept the deviation. Then the agent type θ gets $-\widehat{p} = \sigma - l(b)$ from the deviation while he gets $-p^* < -\widehat{p}$. Therefore, all agent types in $[0,1]$ strictly prefer their payoffs from offering the deviation to their equilibrium payoff. This completes the proof.

Proof of Proposition 11. Suppose that $b < l^{-1}(\sigma)$. Take any informative perfect Bayesian equilibrium and consider a deviation $\widehat{p} = 0$. Suppose that the principal hypothetically assumes that $\widehat{p}$ is from $[0,1]$. Then the principal accepts the offer because accepting the offer gives $-l(b)$ while rejecting it gives $-\sigma$. Notice that, for all $\theta \in [0,1]$, the equilibrium payoff of the agent type θ is strictly less than 0. Thus, all agent types in $[0,1]$ strictly prefer making $\widehat{p}$ to making equilibrium price offers. This implies that the deviation $\widehat{p}$ has consistent interpretation $[0,1]$, which destroys the original equilibrium.

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