Communication in Bargaining over Decision Rights

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Abstract

This paper develops a model of bargaining over decision-rights between an uninformed principal and an informed but self-interested agent. The uninformed principal makes a price offer to the agent who then decides either to accept or to reject the offer. Contrary to the prediction the Coase Theorem provides, actions induced in the unique perfect Bayesian equilibrium do not always satisfy *ex-post* efficiency. Once we introduce explicit communication into the model, however, there exists a truth-telling perfect Bayesian equilibrium, even when the conflict of interest is arbitrarily large. The truth-telling equilibrium outcome is *ex-ante* Pareto superior to that of several dispute-resolution schemes studied in the framework of Crawford and Sobel (1982) and Holmström (1977).

Keywords: decision-rights, information transmission, cheap talk, ex-post efficiency.
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1 Introduction

In many economic situations a party, such as a government or firm (principal), initially has full authority to make a decision but lacks information about the task or project at hand. There is often another, better-informed party (agent), but its interest may be so different from that of the principal that it may not be willing to share useful information with the uninformed principal.[1] This creates an incentive for the principal to delegate his or her decision-rights to the agent in order to make full use of the agent's private information. When the principal delegates decision-making authority to the agent, it may be beneficial for the principal to make some restriction on the set from which the agent can choose an action. The principal's optimal choice of the restricted set of actions is extensively studied in the literature of optimal delegation (Holmström [35, 36], Goltsman, Hörner, Pavlov, and Squintani [28], Alonso and Matouschek [3], Kováč and Mylovanov [38] and Melumad and Shibano [48]).

It is remarkable that most of papers in the delegation literature focus on settings without monetary transfers.[2] Although there are many settings in which the use of monetary transfers is limited or ruled out, sometimes it is more natural to assume that nothing prevents parties from using financial incentives. In practice, the principal can and do use contracts or bargaining mechanisms that include financial incentives in order to transfer decision-rights.[3] For example:

- Workers and managements typically negotiate the right and responsibility to choose how workers spend their time in workplace or how workers participate in the firm's managemental decision-making process. There is a strong empirical evidence that shows a positive relation between degree of delegation and wage levels, controlling for a variety of worker and firm characteristics (Caroli and Van Reenen [14], Black, Lynch and Krivelyova [13], Bauer and Brender [9]). Managements rarely have good information about the ease with which workers could increase their personal productivity. In some settings, workers may also have superior information about changes in workplace organization, job descriptions, or work flows that would increase firm productivity. Managements also may not have very good information about worker preferences, such as the trade-offs

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[1] See Crawford and Sobel [19]. For more general communication mechanisms, see Goltsman, Hörner, Pavlov, and Squintani [28].

[2] There are few exceptions in the literature on optimal delegation. See, for example, Krähmer [39]. Recently, Ambrus and Egorov [4] investigate the effect of money burning on the optimal delegation structure.

[3] In the framework of incomplete contracts (Grossman and Hart [30] and Hart and Moore [33]), Baker, Gibbons and Murphy [8] model the allocation of decision-rights via contracts. They assume that decisions are not contractible ex post, the parties cannot negotiate over the decision after the state is revealed. Instead, the party in control simply takes her self-interested decision.

workers would be willing to make between such matters as safety, work rates, wages, job security, and the like (Bainbridge [7]).

- When launching into a new business partnership, auto manufacturers and their dealers negotiate the right to determine the size and qualification of the sales force, or the right to set prices. Auto manufacturers may not have very good information about consumer preferences so that they get some difficulties to determine price, advertising strategy and so on. Arruñada, Garicano, and Vasquez [6] empirically analyze the allocation of rights and monetary incentives in automobile franchise contracts. Similarly, when an international manufacturer enters a particular national market, it typically lacks relevant information about local market conditions and has difficulties making decisions on pricing, marketing, advertising, distribution and so on. As a result, it sells an exclusive distributorship to a domestic company who is better-informed but lacks authority to make such decisions. If a license agreement is reached through bargaining, the domestic company pays license fees in return for the exclusive right to make decisions about pricing, marketing, advertising, distribution, and so on in the domestic market.[4]

- It is typical that venture capitalists face substantial uncertainty when financing new ventures. (Kaplan and Strömberg [37] and Dessein [21]). Due to this uncertainty, biotechnology companies often *sell* its patent or legal rights to manufacture the final product to pharmaceutical firms who are better-informed through a license and development agreements.[5]

All these examples share the following commonalities: (i) there is a principal who has to make a decision but lacks decision-relevant information or knowledge; (ii) there is another party who is better-informed or more-experienced but it has its own agenda; (iii) these two parties

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[4] For example, I.B.M., the world's largest computer maker in the 1990's, agreed to allow Mitsubishi to sell an I.B.M. mainframe computer under its own name in Japan in April, 1991. See Andrew Pollack, "IBM Model to Be Sold By Mitsubishi," *The New York Times* (April 29, 1991), 17. More recently, tobacco industry leader Philip Morris International announced an agreement with Chinese National Tobacco under which Chinese National Tobacco will manufacture Marlboro cigarettes for marketing in China. See Nicholas Zamiska and Juliet Ye, "Chinese Cigarettes to Go Global" *The Wall Street Journal*, (January 30, 2008) B4.

[5] For instance, *Animas Corporation*, an insulin infusion pump manufacturing company, set up a license and development agreements with the Swiss R&D company, *Debiotech*, for intellectual property related to next-generation insulin pumps and micro-needles. In return for the exclusive worldwide license to make, use, and sell products utilizing the intellectual property portfolio that includes over 70 issued patents, *Animas* paid $12 million in cash and issued 400,000 restricted shares of Animas common stock. See Rick Baron, "Animas acquires technology for disposable insulin micro-pumps and micro-neddles," http://www.bioalps.org/Bioalps/en/Internet/Documents/1996.pdf. Also see Lerner and Merges [43] and Higgins [34] for some empirical evidences.

negotiate the allocation of decision-rights by using various monetary incentives schemes; (iv) the final decision made by the party in control determines both parties' welfare. In order to capture this situation, we follow the framework of Crawford and Sobel [19] and Holmström [35, 36]: there are two parties, uninformed principal (P) and informed but self-interested agent (A), with one dimensional decision-making that affects welfare of both under one dimensional uncertainty.

The novel feature of this model is as follows: we investigate reallocation of decision-rights in settings with monetary transfers. In particular, we consider an uninformed principal's optimal choice of a price offer for decision-rights when an informed agent decides either to accept or to reject the offer. If the price offer is accepted then the agent pays the principal the price for decision-rights and makes a decision. Otherwise, the principal retains decision-rights without making any monetary transfers. We call this game *bargaining over decision-rights*.[6] Our main finding is as follows: it is optimal for the uninformed principal to use the price offer as a device for screening some agent types out. In equilibrium, the principal makes a price offer that is accepted by some agent types but not by all agent types. It means that the principal sometimes retains decision-rights while lacking precise information. As a result, actions taken by the principal without precise information may be inefficient *ex-post* for some realization of the state. That is, sometimes there exists an action that makes the principal better off without making the agent worse off or vice versa, once the true state of the world becomes public.

This result seems to be contrary to Coase [17] who asserts that if the market outcome is inefficient and there are no transaction costs, then the parties concerned will negotiate their way to efficiency. The main obstacle to the efficient bargaining seems to be bargaining costs due to incomplete or asymmetric information. Farrell [22] shows that in the presence of private or incomplete information, voluntary negotiation could not lead to the first-best outcome that maximizes joint surplus. The important issue is how to interpret "no transaction costs" in the presence of private information. In the basic model, we assume that bargaining is *tacit* in the sense that parties can communicate only by making a price offer that directly affects their payoffs. As pointed out by Crawford [18], real bargaining, by contrast, is usually *explicit*, in that parties can furthermore communicate by sending non-binding messages with no direct effects on their payoffs. Thus, it is natural to interpret the absence of transaction costs in bargaining under asymmetric information as the absence of communication costs: people freely get together and communicate with each other without any costs. Therefore, it is impetuous

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[6] A companion model of bargaining is considered by Lim [44] by focusing on the situation where the informed agent has bargaining power so that he makes a take-it-or-leave-it price offer.

to conclude that Coase's assertion is unwarranted in our environment without investigating the impact of communication into the tacit bargaining carefully.

There are several theoretical evidences showing that such cheap talk messages play an important role in coordinating bargainers' expectations so that they can reach agreement and in determining how they share the resulting surplus (Farrell and Gibbons [26] and Matthews [46]). Farrell and Gibbons [26] study a two-stage bargaining game in which talk may be followed by the sealed-bid double auction studied by Chatterjee and Samuelson [15], a well-known model of bargaining under incomplete information. They show that talk can matter in the sense that the cheap talk equilibrium features bargaining outcomes that could not be an equilibrium behavior in the absence of talk. Matthews [46] considers a specific bargaining situation with a veto-threat and shows there exists an equilibrium in which an informed party (proposer) tells the other (chooser) which of two sets contains his type. This equilibrium behavior is not a part of equilibrium behavior in the absence of talk. These results intimate that communication may resolve inefficiency caused by the presence of incomplete or asymmetric information in our model.

How then does introducing talk into the tacit bargaining affect the behaviors of the parties? To answer this question, we devote the second half of our paper to bargaining over decision-making rights with explicit communication. Specifically, we assume the informed agent can send a cheap talk message before bargaining begins.[7] Once we allow parties to communicate via cheap talk before bargaining, there exists a truth-telling equilibrium. The existence of the truth-telling equilibrium is surprising because neither the tacit bargaining nor communication via cheap talk alone allows parties to make full use of the agent's private information to make a decision. In this equilibrium, the principal uses the following trigger type of strategy: "*Tell me the truth and prove your honesty by accepting my price offer. I will make a specific message-independent price offer that must be accepted by a truthful agent. Thus, I consider the rejection of my price offer as evidence for your dishonesty so that I will punish you by taking an action that makes you much worse than telling the truth.*" The *threat* action compels the agent to report the true state in the cheap talk stage and to accept the equilibrium-path price offer from the principal in the bargaining stage. It turns out that the threat action coincides with the unique agent type who rejects the price offer on the equilibrium path with positive probability. Consequently, taking the threat action becomes rational for the principal, and more importantly, to be credible to the agent. In this truth-telling equilibrium, induced actions always satisfy *ex-post* efficiency.

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[7] However, our main result, the existence of the truth-telling equilibrium, does not depend on the exact timing of the game.

We apply two standard cheap-talk refinements, *neologism-proofness* (Farrell [23]) and NITS (Chen, Kartik and Sobel [16]), and show that the existence of the truth-telling equilibrium is robust against those refinements: no matter how large the difference between parties' preferences is, the equilibrium is neologism proof in Farrell [23]'s sense. Moreover, it is the unique neologism-proof equilibrium under some parameter value. Imposing NITS (no incentive to separate), the criterion proposed by Chen, Kartik and Sobel [16] to refine equilibria in cheap-talk games (Crawford and Sobel [19]) leads to the same result. That is, not only the truth-telling equilibrium *always* satisfies NITS, but also it is the unique equilibrium satisfying NITS under some parameter value. We also consider the notion of sequential perfect equilibrium or extensive-form trembling-hand perfection and show that the truth-telling equilibrium is robust against those refinements too.

We also show that the truth-telling equilibrium outcome of the explicit bargaining is *ex-ante* Pareto superior to that of several other protocols studied in the literature, such as communication (Crawford and Sobel [19]), optimal mediation (Goltsman, Hörner, Pavlov, and Squintani [28]), optimal delegation (Holmström [35][36], Alonso and Matouschek [3], Kováč Mylovanov [38] and Melumad and Shibano [48]) and optimal compensation contract (Krishna and Morgan [42]) if parties' interests are substantially misaligned. This might explain why bargaining over decision-rights often takes place between two separately owned companies whose interests diverge widely.

We extend the original model with unidimensional state space to multidimensional state space case. Interestingly, the main result, the existence and characterization of fully revealing equilibria, still holds with multidimensional state space if the space is bounded. The result shows that boundedness, as opposed to the dimensionality, of state space plays an important role for fully revealing equilibria to exist. Contrary to the main finding of the multidimensional cheap talk literature, boundedness of state space is a sufficient condition for the existence of fully revealing equilibria.[8]

The rest of the paper is organized as follows. The next section describes the environment. In section 3, we setup the basic model of bargaining over decision-making authority and show that there is no equilibrium in which an *ex-post* efficient action is taken for any realization of the state. Full characterization of equilibria is provided assuming the parties' prior beliefs are *uniform*. In section 4, we extend the basic model and allow parties to communicate before bargaining by sending cheap talk messages. We show that there exists a truth-telling perfect Bayesian equilibrium in which actions induced are efficient *ex-post*. The robustness of the

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[8] For example, unboundedness of state space is a sufficient condition for fully revealing equilibria to exist in Ambrus and Takahashi [5].

truth-telling equilibrium is also discussed. We devote section 5 to welfare comparisons. We discuss two extensions of the basic model in section 6. First, we consider the model with multidimensional state space and investigate the existence of truth-telling equilibria. Second, we follow a mechanism design approach and show that under certain conditions the explicit bargaining is an optimal bargaining mechanism that maximizes a joint surplus of parties. We conclude in section 7.

2 Basic Model

2.1 Environment

There are two parties, a principal (P) and an agent (A). The principal who initially has decision-making authority has little information about the state of the world $\theta \in \Theta \equiv [0, 1]$. She has a prior distribution F over $[0, 1]$ with an absolutely continuous density function $f > 0$. The agent who has different interests from the principal knows the true state of the world θ but does not have decision-making authority. The payoffs for a given allocation of authority depend on an action y taken by the party who has decision-making authority and the state of the world θ. The payoff functions of the parties are of the form $U^P(y, \theta) = -l(|y - \theta|)$ for the principal and $U^A(y, \theta, b) = -l(|y - (\theta + b)|)$ for the agent.[9] We refer to l as the loss function and assume that $l''(\cdot) > 0$, $l'(0) = 0$ and $l(0) = 0$. This means that the ideal action of the principal is $\overline{y}^P(\theta) = \theta$ and the ideal action of the agent is $\overline{y}^A(\theta, b) = \theta + b$ where $b > 0$ is a parameter that measures how nearly the agent's interest coincides with that of the principal. All of these are common knowledge between parties.

2.2 *Ex-post* Efficient Actions

When utility functions are quasi-linear there exists unique *ex-post* efficient action in between two ideal actions, which maximizes a joint surplus of parties. For example, if both U^A and U^P are quadratic, the ex-post efficient action is the mid-point of θ and $\theta + b$. It is well known that property rights and voluntary private negotiation are not able to achieve this first-best efficient outcomes in the presence of important private information.[10] In this section, we shall argue for some kind of second-best comparison as against comparing things with first-best efficiency.

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[9] A special case is a quadratic utility ($U^P(y, \theta) = -(y - \theta)^2$ and $U^A(y, \theta, b) = -(y - \theta - b)^2$) which we are assumed in most examples and applications. Similar utility functions are assumed in many other paper, for example, Dessein [20]. To see more papers assuming quadratic utilities, see Kováč and Mylovanov [38].

[10] See, for example, Myerson and Satterthwaite [50].

Define a (second-best) *ex-post* efficient action as follows. An action is said to be (second-best) *efficient ex-post* if and only if there is no other feasible action that makes some individual better off without making other individuals worse off after the true state of the world θ is publicly known.

Definition 1. *An action $y \in \mathbb{R}$ is efficient ex-post at θ if there is no other action $z \in \mathbb{R}$ such that*

$$U^P(z,\theta) \geq U^P(y,\theta) \quad \textit{and} \quad U^A(z,\theta,b) \geq U^A(y,\theta,b) \tag{1}$$

with at least one strict inequality.

In our environment, an action y is (second-best) efficient *ex-post* if and only if $y \in [\theta, \theta + b]$ when the realization of the state is θ, as one can see in Figure 1. Notice that most mechanisms considered in the literature on strategic information transmission (Crawford and Sobel [19]) and optimal delegation (Holmstrom [35][36]) lead to efficient actions for some states of the world but not all. The following example demonstrates *ex-post* inefficiency of actions in cheap talk and optimal delegation.

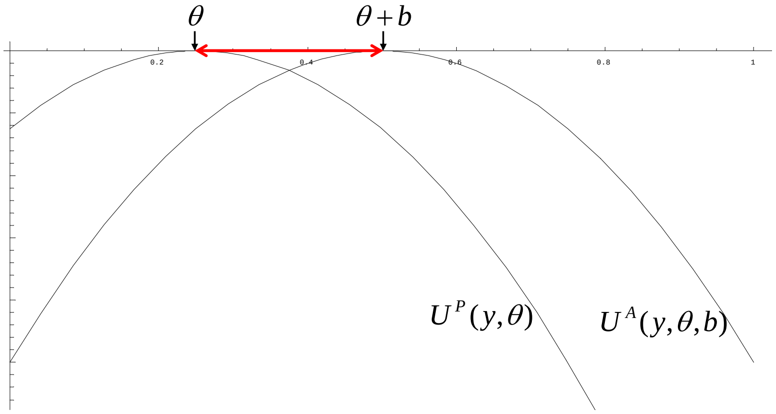

Figure 1: Ex-post Efficient Actions

Example 1. Suppose that F is uniform and utilities are quadratic. Let $b = 1/5$ and the realized state of the world $\theta = 7/8$. As you can see in Figure 1, if an action y is not in $[\theta, \theta + b] = [7/8, 43/40]$, then there exists another action y' such that both parties strictly prefer y' to y. Suppose that parties communicate via cheap talk. In the most informative equilibrium, only two actions, $y_1 = \frac{1}{20}$ and $y_2 = \frac{11}{20}$, are induced.[11] Since $y_1 < \theta$ and $y_2 < \theta$, both actions are inefficient *ex-post*. Alternatively, suppose that the principal optimally proposes the set

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[11] See the leading example of Crawford and Sobel [19].

of admissible actions that the agent can take. In the optimal delegation, the proposed set is $[0, 1-b] = [0, 4/5]$.[12] As the result, the agent cannot take any action $y \in [7/8, 43/40]$.

3 Benchmark: Tacit Bargaining

Consider bargaining over decision-making authority between the informed agent and the uninformed principal. The timing of the game is as follows:

1. The agent privately observes the state of the world $\theta \in \Theta \equiv [0,1]$.
2. The principal makes an offer $p \in \mathbb{R}$ for the authority to take an action.[13]
3. The agent decides whether to reject or accept the offer.
4. If the agent accepts the offer then he pays the price to the principal and takes an action, denoted by y^A. In this case, payoffs become $U^P(y^A, \theta) + p$ and $U^A(y^A, \theta, b) - p$ for the principal and the agent respectively. If the agent rejects the offer, however, the principal takes an action, denoted by y^P, without transferring the decision-making authority. Then payoffs are $U^P(y^P, \theta)$ and $U^A(y^P, \theta, b)$ for the principal and the agent, respectively.

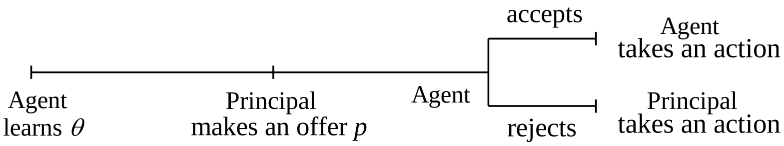

Figure 2: Timing of the game

The equilibrium concept we use is perfect Bayesian equilibrium. For the principal, a strategy consists of a price offer p^* and an action rule y^P. The action rule, denoted by $y^P : \mathbb{R} \to \mathbb{R}$ specifies the principal's action after the rejection of each price offer $p \in \mathbb{R}$ that the he might make. Since the utility function is strictly concave in y, the principal will never use mixed strategies in equilibrium. For the agent, a strategy consists of a decision rule and an action rule. The decision rule, denoted by $d^A : \Theta \times \mathbb{R} \to [0,1]$, specifies the probability of rejection for each price offer $p \in \mathbb{R}$ that the agent might receive. The action rule $y^A : \Theta \times \mathbb{R} \to \mathbb{R}$, specifies the agent's choice of action after he accepts the principal's price offer p. The strategy profile $\{(p^*, y^P), (d^A, y^A)\}$ forms a perfect Bayesian equilibrium if:

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[12] See Holmström [35][36], Melumad and Shibano [48], Alonso and Matouschek [3] and Kováč and Mylovanov [38].

[13] We allow p to be negative, which means that the principal pays $|p|$ to the agent.

(**B1**) p^* solves

$$\max_{p\in\mathbb{R}} \int_0^1 \{d^A(\theta,p)U^P(y^P(p),\theta) + (1-d^A(\theta,p))(p-l(b))\}f(\theta)d\theta$$

(**B2**) for each $p \in \mathbb{R}$ and each $\theta \in [0,1]$, $d^A(\theta,p)$ solves

$$\max_{d^A\in[0,1]} (1-d^A)(-p) + d^A \cdot U^A(y^P(p),\theta,b)$$

(**B3**) for each $\theta \in [0,1]$ and $p \in \mathbb{R}$, $y^A(\theta,p) = \overline{y}^A(\theta) = \theta + b$
(**B4**) for each $p \in \mathbb{R}$, $y^P(p)$ solves

$$\max_{y\in\mathbb{R}} \int_0^1 U^P(y,\theta)\rho(\theta|p)d\theta$$

where $\rho(\theta|p)$ is the principal's updated belief after observing the agent's rejection of p, which is given by Bayes' rule whenever possible.

3.1 Equilibrium

3.1.1 Example: *uniform* distribution

In this section, we illustrate the main idea behind our analysis while assuming that f is *uniform* over $[0,1]$. This setting, together with quadratic utilities, is a leading example of Crawford and Sobel [19] and has been widely used in the literature on strategic information transmission and optimal delegation. We will extend our result to more general distributions in the next subsection. In what follows we first focus on the agent's decision whether to accept a given price offer or not. We will show that the agent's decision rule satisfies an interesting property called *monotonicity.* Next, with the full characterization of the agent's decision rule we show there exists a unique price offer that maximizes the principal's expected utility.

For an arbitrary $p \in \mathbb{R}$, define the set of agent types who accept p with probability one as

$$\Theta(p) = \{\theta \in [0,1] | d(\theta,p) = 0\}.$$

Define the set of agent types who reject the offer p with probability one as

$$\Theta^{-1}(p) = \{\theta \in [0,1] | d(\theta,p) = 1\}.$$

Lemma 1 (Monotonicity)**.** *For any price offer, if there is an agent type θ who accepts the offer with positive probability then all agent types higher than θ have to accept it with probability one.*

Proof. See the appendix. □

To see the intuition of Lemma 1, consider the decision problem of agent type $\theta \in [0,1]$ who observes a price offer p. Define the agent type θ's willingness to pay as follows:

$$W(\theta, p, y^P(p)) = U^A(\overline{y}^A(\theta), \theta, b) - U^A(y^P(p), \theta, b) = l(|y^P(p) - (\theta + b)|), \tag{2}$$

where $y^P(p)$ is the action taken by the principal after the price offer p is rejected and $\overline{y}^A(\theta) = \theta + b$ is the agent type θ's optimal action. The agent type θ accepts the offer only if the gain of getting decision-making authority (or willingness to pay for authority) is at least as big as the loss of it, that is,

$$W(\theta, p, y^P(p)) \geq p.$$

It is not possible that $y^P(p)$ is on the right of θ, because otherwise, the quasi-concavity of U^A implies that the set of agent types who reject the offer should be on the right of θ and the mid-point of the set should be $y^P(p) - b$ but not $y^P(p)$, which is contradicted by **(B4)**. It means that $y^P(p)$ is on the left of θ, and as a result, the agent type $\theta' > \theta$ whose most preferred action is higher than that of the agent type θ is willing to pay more to get authority to make a decision, that is,

$$\frac{\partial W(\theta, p, y^P(p))}{\partial \theta} \geq 0.$$

This means that agent type θ accepts any price offer which is accepted by agent type $\theta' < \theta$.

Lemma 1 implies that for any $p \in \mathbb{R}$, both $\Theta(p)$ and $\Theta^{-1}(p)$ are convex if they are non-empty. Further, $\Theta(p)$ cannot be to the left of $\Theta^{-1}(p)$. These guarantee that for any $p \in \mathbb{R}$ there is at most one agent type who is indifferent between accepting and rejecting the offer. Let $\theta_p \in [0,1]$ denote the agent type if it exists. Then we can write that $\Theta(p) = (\theta_p, 1]$ and $\Theta^{-1}(p) = [0, \theta_p)$. From the indifference condition at θ_p we have

$$p = l(|y^P(p) - \theta_p - b|), \tag{3}$$

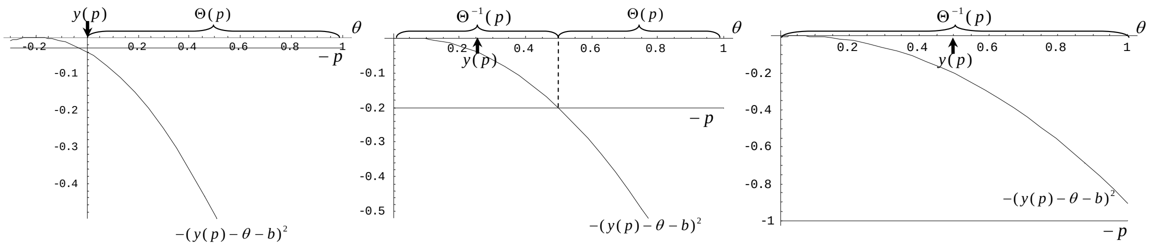
where $y^P(p) = arg \max_y \int_0^{\theta_p} -l(|y - \theta - b|) \cdot \frac{1}{\theta_p} d\theta = \frac{\theta_p}{2}$. Thus, we have

$$p = l(\frac{\theta_p}{2} + b) \quad or \quad \theta_p = 2(l^{-1}(p) - b). \tag{4}$$

Since $\theta_p \in [0,1]$, we have the following corollary.

Corollary 1.

$$\Theta(p) = \begin{cases} [0,1] & \textit{if } p < l(b), \\ (2(l^{-1}(p) - b), 1] & \textit{if } l(b) \leq p \leq l(b + \frac{1}{2}), \\ \emptyset & \textit{if } p > l(b + \frac{1}{2}) \end{cases}$$



(a) low p (b) moderate p (c) high p

Figure 3: The agent's decision rule

In words, all agent types in $[0,1]$ accept a low price offer ($p < l(b)$) with probability one, and once the price offer becomes greater than $l(b)$ then low agent types start rejecting it. As p increases, the set $\Theta(p)$ becomes smaller and finally all agent types in $[0,1]$ reject a high price offer ($p > l(b+\frac{1}{2})$) with probability one.

In Figure 3, we see the clear trade-off between higher price and more rejections that the principal faces. Although a higher price offer gives a higher payoff to the principal if accepted, it does not seem to be optimal for the principal to make a very high offer because it cannot be accepted (Figure 3(a)). Similarly, making a price offer that could be accepted by all agent types does not seem to be optimal either because it is very low (Figure 3(c)). These suggest that the principal's optimal price offer should lie halfway between two extremes (Figure 3(b)). To confirm this idea, consider the principal's optimal price offer as a best response to the agent's strategy. The principal chooses p^* to solve

$$\begin{aligned} \max_{p\in\mathbb{R}} EU^P &= \int_0^{\theta_p} -l(|y^P(p)-\theta|)d\theta + (1-\theta_p)(p-l(b)) \qquad (5)\\ &\quad \text{s.t. } y^P(p) = \frac{\theta_p}{2} \text{ and } \theta_p = 2(l^{-1}(p)-b). \end{aligned}$$

Notice that there exists a unique interior solution of this maximization problem because of the strict concavity of EU^P in p. From the first order condition, we get

$$p^* = l(\frac{1}{4}+b) \quad and \quad \theta_{p^*} = \frac{1}{2}. \qquad (6)$$

This implies that it is optimal for the principal to make the price offer that is acceptable for some agents of high type but not for the remaining agents of low type. This result is summarized in the following proposition.

Proposition 1. *In equilibrium, the principal makes a price offer* $p^* = l(\frac{1}{4}+b)$*. The agent type* $\theta \in [0,\frac{1}{2})$ *rejects the offer with probability one and* $\theta \in (\frac{1}{2},1]$ *accepts the offer with probability one. As a result, ex-post efficiency does not hold.*

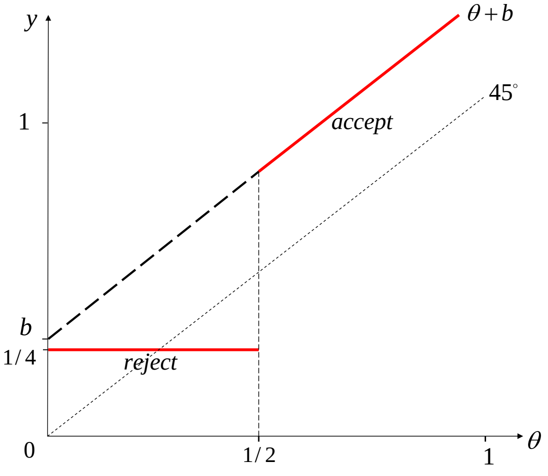


Figure 4: Equilibrium Outcome

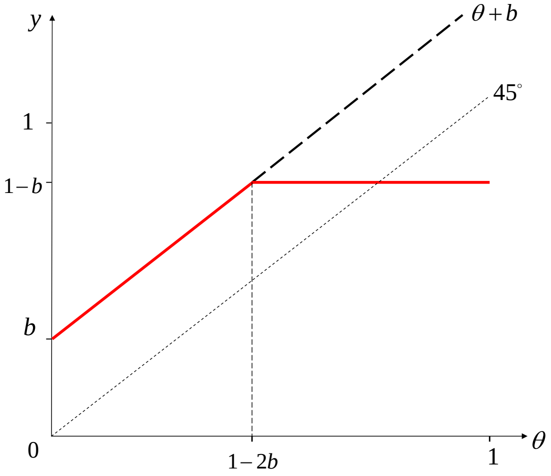


Figure 5: Optimal Delegation

It is interesting to compare this result with the outcome of optimal delegation studied by Holmström [35][36], Melumad and Shibano [48], Alonso and Matouschek [3], Goltsman *et al.* [28] and Kováč and Mylovanov [38]. They studied the uninformed principal's optimal choice of the set of admissible actions that the informed agent can take. According to the optimal delegation rule in the model with a uniform prior and quadratic utility functions, the informed agent can enforce any decision he likes, as long as it does not exceed $1-b$ (see Figure 5). The intuition is as follows: since the informed party's most preferred action is always higher than that of the principal, it pays to impose an upper bound on the allowable actions. In case of the low state, on the other hand, the best way to make use of the informed agent's information is to grant complete freedom of choice of the action to the informed agent.

Although the outcome of the equilibrium in this model in which the agent obtains complete freedom of choice of the action only in case of the high state looks exactly opposite of that of optimal delegation, the underlying intuition is exactly the same as that of optimal delegation. Recall that the type θ agent's willingness to pay is strictly increasing in the distance between $y^P(p)$ and $(\theta+b)$ where $y^P(p)$ is determined by Bayes' rule. This means that the principal can maximize the willingness to pay by choosing $y^P(p)$ and the agent's most preferred action $(\theta+b)$ as distant as possible. Since $b>0$, the principal can maximize this distance by making the price offer that the agent rejects in case of a low state and accepts in case of high state so that $y^P(p)$ is low and $(\theta+b)$ is high. Clearly, we should get exactly the opposite outcome in which the informed agent gets a complete freedom to choose the action in case of low state if the agent's most preferred action is always lower than that of the principal.

3.1.2 General Case

In this section, we extend the analysis in the previous section to more general distributions. Recall that in the previous section the monotonicity of the agent's decision rule allows us to have a unique optimal price offer for the principal. The following regularity condition on the parties' prior belief f is necessary for us to have the same monotonicity of the agent's decision rule and as a result, ensures that all results we got in the previous section are preserved.

Condition 1. *For a given value of* $b > 0$*,*

$$y(\underline{\theta}, \overline{\theta}) - b < \frac{\underline{\theta} + \overline{\theta}}{2} \tag{7}$$

for any $\underline{\theta}$ *and* $\overline{\theta}$ *with* $0 \leq \underline{\theta} \leq \overline{\theta} \leq 1$*, where*

$$y(\underline{\theta}, \overline{\theta}) = \begin{cases} argmax \int_{\underline{\theta}}^{\overline{\theta}} U^P(y, \theta) f(\theta) d\theta & if\ \underline{\theta} < \overline{\theta}, \\ \overline{\theta} & if\ \underline{\theta} = \overline{\theta}. \end{cases}$$

In words, this condition implies that for any interval subset of Θ, an action that maximizes the expected payoff for a principal who believes that agent type is in the interval is not lopsided too much toward the right of the interval. Any prior f satisfies this regularity condition if $b \geq 1/2$. Moreover, this condition holds for any $b > 0$ if f is non-increasing in θ. In particular, it is satisfied in the setting with *uniform* distribution considered in the previous subsection.

Under this regularity condition, the agent types' decision rule satisfies the *monotonicity.*[14] As we already saw in the example with uniform distribution, the monotonicity makes it optimal for the principal to use her price offer as a screening device. This result is summarized in the following proposition.

Proposition 2. *Under Condition 1, there exists a unique perfect Bayesian equilibrium. In the equilibrium, the principal makes a price offer accepted by positive measure of agent types but not all.*

Proof. See the appendix. □

4 Explicit Bargaining

In this section, we explore how introducing explicit communication into the basic model affects its outcomes. The timing of the game is as follows:

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[14] In the proof of Lemma 1, we first show that under condition 1 the agent's decision rule satisfies the monotonicity. The proof is completed by pointing the fact out that the condition 1 holds with the uniform distribution.

1. The agent privately observes the state of the world $\theta \in \Theta \equiv [0,1]$.

2. The agent sends a message $m \in M$ to the principal.

3. After observing the message from the agent, the principal makes a price offer $p \in \mathbb{R}$ for authority to take an action.

4. The agent decides whether to accept or reject the offer.

5. If the agent accepts the offer then he pays the price offered by the principal and takes an action, denoted by y^A. In this case, payoffs become $U^P(y^A,\theta)+p$ and $U^A(y^A,\theta,b)-p$ for the principal and the agent respectively. If the agent rejects the offer, however, the principal takes an action, denoted by y^P, without transferring the decision-making authority. Then payoffs are $U^P(y^P,\theta)$ and $U^A(y^P,\theta,b)$ for the principal and the agent, respectively.

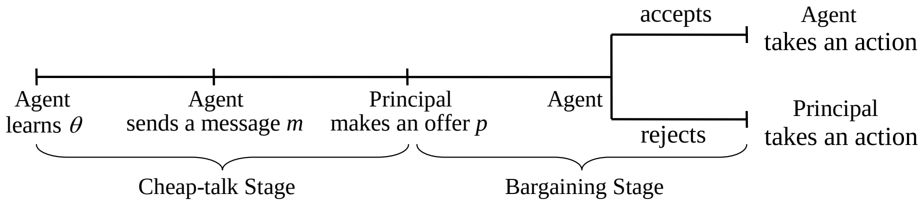

Figure 6: Communication before Bargaining

Again, the equilibrium concept we use is perfect Bayesian equilibrium. For the agent, a strategy consists of a message rule, a decision rule and an action rule. The message rule $\mu : \Theta \to \Delta(M)$ specifies the choice of message for each type $\theta \in \Theta$. The decision rule, denoted by $d : \Theta \times M \times \mathbb{R} \to [0,1]$, specifies the probability of rejection for each price offer $p \in \mathbb{R}$ that the agent who sent the message m might receive. The action rule $y^A : \Theta \times M \times \mathbb{R} \to \mathbb{R}$, specifies the action taken by the agent type θ who sent a message m and accepted the principal's price offer p. For the principal, a strategy consists of a price rule and an action rule. The price rule $p^* : M \to \mathbb{R}$ specifies the principal's choice of price offer for each message $m \in M$ that the principal might receive. The action rule, denoted by $y^P : M \times \mathbb{R} \to \mathbb{R}$ specifies the action taken by the principal who observed a message m and her price offer p was rejected. The strategy profile $\{(\mu, d, y^A), (p^*, y^P)\}$ and the principal's posterior beliefs ρ_1 and ρ_2 form a perfect Bayesian equilibrium if:

(**CB1**) for each $\theta \in [0,1]$, $\int_M \mu(m|\theta)dm = 1$ and if $m^* \in M$ is in the support of $\mu(\cdot|\theta)$ then m^* solves

$$\max_{m \in M} d(\theta, m, p^*(m))U^A(y^P(m, p^*(m)), \theta, b) - p^*(m)(1 - d(\theta, m, p^*(m)))$$

(**CB2**) for each $m \in M$, $p^*(m)$ solves

$$\max_{p \in \mathbb{R}} \int_0^1 \{d(\theta, m, p)U^P(y^P(m,p), \theta) + (1 - d(\theta, m, p))(p - l(b))\}\rho_1(\theta|m)d\theta$$

(**CB3**) for each $\theta \in [0,1]$, $m \in M$, and $p \in \mathbb{R}$, $d(\theta, m, p)$ solves

$$\max_{d \in [0,1]} (1-d)(-p) + d \cdot U^A(y^P(m,p), \theta, b)$$

(**CB4**) for each $\theta \in [0,1]$, $m \in M$, and $p \in \mathbb{R}$, $y^A(\theta, m, p) = \theta + b$

(**CB5**) for each $m \in M$ and $p \in \mathbb{R}$, $y^P(m,p)$ solves

$$\max_{y \in \mathbb{R}} \int_0^1 U^P(y, \theta)\rho_2(\theta|m,p)d\theta$$

(**CB6**)

$$\rho_1(\theta|m) = \frac{\mu(m|\theta)}{\int_0^1 \mu(m|\theta')d\theta'} \qquad \text{and} \qquad \rho_2(\theta|m,p) = \frac{d(\theta, m, p)\rho_1(\theta|m)}{\int_0^1 d(\theta', m, p)\rho_1(\theta'|m)d\theta}$$

where $\rho_1(\theta|m)$ is the principal's updated belief after observing the message m from the agent and $\rho_2(\theta|m,p)$ is the updated belief of the principal receiving the message m and observing the rejection of the price offer p.

4.1 Truth-telling Equilibrium

Is it possible that communication before bargaining is informative and as a result, improves efficiency of bargaining? Surprisingly, there exists a truth-telling perfect Baysian equilibrium once we allow parties to communicate before bargaining.

Consider the following strategy profile: the principal makes a price offer $l(b)$ regardless of the message he observed and takes an action $y = \overline{y}^P(0) = 0$ if the offer is rejected. The agent fully reveals his private information by sending a truth-telling message in the cheap talk stage and accepts any offer less than or equal to $l(b)$ with probability one but rejects any other offer with probability one. If the principal makes any price offer $p \neq l(b)$ and it is rejected, then the principal takes an action $y = \theta$. It is easy to see that the agent types' strategy is a best response to the principal's strategy. First, no agent type has an incentive to deviate in cheap talk stage because the principal's price offer is message-independent. Second, no agent type has an incentive to reject the offer in the bargaining stage because for all $\theta \in [0,1]$

$$\underbrace{-l(b)}_{\textit{from accepting } l(b)} \geq \underbrace{-l(|0 - \theta - b|)}_{\textit{from rejecting } l(b)} = -l(\theta + b).$$

Given the agent's strategy specified above, the principal's best response is to make an offer $l(b)$, the highest price offer accepted by agent types who tells the truth in the cheap talk stage. The principal does not have an incentive to make any offer $p < l(b)$ because such a price offer will be accepted by all agent types and gives a strictly less payoff to the principal than making the price offer $l(b)$. The principal does not have a strict incentive to make any offer $p > l(b)$ because such a price offer will be rejected by all agent types and the action taken by the principal after $p > l(b)$ is rejected will give him exactly the same payoff as he could get by making the offer $p = l(b)$. After the price offer $l(b)$ is rejected, the principal believes that the true state of the world is $\theta = 0$ with probability one. This is a reasonable belief in the sense that the agent with type $\theta = 0$ is the only type who is indifferent between accepting and rejecting the offer $l(b)$, and all other agent types strictly prefer accepting the offer. We will discuss this issue more carefully in section 4.2.3.

Proposition 3 (Truth-telling equilibrium). *For any $b > 0$, there exists a perfect Bayesian equilibrium in which the informed agent fully reveals his private information by sending truth-telling messages in cheap talk stage.*

Proof. The proof is constructive. Consider the following strategies and belief:
i) The agent type θ fully reveals his private information by sending a message θ.
ii) For any $m \in M$, the principal makes the price offer $l(b)$.
iii) For any $\theta \in [0,1]$, the agent accepts the offer p with probability one if $p \leq l(b)$ but rejects p with probability one if $p > l(b)$, regardless of the message he sent.
iv) If a price offer $p = l(b)$ is rejected then the principal takes an action $y = 0$ regardless of the message she received. If a price offer $p \neq l(b)$ is rejected then the principal who received a message m takes an action $y = m$.
v) For any $m \in M$, $\rho_1(\theta|m) = \begin{cases} 0 & \forall\theta \in [0,1] \setminus m, \\ 1 & \textit{if } \theta = m. \end{cases}$
vi) For any $m \in M$ and any $p = l(b)$, $\rho_2(\theta|m,p) = \begin{cases} 0 & \forall\theta \in (0,1], \\ 1 & \textit{if } \theta = 0. \end{cases}$
vii) For any $m \in M$ and any $p \neq l(b)$, $\rho_2(\theta|m,p) = \begin{cases} 0 & \forall\theta \in [0,1] \setminus m, \\ 1 & \textit{if } \theta = m. \end{cases}$

First, consider the agent's incentive. Under the principal's strategy and beliefs above, the agent has no incentive to deviate in his message rule because the principal makes the message-independent price offer $l(b)$. For any $m \in M$, any agent type $\theta \in [0,1]$ accepts an offer p with probability one if $p \leq l(b)$ since he gets $-p$ which is greater than or equal to $-l(b)$ from accepting the offer, but the expected payoff of the agent type θ from rejecting the offer is

$$-l(|0 - \theta - b|) = -l(\theta + b) \leq -l(b), \quad \forall\theta \in [0,1].$$

Any agent type $\theta \in [0,1]$ who reveals his private information fully rejects an offer p with probability one if $p > l(b)$ since he gets $-p$ which is less than $-l(b)$ from accepting the offer, but the expected payoff of the agent type θ from rejecting the offer is $-l(b)$.

Second, consider the principal's incentive. Under the agent's strategy and beliefs above, the principal's optimal behavior after observing a truthful message (or a message θ) is to make the price offer $l(b)$, because any offer less than $l(b)$ will be accepted by all types of agent with probability one and give her the expected payoff strictly less than 0, the principal's expected payoff from making the offer $l(b)$ and any offer greater than $l(b)$ will be rejected with probability one and induces the principal's action θ which gives the principal the expected payoff 0.

By the construction, no price offer is rejected with positive probability on the equilibrium path so that we cannot use Bayes' rule to determine beliefs that the principal has after price offers are rejected. The principal's action rule specified above is sequentially rational under the beliefs we take. This completes our proof. □

In this truth-telling equilibrium, the informed agent accepts the equilibrium price offer $l(b)$ with probability one so that the final outcome is always efficient *ex-post*. This is surprising because neither the tacit bargaining nor communication via cheap talk alone allow parties to make full use of the agent's private information to make a decision. This result is similar to Farrell and Gibbons [24] in the sense that not only information conveyed by cheap talk in equilibrium, but the equilibrium outcomes differ from any that could occur in an equilibrium without talk.

What is the role of communication in this equilibrium? In fact, the principal completely ignores the messages she got from the agent in the cheap-talk stage on the equilibrium path. Nonetheless, the role of communication is clear in this equilibrium. The principal uses her information she got from communication when any off-the-equilibrium-path price offer $p \neq l(b)$ is rejected, and takes an action $y = \theta$. Hence, the agent types who send truthful messages in the cheap-talk stage would not want to accept any price offer $p > l(b)$. This leads the principal not to make any price offer $p > l(b)$ in this equilibrium. As a result, making a price offer $l(b)$ is optimal for the principal. Recall that without communication, making the price offer $l(b)$ is not optimal. Since almost all of agent types still accept a price offer $l(b) + \varepsilon$ with an arbitrarily small $\varepsilon > 0$, the principal has an incentive to make an offer $l(b) + \varepsilon$ in the case without communication.

It is remarkable to see that the existence of this equilibrium is robust against the exact timing of the game. To be more precise, consider the game in which bargaining comes first and communication comes next under the contingency that an agreement is not reached. Then there exists the following perfect Bayesian equilibrium in this game which is outcome equivalent to the truth-telling equilibrium in the original model. The construction of the equilibrium is almost the same as before: the principal makes a price offer $l(b)$ and takes an action $y = 0$ regardless of the message she received from the agent if the offer is rejected. All agent types in Θ always accept the offer $l(b)$ with probability one and fully reveal their private information by sending truth-telling messages off the equilibrium path (i.e. when any offer is rejected.) Since it is straightforward to see that this strategy profile satisfies the mutual best response under some belief derived by Bayes' rule, I skip the detailed proof.

4.2 Robustness of the Truth-telling Equilibrium

In this section, we apply two equilibrium refinements for cheap-talk models- neologism-proofness developed by Farrell [23] and NITS (no incentive to separate) developed by Chen, Kartik, and Sobel [16]. We show that the truth-telling equilibrium satisfies both neologism-proofness and NITS condition for any $b > 0$ whereas the babbling equilibrium does not satisfy either of them for some parameter value of b. Moreover, we discuss robustness of the truth-telling equilibrium against support restriction, the assumption used by several papers such as Grossman and Perry [29][31], Harrington [32], Kreps and Wilson [40] and Rubinstein [53]. The extensive-form trembling hand perfection by Selten [54] and sequential equilibria by Kreps and Wilson [41] will be discussed.

4.2.1 Neologism-proofness

There are several papers (see Gertner, Gibbons, and Scharfstein [27], Farrell and Gibbons [24][25], and Matthews [46]) that use neologism-proofness, developed by Farrell [23] to refine equilibrium outcomes of cheap talk games, to refine their equilibrium outcomes. In what follows, we show that the truth-telling perfect Bayesian equilibrium is always neologism proof and for some parameter values, it is a unique neologism-proof equilibrium.

According to Farrell [23], assume that for every non-empty subset X of Θ, and for every perfect Bayesian equilibrium of the game, there exists a message $m(X)$ that is unused in the equilibrium and whose literal meaning is that $\theta \in X$. If the principal observes the message $m(X)$, then she hypothesizes that some members of specified subset X are responsible for the message and makes a price offer that is a best response under the posterior belief derived by Bayes' rule from her prior. For example, by Lemma 1 and the tacit bargaining analysis, for any convex $X \subseteq \Theta$ it is the principal's best response against getting the message $m(X)$ to make a price offer which is rejected by agent types in the low half of X and accepted by remaining agent types in X. The parties' behaviors in the remainder of the game satisfy sequential rationality and the final payoffs for the agent types from sending the message $m(X)$

are determined. Let $P(X)$ denote the set of all agent types that strictly prefer their payoffs from sending the message $m(X)$ to their equilibrium payoffs. We say that a subset X is *self-signaling* if $P(X) = X$. The neologism $m(X)$ is credible if X is self-signaling. If there is a credible neologism available in an equilibrium, we say that such an equilibrium is not *neologism proof*.

The notion of neologism-proofness has a refining power in our model. To see this, suppose that parties' prior belief is *uniform* over $[0, 1]$ and utilities are quadratic. Notice that in the babbling equilibrium, it might be the case that some agent types prefer to reveal their types because either the unique action induced in equilibrium is too small ($\frac{1}{4}$) for them or the amount of money they should pay to the principal ($p = (\frac{1}{4} + b)^2$) is too high (See Figure 7). We therefore investigate if there is a self-signaling subset of the form $X = [\widetilde{\theta}, 1]$ with $\widetilde{\theta} > 0$. Suppose that $X = [\widetilde{\theta}, 1]$ send a neologism to the principal. Then, by Lemma 1 and analysis on the tacit bargaining, the principal's optimal response is to make a price offer, p', which is rejected by $[\widetilde{\theta}, \frac{\widetilde{\theta}+1}{2}]$ but accepted by $(\frac{\widetilde{\theta}+1}{2}, 1]$. From the indifference condition at $\frac{\widetilde{\theta}+1}{2}$, we have $p' = (\frac{1-\widetilde{\theta}}{4} + b)^2$. For X to be self-signaling, it is necessary and, for $\widetilde{\theta} \in (0, 1)$, sufficient that i) the agent type $\widetilde{\theta}$ is indifferent between sending the neologism inducing the action $\frac{3\widetilde{\theta}+1}{4}$ and sending his equilibrium message inducing the action $\frac{1}{4}$ and ii) the agent type 0 does not want to deviate to the neologism $m(X)$. This requires that

$$\frac{1}{4} - \widetilde{\theta} - b = -\frac{3\widetilde{\theta} + 1}{4} + \widetilde{\theta} + b \tag{8}$$

and

$$-(\frac{1}{4} - b)^2 \geq -p' = -(\frac{1 - \widetilde{\theta}}{4} + b)^2. \tag{9}$$

If the equation (8) gives a value of $\widetilde{\theta}$ in the range of $(0, 1)$ and the value of $\widetilde{\theta}$ satisfies the inequality (9), then we have constructed a self-signaling subset X. It is immediately clear that $\widetilde{\theta}$ satisfies both conditions if and only if $\frac{1}{24} \leq b < \frac{1}{4}$. Therefore, if $\frac{1}{24} \leq b < \frac{1}{4}$, then any babbling equilibrium is not neologism proof.

It is well-known that a neologism-proof equilibrium may select only a pooling equilibrium (Gertner, Gibbons, and Scharfstein [27]). Moreover, there might be no neologism-proof equilibrium in some models (Matthews [46]). However, truth-telling is always neologism proof in our model.

Proposition 4. *For any $b > 0$, the truth-telling perfect Bayesian equilibrium is neologism proof.*

Proof. See the appendix. □

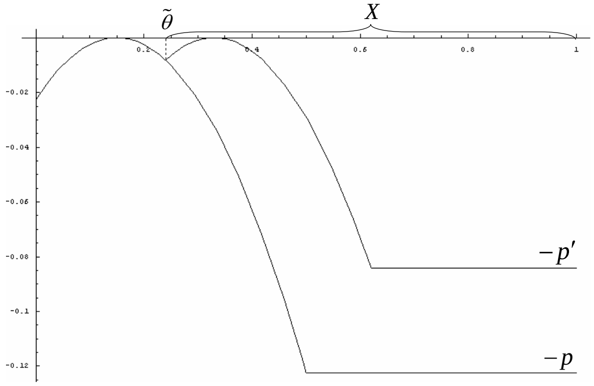

Figure 7: neologism-proofness

The proof in the appendix shows that for any non-empty subset X of Θ there must be an agent type $\theta \in X$ such that sending the neologism $m(X)$ generates a payoff less than $-l(b)$, the payoff from the truth-telling equilibrium.

4.2.2 NITS (No Incentive To Separate)

Chen, Kartik, and Sobel [16] pose a criterion to select equilibria in Crawford and Sobel [19] cheap-talk games: NITS, for *no incentive to separate*. An equilibrium satisfies NITS if the agent of the lowest type weakly prefers the equilibrium outcome to credibly revealing his type. They show that equilibria satisfying NITS always exist in Crawford and Sobel [19], and the most informative equilibrium outcome is the unique equilibrium satisfying NITS under the monotonicity condition M in Crawford and Sobel [19]. In this section, we apply NITS to our model and show that the criterion is selective; under some value of b the babbling equilibrium does not survive, while the truth-telling equilibrium does survive for any $b > 0$.

Suppose that parties' prior belief is *uniform* over $[0, 1]$ and utilities are quadratic. Notice that in the babbling equilibrium, the agent of the lowest type gets $-(\frac{1}{4} - 0 - b)^2$. Thus, the babbling equilibrium does not satisfy NITS if and only if

$$-(\frac{1}{4} - b)^2 < -b^2,$$

which is equivalent to $b < \frac{1}{8}$.

It is straightforward that NITS holds in the truth-telling equilibrium, in which all agent types reveal their types fully.

Proposition 5. *For any $b > 0$, the truth-telling perfect Bayesian equilibrium satisfies NITS.*

4.2.3 Support Restriction and Trembling-hand Perfection

One might be tempted to argue that the truth-telling equilibrium is not very reasonable because it does not satisfy "support restriction", the assumption used by several papers such as Grossman and Perry [29][31], Harrington [32], Kreps and Wilson [40] and Rubinstein [53]. This restriction requires that the support of beliefs at an information set should be contained in the supports of beliefs at preceding information sets.

In our truth-telling equilibrium the principal has probability one beliefs after getting messages in the cheap-talk stage and switches away from these beliefs to the new belief that assigns probability one to the type $\theta = 0$ after observing "rejection" of the equilibrium price offer which takes place off-the-equilibrium path, and therefore the equilibrium violates the support restriction. There are two responses to the support restriction. First, it has been shown, however, that not only the support restriction may be based on a wrong interpretation of the concept of a belief in some games but also violations of the support restriction may represent a sensible reasoning process which supports interesting equilibrium behaviors by Madrigal, Tan and Werlang [45] and Nöldeke and van Damme [51]:

> ... violations of the support restriction may very well reflect the fact that once a deviation from equilibrium behavior has been observed, a reassessment of all previous beliefs - which were based on the assumption that equilibrium strategies are followed - is called for. In this light such "switching beliefs" is not an unfortunate problem, which cannot be avoided in some cases, but actually is a natural consequence of observing a deviation. (Nöldeke and van Damme [51], p. 9.)

Second, there exists another truth-telling equilibrium in which there is no such a problem off the equilibrium path. Recall that in the previous equilibrium the agent type $\theta = 0$ is indifferent between accepting and rejecting the equilibrium-path price offer $p = l(b)$. Now, consider the strategy profile that has only one difference from the previous construction: on the equilibrium path, the principal makes a message-independent price offer $l(b)$ and all agent types (0,1] accept it with probability 1 but the agent type 0 rejects it with probability 1.[15] Except this, all of the strategy profile are exactly the same as before. It is straightforward that this strategy profile forms a perfect Bayesian equilibrium. Importantly, "rejection" of $p = l(b)$ is not an off-the-equilibrium-path event anymore in this construction. That is, there is no switching away from beliefs so that this equilibrium satisfies the support restriction.

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[15] It is not necessary in this construction that the agent type 0 rejects $l(b)$ with probability 1. If the agent type $\theta = 0$ accepts $p = l(b)$ with a positive probability, then we have an equilibrium in which "rejection" of $p = l(b)$ is not an off-the-equilibrium-path event anymore.

One may also wonder if the truth-telling equilibrium is robust against the extensive-form trembling-hand perfection by Selten [54] or sequential equilibria by Kreps and Wilson [41]. While the definition of trembling-hand perfect equilibria or sequential equilibria does not directly apply to our game[16], the truth-telling equilibrium does not violate any possible implementation of sequential equilibria or trembling-hand perfect equilibria. To see this, suppose that some trembles are allowed in the bargaining stage so that the principal makes some rejected offers with a positive probability. As a result, beliefs after every history can be derived by Bayes' rule. Then, regardless of the trembles made, the principal facing the price offer by trembles sticks to the belief he updated through the cheap-talk stage and thus taking an action $y = \theta$ is optimal.

5 Welfare Comparison

5.1 Comparisons to Other Schemes

In this section, we demonstrate the benefit from trade of decision-rights by comparing the equilibrium outcomes of our model to those of several dispute resolution schemes studied in the framework of Crawford and Sobel [19]: communication (Crawford and Sobel [19]), optimal mediation (Goltsman, Hörner, Pavlov, and Squintani [28]), optimal delegation (Holmström [35][36], Melumad and Shibano [48], Alonso and Matouschek [3] and Kováč and Mylovanov [38]) and optimal compensation contract (Krishna and Morgan [42]). For the comparison, we assume that f is *uniform* and utility functions are *quadratic* as follows:

$$U^P(y,\theta) = -(y-\theta)^2 \quad and \quad U^A(y,\theta,b) = -(y-\theta-b)^2.$$

Crawford and Sobel [19] consider a situation in which the principal has no commitment power at all and sends cheap-talk messages to the agent. It is shown that all equilibria in their model are interval partitional so that there is only a finite number of actions chosen in equilibrium, each associated with an interval of states. With uniform quadratic assumption, they show that the number of distinct equilibrium outcomes, denoted by $N_{CS}(b)$, is

$$N_{CS}(b) = \left\langle -\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2}{b}} \right\rangle \tag{10}$$

where $\langle z \rangle$ denotes the smallest integer greater than or equal to z. Moreover, there is a Pareto ranking among $N_{CS}(b)$ equilibria so that, for any $b > 0$, the number of elements of the partition associated with the *Pareto dominant* equilibrium, which we will call the *best* equilibrium,

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[16] For more details, see Simon and Stinchcombe [55]

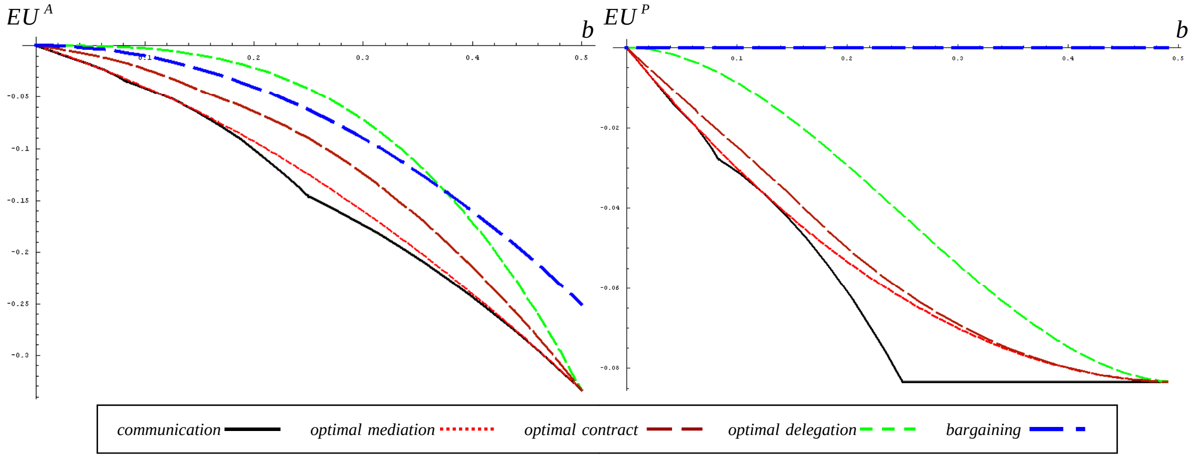

Figure 8: (a) Agent's expected payoff (b) Principal's expected payoff

is $N_{CS}(b)$. The expected payoff of the principal in this *best* equilibrium is

$$EU^P_{CS}(b) = -\frac{1}{12N_{CS}(b)^2} - \frac{b^2(N_{CS}(b)^2 - 1)}{3} \tag{11}$$

while the *ex-ante* expected payoff for the informed agent is

$$EU^A_{CS}(b) = EU^P_{CS}(b) - b^2. \tag{12}$$

Recently, Goltsman, Hörner, Pavlov, and Squintani [28] allow the parties to use any communication protocol, including the ones that call for a neutral trustworthy mediator. According to the optimal mediation rule, the parties' expected payoffs are

$$EU^P_{mediation}(b) = -\frac{b(1-b)}{3} \quad \text{and} \quad EU^A_{mediation}(b) = EU^P_{mediation}(b) - b^2. \tag{13}$$

Holmström [35][36], Melumad and Shibano [48], Alonso and Matouschek [3] and Kováč and Mylovanov [38] study the principal's optimal choice of the set of admissible actions that the agent can take and show that under the optimal delegation scheme, the principal restricts project choices of the agent to be from 0 up to a maximum of $1-b$. Under this scheme, the parties' expected payoffs are

$$EU^P_{delegation}(b) = -\frac{b^2(3-4b)}{3} \quad \text{and} \quad EU^A_{delegation}(b) = -\frac{8b^3}{3}. \tag{14}$$

In these papers, the principal also has imperfect commitment power so that she can only commit on the *ex-ante* allocation of decision-rights. Moreover, the monetary transfer is impossible.

Krishna and Morgan [42] consider the situation in which the principal can commit to pay the agent for his advice but retains decision-making authority. They fully characterize the optimal compensation contract: the optimal compensation contract involves separation in low states and a finite number of pooling intervals in high state, and the principal never pays for imprecise information. In this optimal compensation contract, the expected payoffs for the principal and the agent are

$$EU^P_{contract}(b) = -\int_0^{a_0} (2b(a_0-\theta)+t_0)d\theta - \frac{1}{12}\sum_{i=1}^{K}\left(\frac{1}{K}-\frac{a_0}{K}-2b(K-2i+1)\right)^3 \tag{15}$$

and

$$EU^A_{contract}(b) = EU^P_{contract}(b) - b^2 + 2\int_0^{a_0}(2b(a_0-\theta)+t_0)d\theta \tag{16}$$

where

$$K = \Big\langle -\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{3}{2b}}\Big\rangle,$$

$$a_0 = \frac{3}{4}-\frac{1}{4}\sqrt{4+\frac{1}{3}(3-8bK(K-1))(8bK(K+1)-3)} \quad and$$

$$t_0 = \frac{(1-a_0-2K(K-1)b)(2bK(K+1)-(1-a_0))}{4K^2}.$$

Figure 8 illustrates the comparison. As the figure shows, the truth-telling equilibrium outcome of explicit bargaining is *ex-ante* Pareto superior to communication, optimal mediation rule and optimal contract for any $b > 0$. Furthermore, it is *ex-ante* Pareto superior to all other schemes (including optimal delegation) when $b > .375$. This might explain why bargaining over decision-rights often takes place between two separately owned companies whose interests diverge widely.

It is impertinent to interpret this welfare comparison as a result showing that bargaining mechanism we considered is superior to all other schemes considered in the literature. It is more appropriate to say that the higher *ex-ante* utilities for both parties comes from different assumptions on the principal's commitment power rather than from the superiority of the mechanism. Unlike the most papers in the literature on communication and optimal delegation, this model assumes not only the principal can commit on the *ex-ante* allocation of decision-rights but also monetary transfer is available. The welfare comparison shows the benefit of using monetary transfer to trade decision-rights in our environment though. The use of monetary incentives allows parties to make full use of the agent's private information when making a decision, and as a result, the truth-telling outcome of explicit bargaining is *ex-ante* Pareto superior to the outcomes of several dispute-resolution schemes studied in the literature.

5.2 Comparisons to Bargaining with Agents Making Offers

Lim [44] considers bargaining over decision-making authority in which the informed agent makes a price offer (*A-offer Bargaining*) and shows that there are continuum of perfect Bayesian equilibria, each of which yields an *ex-post* efficient outcome. Although there is no general Pareto ranking among equilibria, Lim [44] shows that there exists the principal optimal equilibrium and the agent optimal equilibrium. Moreover, the principal optimal equilibrium gives the lowest payoff to the agent among all equilibria of the model and vice versa. By using the refinement of perfect sequential equilibrium (Grossman and Perry [29]), Lim [44] gets a unique equilibrium outcome, which coincides with the agent optimal equilibrium.

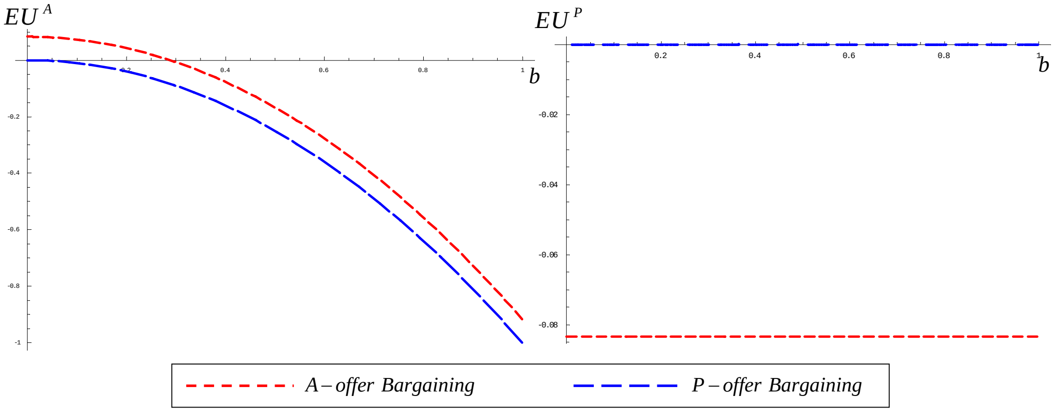

Figure 9: (a) Agent's expected payoff (b) Principal's expected payoff

Since the uninformed principal makes a price offer in our model (*P-offer Bargaining*), the equilibrium outcomes are quite different from those of *A-offer Bargaining.* Interestingly, the truth-telling equilibrium outcome of *P-offer Bargaining* coincides with the outcome of the principal optimal equilibrium of *A-offer Bargaining.* Figure 9 illustrates this result. It tells us that although bargaining over decision-rights can lead to a Pareto-efficient outcome regardless of who has bargaining power, allocation of initial bargaining power plays an important role in determining how they share the resulting surplus.

6 Discussion and Extensions

6.1 Multidimensional State Space

In this section, we extend our model to the case where the state space is multidimensional. We show that the boundedness, as opposed to dimensionality, of the state space plays a crucial role for the existence of the truth-telling equilibrium. Suppose that the state space Θ is a closed subset of $\mathbb{R}^d$. Similarly, the action space Y is a subset of $\mathbb{R}^d$. $b \in \mathbb{R}^d$ is a bias of the agent. For state θ and action y, the principal's utility is $-\sum_{j=1}^{d} l(|y_j - \theta_j|)$ and the agent's utility is $-\sum_{j=1}^{d} l(|y_j - \theta_j - b_j|)$ where y_j, θ_j, and b_j are jth coordinate of y, θ, and b, respectively.

It is straightforward to see that the construction of truth-telling equilibrium from the previous section can be extended to the current model with multidimensional state space, if the state space is bounded. Let $\underline{W}_j$ and $\overline{W}_j$ denote minimum and maximum of j's coordinate of Θ, respectively. Define $\theta^* = (\theta_1^*, \theta_2^*, ..., \theta_d^*)$ such that $\theta_j^* = \underline{W}_j$ if $b_j \geq 0$ and $\theta_j^* = \overline{W}_j$ otherwise. The following strategy profile constitutes a perfect Bayesian equilibrium: (i) All agent types fully reveal their types in the cheap talk stage. (ii) All agent types but θ^* accept any offer $p \leq \sum_{j=1}^{d} l(|b_j|)$ with probability one and otherwise, reject with probability one. (iii) The agent type θ^* rejects the offer $p = \sum_{j=1}^{d} l(|b_j|)$ with positive probability and behaves in the same way as other agent types for any other price offers. (iv) The principal makes a price offer $\sum_{j=1}^{d} l(|b_j|)$ regardless of the message he received from the agent. (v) If the offer $p = \sum_{j=1}^{d} l(|b_j|)$ is rejected, the principal believes that the type is θ^* with probability one and takes an action $y = \overline{y}^P(\theta^*) = \theta^*$. (vi) If the offer $p \neq \sum_{j=1}^{d} l(|b_j|)$ is rejected, the principal believes messages from agent types and takes an action $y = \theta$. (vii) If any offer is accepted, the agent type θ takes his ideal action $\theta + b$.

This equilibrium is graphically illustrated in Figure 10. In this figure, the solid circle represents the indifference curve of the agent at the threat action θ^* (in this example, it is 0) that would be taken by the principal in case that the equilibrium price offer $\sum_{j=1}^{d} l(|b_j|)$ is rejected. The distance between θ and $\theta + b$ represents the equilibrium price offer $\sum_{j=1}^{d} l(|b_j|)$. As you can see in this figure, any agent type $\theta \neq 0$ has to accept the equilibrium price offer since the distance between θ and $\theta + b$ is shorter than the radius of the circle with the origin of $\theta + b$. Remarkably, those two coincide with each other only if $\theta = \theta^* = 0$, which means that the agent type θ^* is indifferent between accepting and rejecting $\sum_{j=1}^{d} l(|b_j|)$. Taking the action θ^* turns out to be rational for the principal (and as a result credible to the agent) under the belief that the agent type θ^* is the only type who randomizes between accepting and rejecting the price offer. Again, truth-telling is optimal for the agent since the principal makes a message-independent price offer $\sum_{j=1}^{d} l(|b_j|)$. The dotted circle represents the indifference curve of the agent at the action $y = \theta$. Truth-telling guarantees the agent the utility level in the dotted indifference curve so that truthful agent will always reject a price offer higher than $\sum_{j=1}^{d} l(|b_j|)$. This makes it optimal for the principal to make such an offer $\sum_{j=1}^{d} l(|b_j|)$ regardless of messages.

Figure 10 also highlights the importance of the boundedness of the state space. Especially,

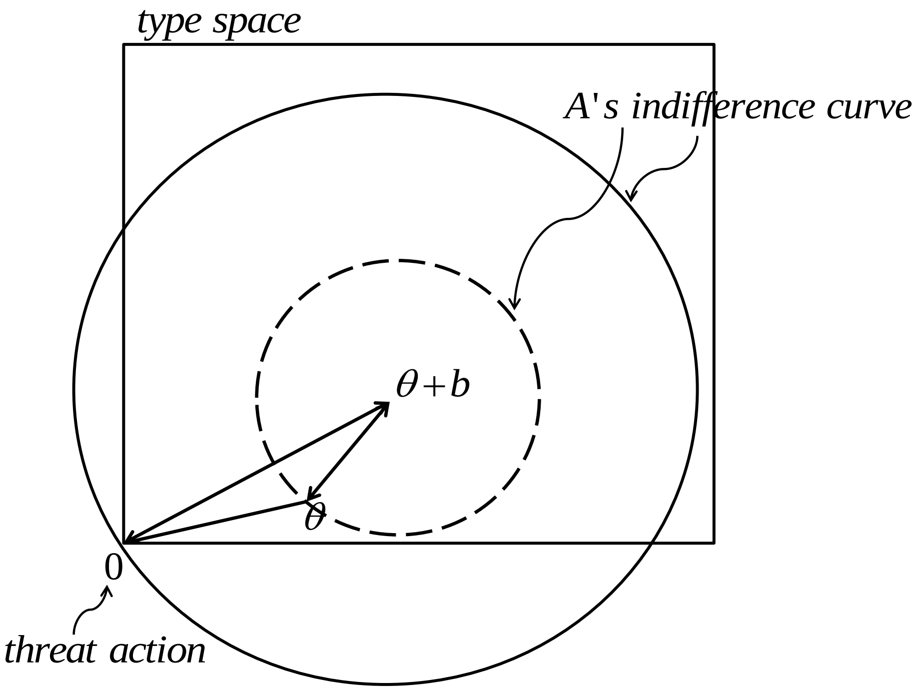

Figure 10: Constructing a truth-telling equilibrium

the construction of the credible threat action heavily relies upon boundedness of the state space. If each of the unidimensional subspaces of the entire state space is neither bounded below nor bounded above, then it is impossible to construct any credible threat action or equivalently an agent type who is indifferent between accepting and rejecting some message-independent price offer. Indeed, boundedness of the state space is a sufficient condition for truth-telling equilibrium to exist. This result is summarized in the following proposition. The proof is omitted and replaced by Figure 10 since it is a straightforward extension of the unidimensional case.

Proposition 6. *Suppose that* $\Theta \subset \mathbb{R}^d$ *is bounded and convex. For any* $b \in \mathbb{R}^d$*, there exists a truth-telling equilibrium.*

It is interesting to compare this result to the main finding of multi-dimensional cheap talk literature. Battaglini [10] shows that full revelation of information in all states of nature is generically possible if there is more than one sender and the state space is multidimensional. Ambrus and Takahashi [5] further investigate the existence of fully revealing equilibrium and find that boundedness, as opposed to dimensionality, of the state space plays an important role in determining the qualitative feature of a cheap talk model. They show that if the state space is bounded and biases are large enough then there is no fully revealing perfect Bayesian equilibrium. It is worth emphasizing that although the boundedness of state space also plays an important role for fully revealing equilibria to exist in our model, the role is not the same as in Ambrus and Takahashi [5]. If the state space is bounded, then there exists a fully revealing equilibrium even when conflict of interest is arbitrarily large. Unlike Battaglini [10] and Ambrus and Takahashi [5], multiple agents are not necessary even though the construction does not rely on the fact that there is only one agent. The direction of biases does not matter for this result.

6.2 Optimal Bargaining Mechanism

One important observation is that bargaining over decision-rights that we have considered is not the only bargaining mechanism for decision-rights in our environment. For example, after the principal's price offer is rejected, the principal may want to make another price offer instead of taking action by herself. It may be possible that the agent makes a price offer to buy decision-rights after he rejects a price offer from the principal in the first round. By a bargaining mechanism, we mean any kind of scheme by which principal and agent make offers directly, indirectly, once, repeatedly, sequentially, simultaneously, alternatively and so on. Accordingly, we have *infinitely many* alternatives as a bargaining mechanism for decision-rights. In this section, we follow a mechanism design approach and show that the explicit bargaining we have considered is an optimal mechanism among all feasible mechanisms in the sense that it achieves the upper bound of the *ex-ante* social welfare. Although the analysis in this section is restricted to the quadratic utility functions, I believe that one can get a similar result for a broader class of utility functions.[17]

A *bargaining mechanism* is one in which the informed agent send a message to a mediator who then credibly commits to the final allocation of decision-rights and the monetary transfers. We restrict our attention to a *direct bargaining mechanism* in which the informed agent reports the true state of the world to a mediator who then determines the final allocation of decision-rights and the monetary transfers. This means that a direct mechanism is characterized by two outcome functions, denoted by $x(\cdot)$ and $p(\cdot)$, where $x(\theta)$ is the probability that the decision-right is transferred to the agent and $p(\theta)$ is the expected payment from the agent to the principal if θ is the reported state of the world from the agent.

Bester and Strausz [12] shows that the standard revelation principle by Myerson [49] may fail if the mechanism designer is not able to credibly commit to the outcome of the mechanism. In our case, however, it is straightforward to see that the standard argument for the revelation principle holds. Consider an indirect mechanism (x, p) where the agent follows a message rule $m : \Theta \to M$ in an equilibrium of the mechanism where M is a Borel-measurable message space. In this mechanism, 1) the agent takes an action $\theta + b$ and $p(m(\theta))$ is transferred from the agent to the principal with probability $x(m(\theta))$ and 2) the principal takes an action based on the information she updated from observing $p(m(\theta))$ and $x(m(\theta))$ with probability $1-x(m(\theta))$. Define a direct mechanism $(x', p') \equiv (x \circ m, p \circ m)$. The outcome of this mechanism is that 1) the agent takes an action $\theta + b$ and $p'(\theta)$ is transferred from the agent to the principal with probability $x'(\theta)$ and 2) the principal takes an action based on the information she updated from observing $p'(\theta)$ and $x'(\theta)$ with probability $1 - x'(\theta)$. By definition of the direct mechanism, $p(m(\theta)) = p'(\theta)$ and $x(m(\theta)) = x'(\theta)$ for any state of the world $\theta \in \Theta$. Furthermore, this implies that for any state of the world the actions taken by the principal under two mechanisms are exactly the same since the same information is transmitted from the agent to the principal through the mechanisms. Therefore, these two mechanisms are outcome equivalent. By invoking this version of revelation principal, we restrict our attention to a direct mechanism without loss of generality.

Our goal is to find a mechanism that maximizes a social welfare which is defined as the sum of expected payoff of the principal and of the agent. By focusing on this mechanism, we are able to see if a bargaining mechanism can achieve the first-best efficient outcome in our environment. Recall that in the quasi-linear environment there exists a unique first-

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[17] However, the result does not depend on the distribution function.

best outcome that maximizes the joint surplus of parties. In case of quadratic utility, it is the midpoint of θ and $\theta + b$. At the first-best outcome, the social welfare is $-\frac{b^2}{4}$. Does any bargaining mechanism achieve this first-best outcome? Otherwise, what is the efficiency bound?

Given a mechanism (p, x), define $\mathcal{U}$ as a social welfare. Then

$$\begin{aligned}
\mathcal{U} &= EU^A + EU^P \\
&= \int_\Theta \{x(\theta) \cdot U^A(\overline{y}^A(\theta), \theta, b) + (1 - x(\theta)) \cdot U^A(\overline{y}^P(p(\theta)), \theta, b) - p(\theta)\} f(\theta) d\theta \\
&\quad + \int_\Theta \{x(\theta) \cdot U^P(\overline{y}^A(\theta), \theta) + (1 - x(\theta)) \cdot U^P(\overline{y}^P(p(\theta)), \theta) + p(\theta)\} f(\theta) d\theta \\
&= \int_\Theta \{-x(\theta) \cdot l(b) + (1 - x(\theta)) \cdot [U^P(\overline{y}^P(p(\theta)), \theta) + U^A(\overline{y}^P(p(\theta)), \theta, b)]\} f(\theta) d\theta. \quad (17)
\end{aligned}$$

We say that a mechanism (p, x) is optimal if it maximizes the social welfare. That is, an optimal mechanism is a solution for the following optimization problem:

$$\max_{p(\cdot), x(\cdot)} \mathcal{U}$$

subject to the incentive compatibility constraint.

Equation (17) shows that an optimal mechanism should assign decision-rights to the principal if and only if the sum of interim utilities resulting from the action $\overline{y}^P(p(\theta))$ is greater than $-l(b)$ where $\overline{y}^P(p(\theta))$ is an action taken by the principal who updates her belief after observing $p(\theta)$. Thus, if a mechanism achieves a social welfare $\mathcal{U} > l(b)$, there exists a nonempty set of agent types, denoted by S, such that

$$\forall \theta \in S \quad x(\theta) = \overline{x} \quad and \quad p(\theta) = \overline{p}, \tag{18}$$

and

$$\int_S [U^P(\overline{y}^P(\overline{p}), \theta) + U^A(\overline{y}^P(\overline{p}), \theta, b)] f(\theta) d\theta > -l(b) \tag{19}$$

where

$$\begin{aligned}
\overline{y}^P(\overline{p}) &= argmax_y \int_\Theta U^P(y, \theta) \frac{q(\overline{p}|\theta) f(\theta)}{\int_\Theta q(\overline{p}|\theta') f(\theta') d\theta'} d\theta \\
&= argmax_y \int_S U^P(y, \theta) f(\theta) d\theta.
\end{aligned}$$

Let $\overline{\mathcal{U}}$ denote the upper bound of the social welfare. The next proposition shows that when the utility function is quadratic the upper bound of the social welfare is $-l(b) = -b^2$ in any bargaining mechanism.

Proposition 7. *Suppose that the utility function is quadratic. Then* $\overline{\mathcal{U}} = -l(b) = -b^2$.

Proof. See the appendix. ☐

The intuition of this result is straightforward. A bargaining mechanism determines the final allocation of decision-rights but has no effect on the incentive in the decision-making stage. That is, the final decision depends only on the decision-making party's own interest and private information the party possess. It is well-known from the cheap-talk literature that more precise information is always beneficial *ex-ante* not only to the principal but also to the agent. Therefore, the social welfare cannot be higher than $-l(b)$, the social welfare that results from the most informative decision-making. Recall that the explicit bargaining we have considered in the previous sections leads to the social welfare $-l(b)$ in the truth-telling equilibrium. This leads to the following corollary.

Corollary 2. *When the utility function is quadratic, the explicit bargaining is an optimal mechanism.*

Notice that the efficiency of bargaining mechanisms is bounded away from the first-best efficiency. Therefore, one can interpret this result as theoretical supports reinforcing the previous finding that property rights and voluntary private negotiation are not able to achieve this first-best efficient outcomes when information is asymmetrically distributed.

7 Conclusion

This paper studies bargaining over decision-making rights between an informed but self-interested agent and an uninformed principal in which the uninformed principal makes a price offer to the agent who then decides either to accept or to reject it. We show that the unique perfect Bayesian equilibrium outcome does not satisfy *ex-post* efficiency. Once we introduce explicit communication into the model, however, there exists a truth-telling perfect Bayesian equilibrium, which is not only efficient *ex-post* but also neologism proof. Moreover, it is the unique neologism-proof equilibrium if parties' preferences are sufficiently similar.

We compare the equilibrium outcome of our model to that of some dispute resolution schemes studied in the framework of Crawford and Sobel [19] and and Holmström [35] and show that it is *ex-ante* Pareto superior to all other schemes when the parties' interests diverge substantially. This might explain why bargaining over decision-rights often takes place between two separately owned companies whose interests diverge widely. Although bargaining over decision-rights can lead to a Pareto-efficient outcome regardless of who has bargaining power, allocation of initial bargaining power plays an important role in determining how they share the resulting surplus.

A Appendix. Proofs

Proof of Lemma 1. First, any price offer $p < 0$ is accepted by all agent types because for any $\theta \in [0,1]$, $U^A(y,\theta,b) < -p$ for any action $y \in \mathbb{R}$. Thus, in the remainder of the proof, take $p \geq 0$. Let $\overline{y}$ denote the principal's action induced by the offer p. Suppose that an agent type $\overline{\theta} \in [0,1]$ accepts the price offer p with positive probability in equilibrium. Then we have $U^A(\overline{y},\overline{\theta},b) \leq -p$. By continuity, there exists $\theta_p \in [0,1]$ such that $U^A(\overline{y},\theta_p,b) = -p$. (Otherwise, we have $U^A(\overline{y},\theta,b) < -p$ for any $\theta \in [0,1]$ so that all agent types accept the offer, which means the proof is done.) Now, suppose that $\theta_p > \overline{\theta}$. Then by quasi-concavity of U^A, the set of agent types who reject p with probability one is $(\theta_p, \theta']$ with $\theta_p < \theta' \leq 1$. Since the agent type θ' rejects the offer, we have $U^A(\overline{y},\theta',b) \geq -p$. However, by **(B4)** and Bayes' rule, we have $\overline{y} = y(\theta_p,\theta')$ and by Condition 1, $U^A(\overline{y},\theta',b) < U^A(\overline{y},\theta_p,b) = -p$, which leads to a contradiction. Thus, we have $\theta_p \leq \overline{\theta}$. Then, by the strict-concavity of U^A, we get $U^A(\overline{y},\theta,b) < -p$ for all $\theta > \theta_p$. This implies that under Condition 1, the monotonicity holds. We complete our proof by pointing out that the uniform prior satisfies Condition 1.

Proof of Proposition 2. Lemma 1 implies that for any $p \in \mathbb{R}$, both $\Theta(p)$ and $\Theta^{-1}(p)$ are convex if they are non-empty. Further, $\Theta(p)$ cannot be to the left of $\Theta^{-1}(p)$. These guarantee that for any $p \in \mathbb{R}$ there is at most one agent type who is indifferent between accepting and rejecting the offer. Let $\theta_p \in [0,1]$ denote the agent type if it exists. Then we can write that $\Theta(p) = (\theta_p, 1]$ and $\Theta^{-1}(p) = [0, \theta_p)$. From the indifference condition at θ_p we have

$$p = l(|y^P(p) - \theta_p - b|), \tag{20}$$

where

$$y^P(p) = arg\max_y \int_0^{\theta_p} -l(|y-\theta|) \cdot \frac{f(\theta)}{F(\theta_p)} d\theta = y(0,\theta_p). \tag{21}$$

Notice that $y(0,\theta_p) < \theta_p$. Then from (20), we have

$$\theta_p = y(0,\theta_p) + l^{-1}(p) - b. \tag{22}$$

Then the principal chooses p^* to solve

$$\begin{aligned} \max_{p\in\mathbb{R}} EU^P \quad &= \quad \int_0^{\theta_p} -l(|y^P(p) - \theta|)d\theta + (1-\theta_p)(p - l(b)) \\ &\quad\;\; \text{s.t. (22)}. \end{aligned} \tag{23}$$

Since, from (21), $\frac{\partial y^P(p)}{\partial \theta_p} = \frac{\partial y(0,\theta_p)}{\partial \theta_p}$ and f has a full support, $0 < \frac{\partial y^P(p)}{\partial \theta_p} < 1$. From (22), we have

$$\frac{\partial \theta_p}{\partial p} = \frac{\partial y^P(p)}{\partial \theta_p} \cdot \frac{\partial \theta_p}{\partial p} + \frac{\partial l^{-1}(p)}{\partial p}.$$

After some rearrangement, we get

$$\frac{\partial \theta_p}{\partial p} = \frac{1}{1 - \frac{\partial y^P(p)}{\partial \theta_p}} \cdot \frac{\partial l^{-1}(p)}{\partial p}.$$

Since $\frac{\partial l^{-1}(p)}{\partial p} \geq 0$, we have $\frac{\partial \theta_p}{\partial p} \geq 0$. This, together with $0 < \frac{\partial y^P(p)}{\partial \theta_p} < 1$, implies that $\frac{\partial y^P(p)}{\partial p} > 0$. Taking a derivative in (23) w.r.t. p yields

$$\frac{\partial EU^P}{\partial p} = \frac{\partial \theta_p}{\partial p} \cdot (-l(|y(0, \theta_p) - \theta_p|) - p + l(b)) + (1 - \theta_p). \tag{24}$$

At $\theta_p = 0$, we have

$$\frac{\partial EU^P}{\partial p}\Big|_{\theta_p=0} = \frac{\partial \theta_p}{\partial p}\Big|_{\theta_p=0} \cdot (l(b) - l(b)) + (1 - 0) = 1 > 0. \tag{25}$$

At $\theta_p = 1$, we have

$$\frac{\partial EU^P}{\partial p}\Big|_{\theta_p=1} = \frac{\partial \theta_p}{\partial p}\Big|_{\theta_p=1} \cdot (-l(|y(0,1) - 1|) - l(|1 - y(0,1) + b|) + l(b)) < 0. \tag{26}$$

Taking a derivative in (24) w.r.t. p yields

$$\begin{aligned} \frac{\partial^2 EU^P}{\partial p^2} \quad = \quad & \frac{\partial^2 \theta_p}{\partial p^2} \cdot (-l(|y(0, \theta_p) - \theta_p|) - p + l(b)) \\ & + \frac{\partial \theta_p}{\partial p} \cdot (-l'(|y(0, \theta_p) - \theta_p|) \cdot (-\frac{\partial y(0, \theta_p)}{\partial p} + \frac{\partial \theta_p}{\partial p}) - 1) - \frac{\partial \theta_p}{\partial p}. \end{aligned} \tag{27}$$

It is routine to verify that

$$\frac{\partial^2 EU^P}{\partial p^2} < 0 \quad \textit{if}\ \theta_p \in [0, 1].$$

Therefore, by continuity, the principal's optimal price offer p^* is unique and $\theta_{p^*} \in [0, 1]$. This completes our proof.

Proof of Proposition 4. Note that for any $\theta \in \Theta$, the agent's payoff in the truth-telling equilibrium is $-l(b)$. Any singleton subset of Θ could not be self-signaling because sending a neologism message by himself reveals his true type to the principal so that the agent type in the set could not get more than $-l(b)$. Thus, suppose that an arbitrary non-singleton subset $\widehat{\Theta}$ of Θ sends a neologism message $\widehat{m}$ to the principal. Let $\widehat{p}$ denote the price offer induced by $\widehat{\Theta}$. Then $\widehat{p} \geq l(b)$ because, otherwise, all agent types in Θ would be strictly better off by accepting the offer $\widehat{p}$ which implies $\widehat{\Theta} = \Theta$. However, by Lemma 1, making the price offer $\widehat{p} < l(b)$ is never optimal for the principal who believes that $\widehat{\Theta} = \Theta$. Thus, the price offer induced by $\widehat{\Theta}$ is greater than or equal to $l(b)$. In order for the neologism $\widehat{m}$ to be credible, all agent types in $\widehat{\Theta}$ should reject $\widehat{p}$ since accepting $\widehat{p}$ gives them at most $-l(b)$. However, rejecting $\widehat{p}$ cannot give all agent types in $\widehat{\Theta}$ higher payoff than $-l(b)$ either because the principal's action induced by $\widehat{\Theta}$, denoted by $y^P(\widehat{p})$, is always in the interior of $C(\widehat{\Theta})$, convex hull of $\widehat{\Theta}$, and as a result, there always exist some agent types to the right of $y^P(\widehat{p})$ who get strictly less payoff than $-l(b)$. Therefore, $\widehat{m}$ cannot be a credible neologism.

Proof of Proposition 7. It suffices to show that for an arbitrary subset S of Θ such that for any $\theta \in S$ $x(\theta) = \overline{x}$ and $p(\theta) = \overline{p}$,

$$\int_S [U^P(\overline{y}^P(\overline{p}), \theta) + U^A(\overline{y}^P(\overline{p}), \theta, b)] f(\theta) d\theta \leq -l(b) = -b^2. \tag{28}$$

It is well-known from cheap-talk literature that

$$\int_S U^A(\overline{y}^P(\overline{p}), \theta, b) f(\theta) d\theta = \int_S U^P(\overline{y}^P(\overline{p}), \theta) f(\theta) d\theta - b^2.$$

Therefore, we get

$$\int_S [U^P(\overline{y}^P(\overline{p}), \theta) + U^A(\overline{y}^P(\overline{p}), \theta, b)] f(\theta) d\theta = \int_S 2U^P(\overline{y}^P(\overline{p}), \theta) f(\theta) d\theta - l(b) \leq -l(b) = -b^2.$$

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