University of Pittsburgh
Swanson School of Engineering

Mechanical Design II
Professor Stephen Ludwick

Design Project #2

Design of a Power Transmission Shaft

May 26, 2011

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1. Project Definition

The goal of this project is to design a shaft to transmit power from a motor to a sheave. The shaft should be supported by two pillow blocks, attach to a motor with a coupling, and transmit the power to the other end of the shaft where there is a sheave. The shaft should be designed to be fully functional (e.g. all parts should be attached properly). Analysis of the shaft should be done for both static and dynamic loading. Confirmation and comparison on hand results should be done using ANSYS Workbench.

2. Functional Requirements

1. Shaft should rotate within bearings
2. Rotation speed of at least 1500RPM
3. Transmit 400W of power
4. Requires two pillow blocks; each must be manufactured by Misumi and have the part number BGHK-6802 ZZ-30 (15mm bore)
5. The pillow blocks must be 300mm apart
6. Infinite Life (~10^6 cycles)
7. Must not fail due to static loading

3. Parts Specification

Parts selection was based on ease of purchase/ability to find the part. The final dimensions of the shaft were determined by the dimensions of the attachment parts. The diameter of the pillow blocks determined the diameter of the middle of the shaft between the pillow blocks (24mm) as well as the diameter of the shaft on the outsides of the pillow blocks because it had to be the same diameter as the bore of the couplings (15mm). The outside diameter of the shaft where the coupling is attached determined the bore of the coupling (15mm). The coupling chosen was a slit style making it flexible and able to be attached to different motor configurations. The coupling attaches to the shaft by clamping and therefore doesn’t need any external constraints to keep it in place. In choosing the sheave its diameter had to be 15mm or smaller. A diameter of 12mm was chosen, requiring another fillet in the shaft but ensuring the pulley was installed in the correct axial location. The pulley is attached with two set screws and therefore also does not need any external constraints to keep it in place. The list of parts, suppliers, and costs can be found in *Appendix A*.

4. Material Selection and Design Specifications

The material chosen for this design is 1018 steel. It is readily available in round shaft form and can be easily machined. The final shaft length of 410mm was determined from component dimensions. Distances were assumed in order to perform calculations.

5. Calculations

See Appendix B. From the calculations it was determined that the shaft would most likely fail at pillow block #1 (next to the coupling) during static loading and that it would most likely fail at the sheave during dynamic loading. The factors of safety/life cyclic analysis concluded that the shaft would not fail though.

6. CAD (SolidWorks)

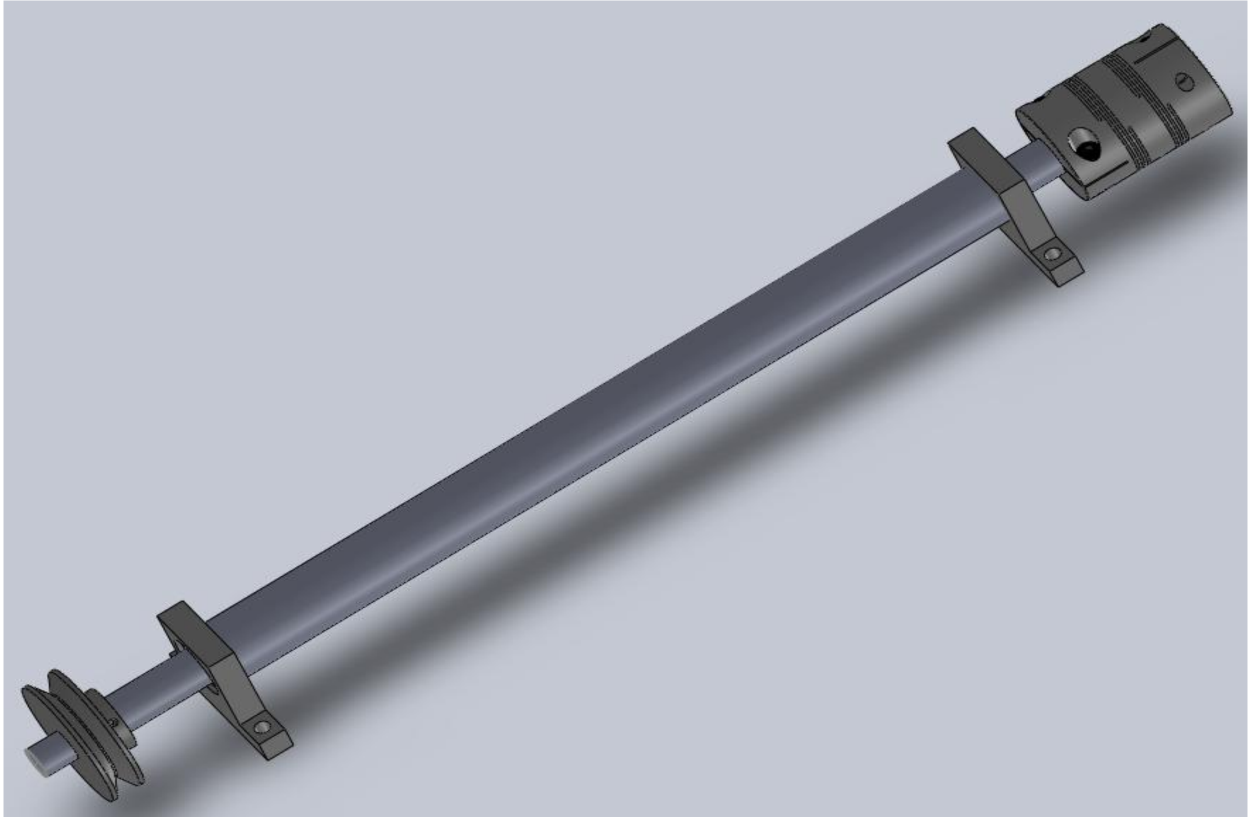


Figure1. Assembled View

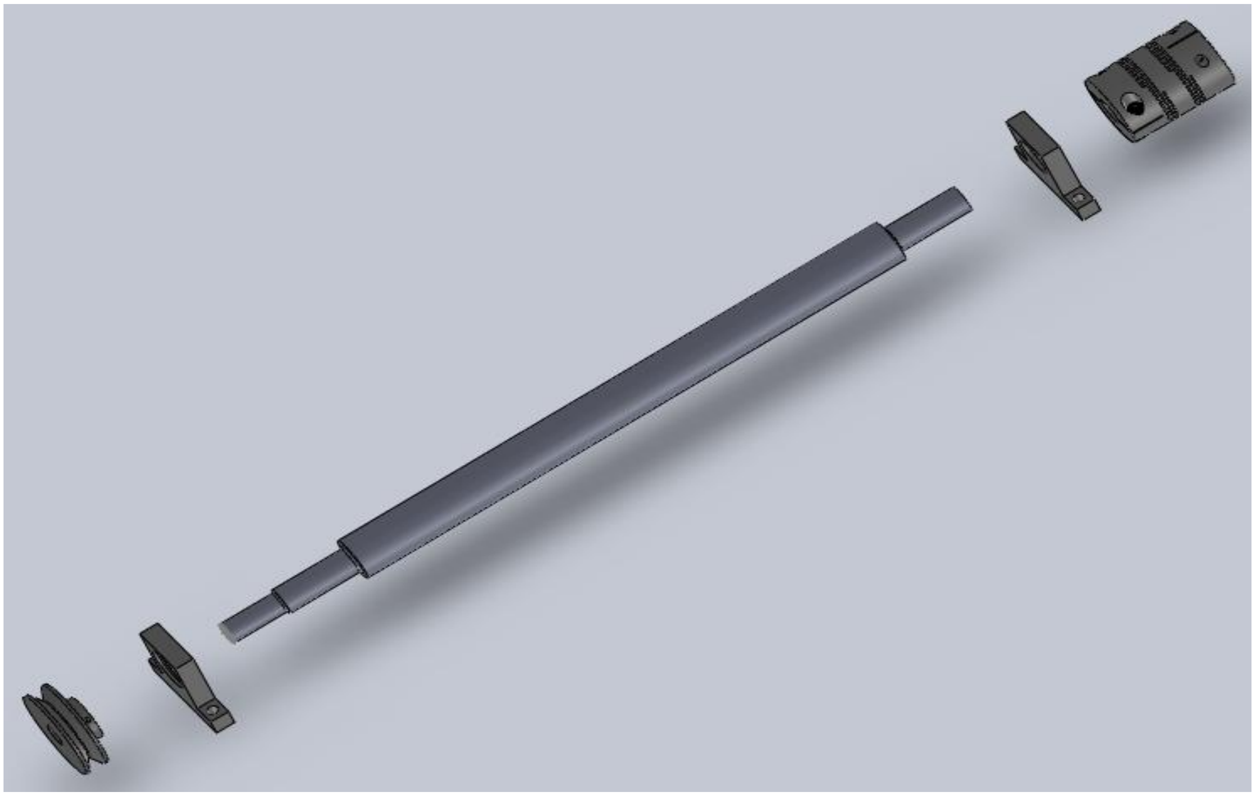


Figure 2. Exploded View

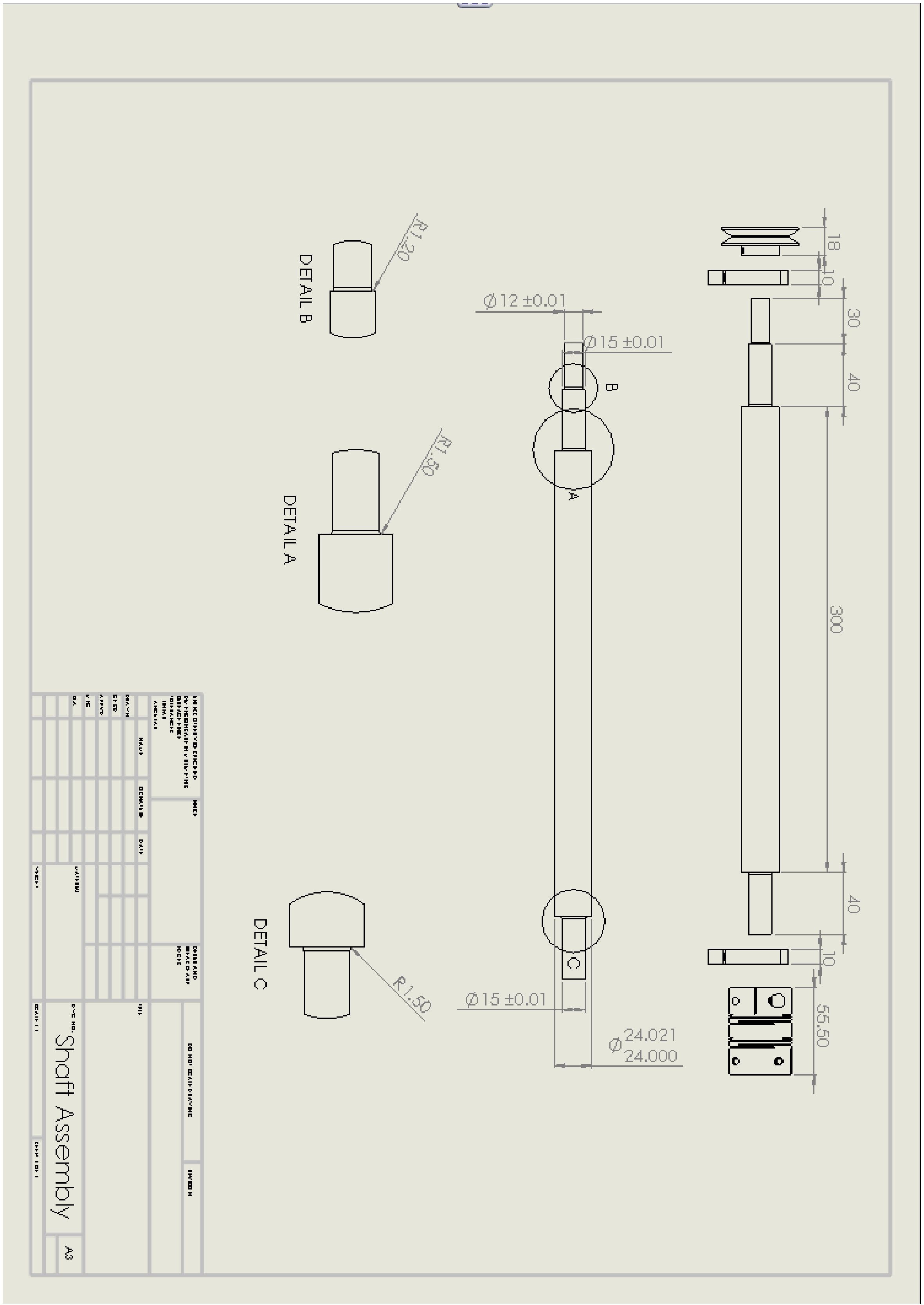


Figure 3. Assembly Drawing

7. Finite Element Analysis

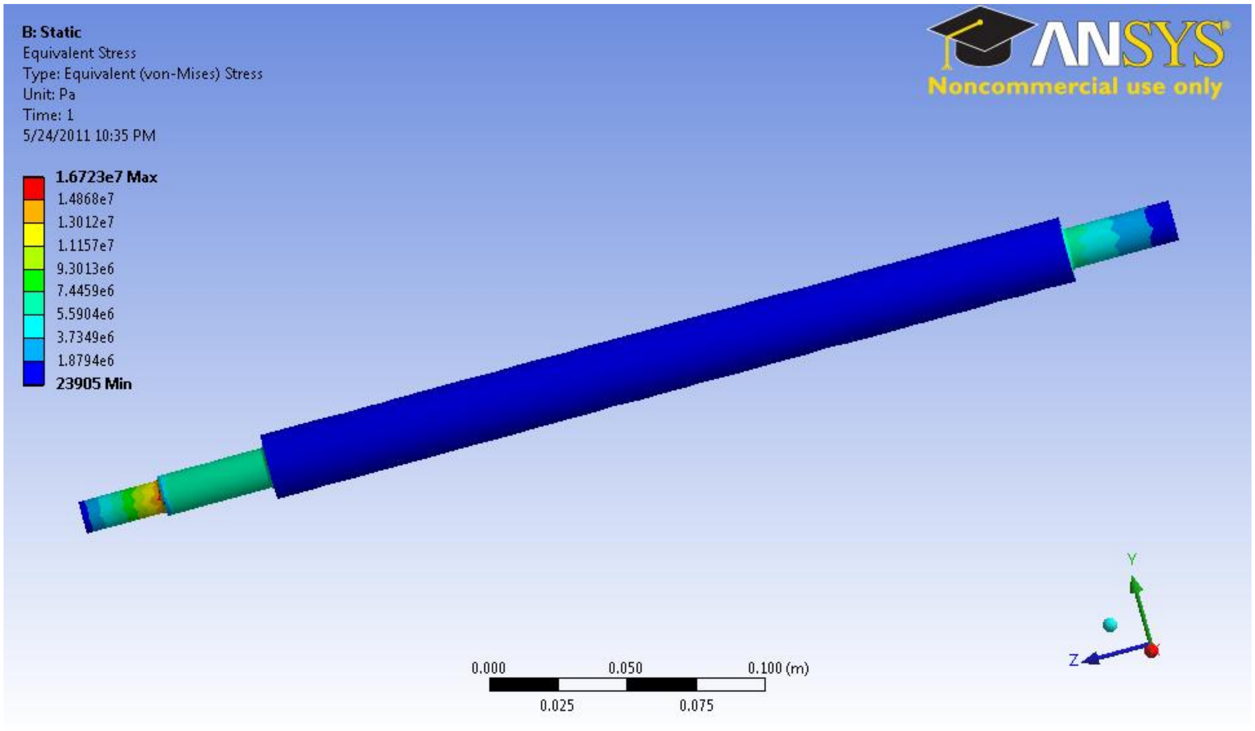


Figure 4. Equivalent Stress Analysis

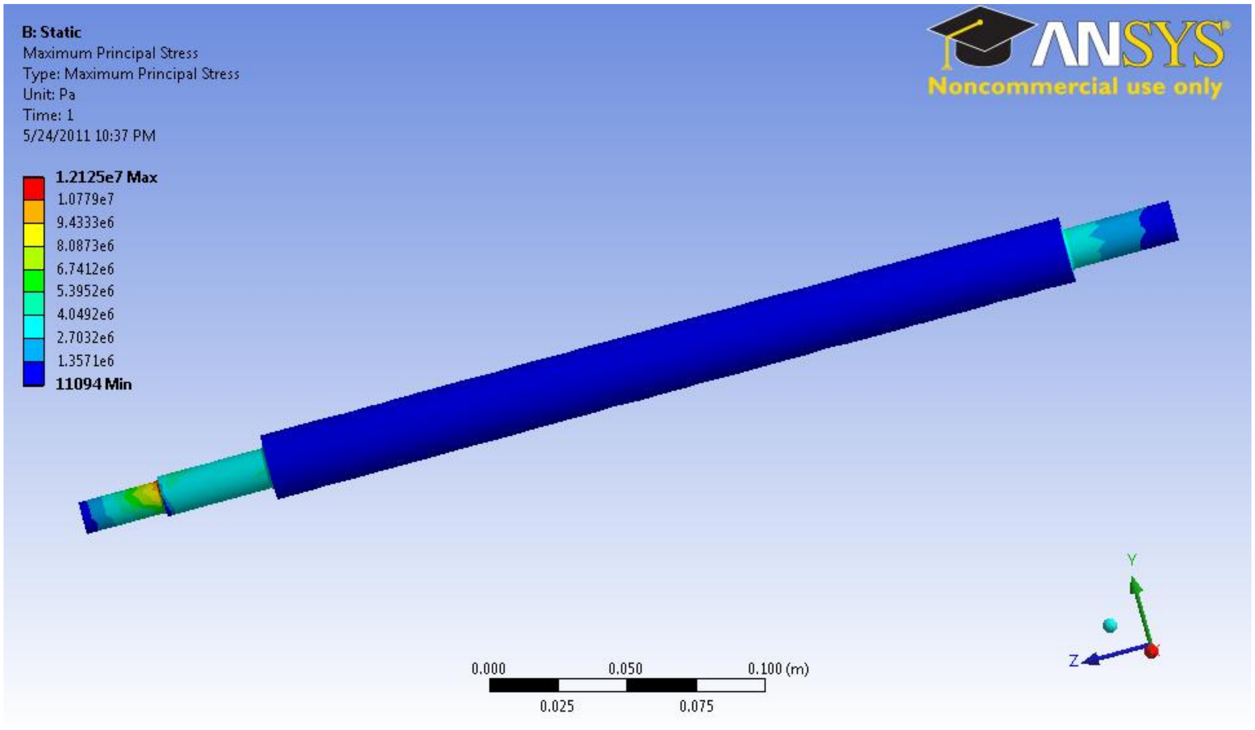


Figure 5. Maximum Principal Stress Analysis

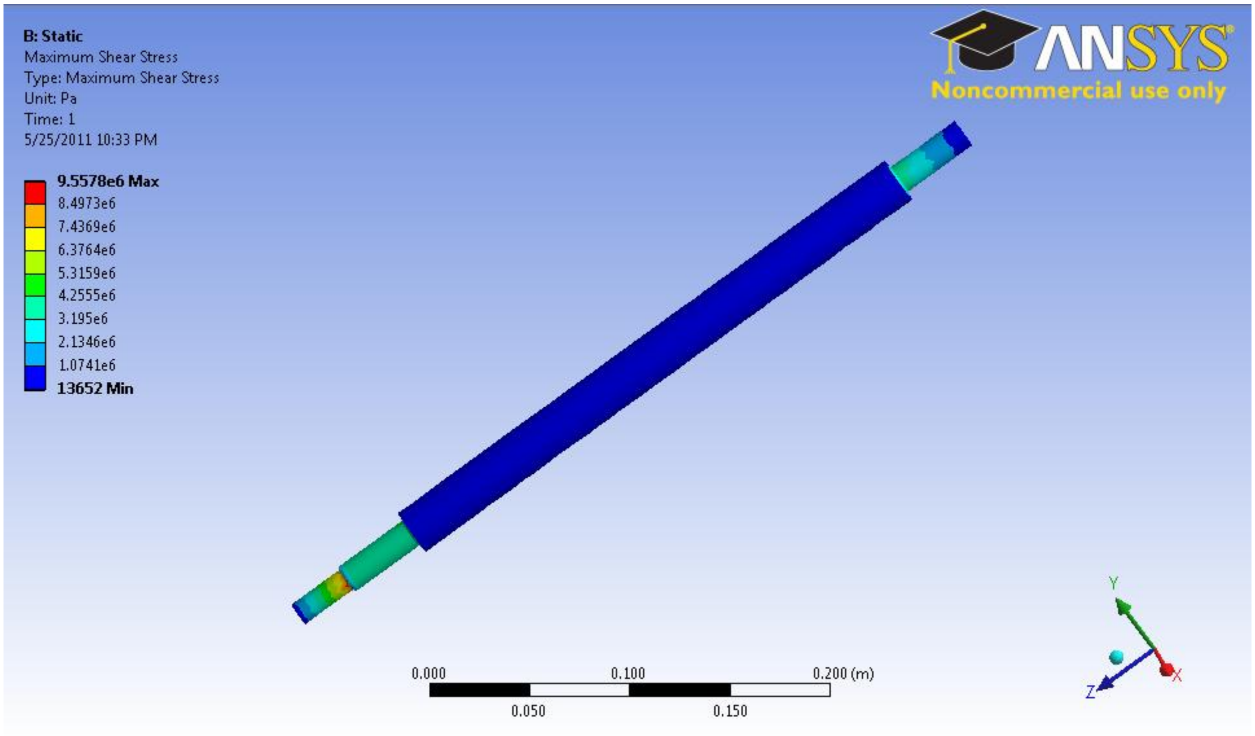


Figure 6. Maximum Shear Stress Analysis

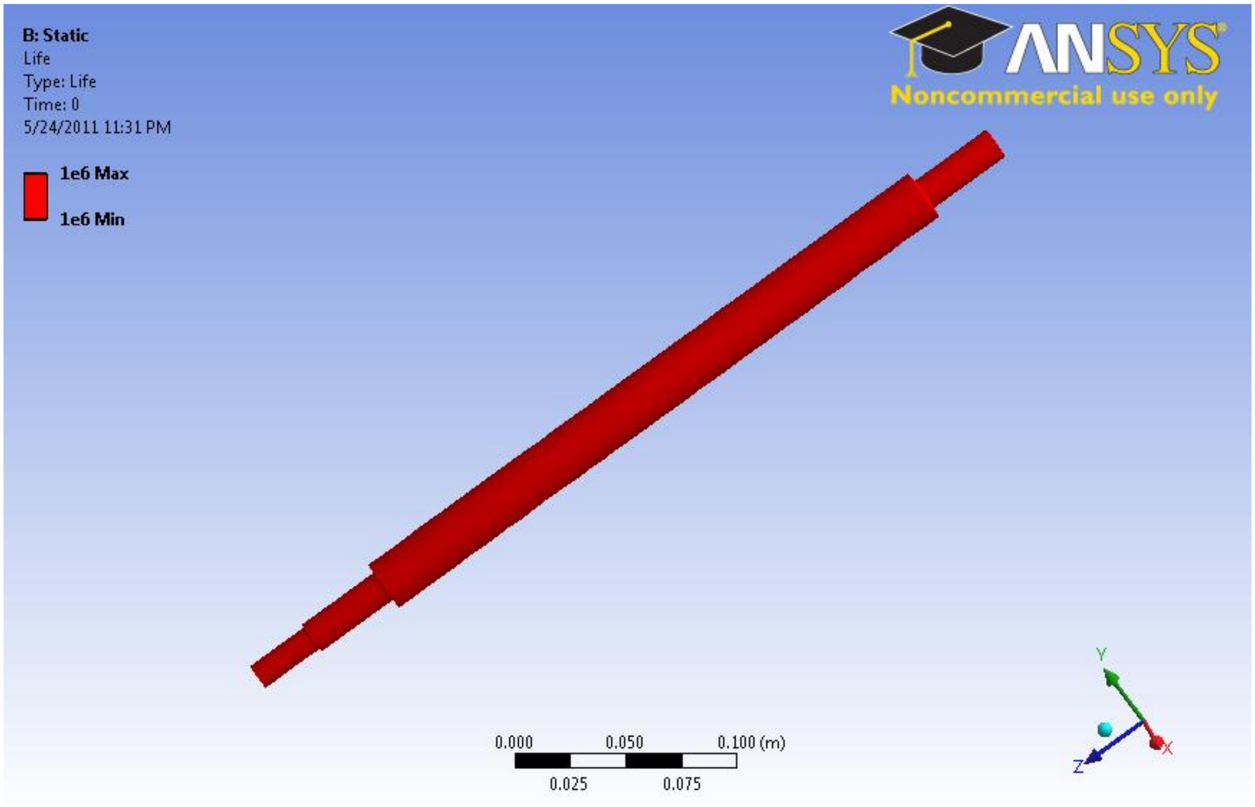
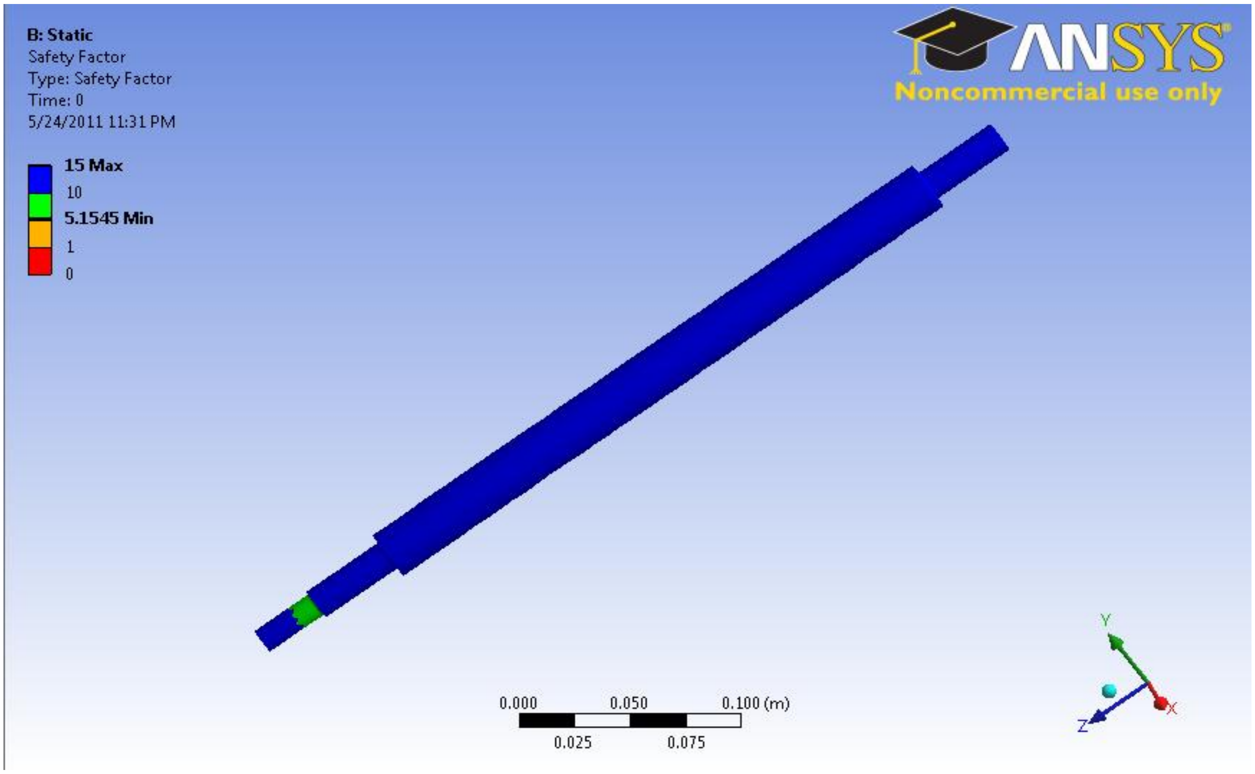
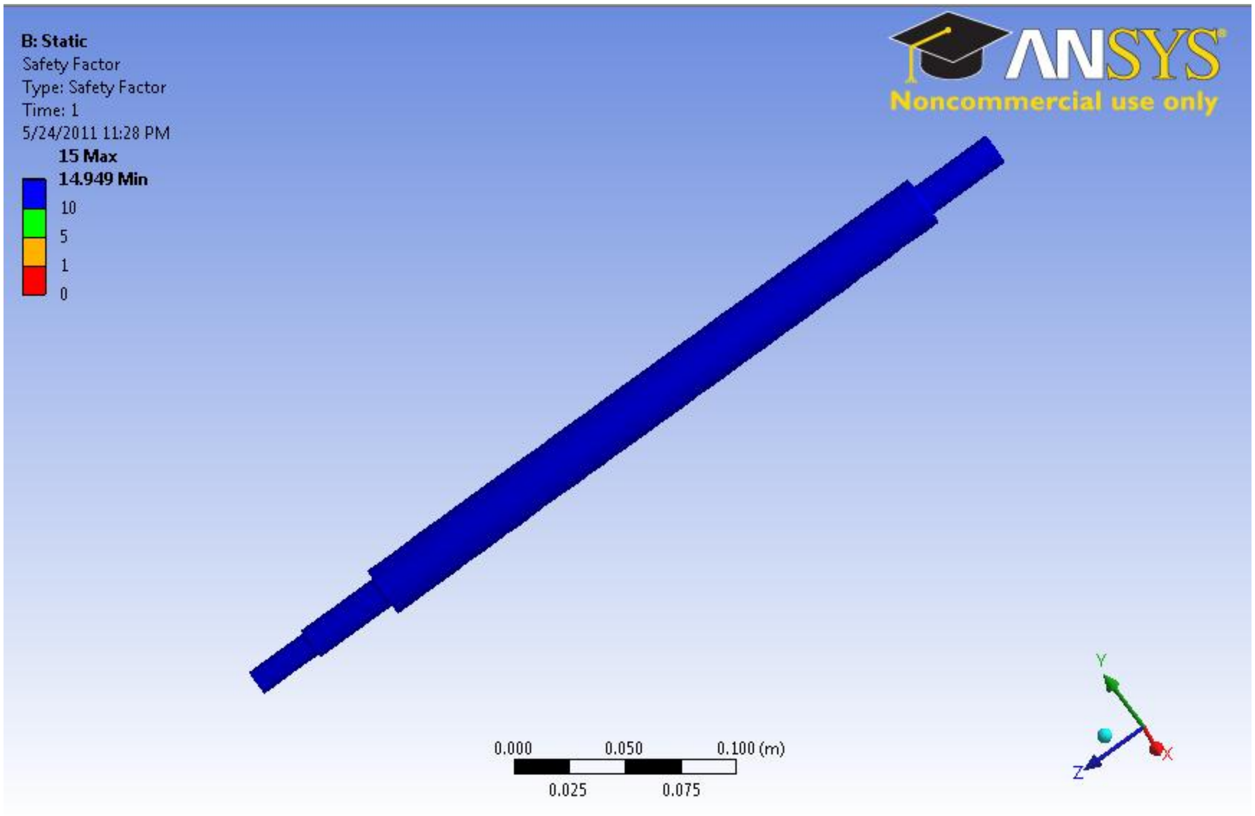


Figure 7. Life Cycle Analysis



B: Static
Safety Factor
Type: Safety Factor
Time: 0
5/24/2011 11:31 PM

Figure 8. Dynamic Safety Factor Analysis



B: Static
Safety Factor
Type: Safety Factor
Time: 1
5/24/2011 11:28 PM

Figure 9. Static Safety Factor Analysis

ANSYS Workbench was used to conduct the finite element analysis for the designed shaft. The shaft was analyzed as it is in the free body diagram. Also, the loads were applied in the general areas where they act on the shaft, since exact distances could not be determined using the finite element software.

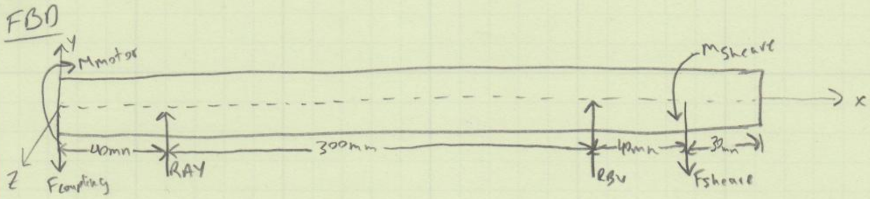
Figure 4 depicts the equivalent stress analysis across the shaft. As shown, the greatest amounts of stress are found at the stress concentrations of the shaft (near the fillets). This is validated by the principle stress analysis in *Figure 5* and the shear stress analysis in *Figure 6*. The software displays the maximum Von-Mises stress to be 16.7MPa while the hand calculations differ from that by +0.6MPa. The software displays the maximum principal stress of 12.1MPa while the hand calculations differ from that value by 20%. . The software displays the maximum shear stress of 9.6MPa while the hand calculations differ from that by -0.75MPa. Figures 8 and 9 show the safety factors of static and dynamic analysis. Although the values to not match the hand calculations exactly, they still prove that the shaft will not fail. Error could be due to assignment of forces on a general area in the software, instead of the actual distances being verified. Fatigue analysis was then conducted on the shaft, by alternating the loads acting on the beam to determine the life cycle. These results verify the results obtained in the hand calculations, which is that the shaft has an infinite life.

Appendix A

Supplier	Part	Quantity	Price
OnlineMetals.com	Mild Steel 1018; Cold Finish Round; 1" Diameter; Cut to: 24"	1	$12.26
MisumiUSA.com	BGHFB6802ZZ-30; 1045 Steel Housing w/ Bearing	2	$113.40
MisumiUSA.com	CPLCN40-15-15; Slit,Clamping, Aluminum Alloy Coupling	1	$83.60
MisumiUSA.com	VMPN50-7-P12; 1045 Carbon Steel V-Groove Pulley	1	$18.30
		Total:	$227.56

Appendix B

Shaft Outline + FBD	Appendix A		1



$\Sigma F_x = 0$

$\Sigma F_y = 0 = R_{Ay} + R_{By} - F_{sheave} - F_{coupling}$

$\Sigma F_z = 0$

$\Sigma M_x = 0 = \Sigma M_y$

$\Sigma M_z = 0 = R_{Ay}(0.04) + R_{By}(0.34) - F_{sheave}(0.38)$

Assume:

- negligible weights
- negligible reaction forces on coupling

$$\text{Power} = \frac{2\pi \omega T}{60} \Rightarrow T = \frac{P\,60}{2\pi\omega} = \frac{(400W)(60)}{2\pi(1500RPM)} = 2.546\ N\cdot m$$

$$F_{sheave} = \frac{M_{sheave}}{r_{sheave}} = \frac{2.546}{0.05m} = 50.92\ N$$

$$F_{coupling} = \frac{M_{coupling}}{r_{coupling}} = 0.04m = \frac{2.546}{0.04} = 63.65\ N$$

$$\begin{cases} 0 = R_{Ay} + R_{By} - 50.92 - 63.65 \\ 0 = 0.04R_{Ay} + 0.34R_{By} - 19.35 \end{cases} \Rightarrow \begin{matrix} R_{Ay} = 65.346\ N \\ R_{By} = 49.224\ N \end{matrix}$$

Material: AISI 1018 CD Steel

$S_{ut} = 440\ MPa$

$S_y = 370\ MPa$

Figure 10. Shaft Outline and FBD

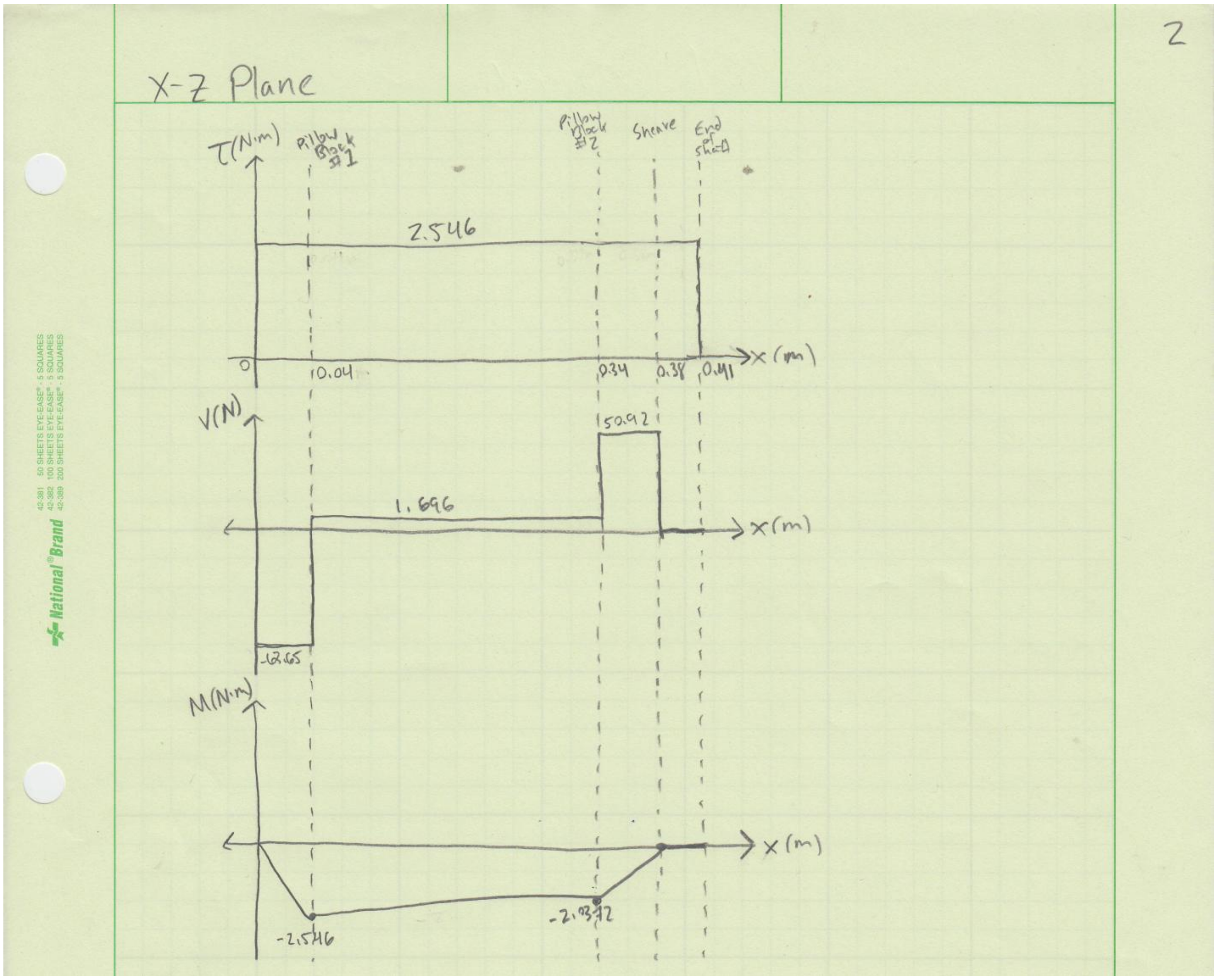
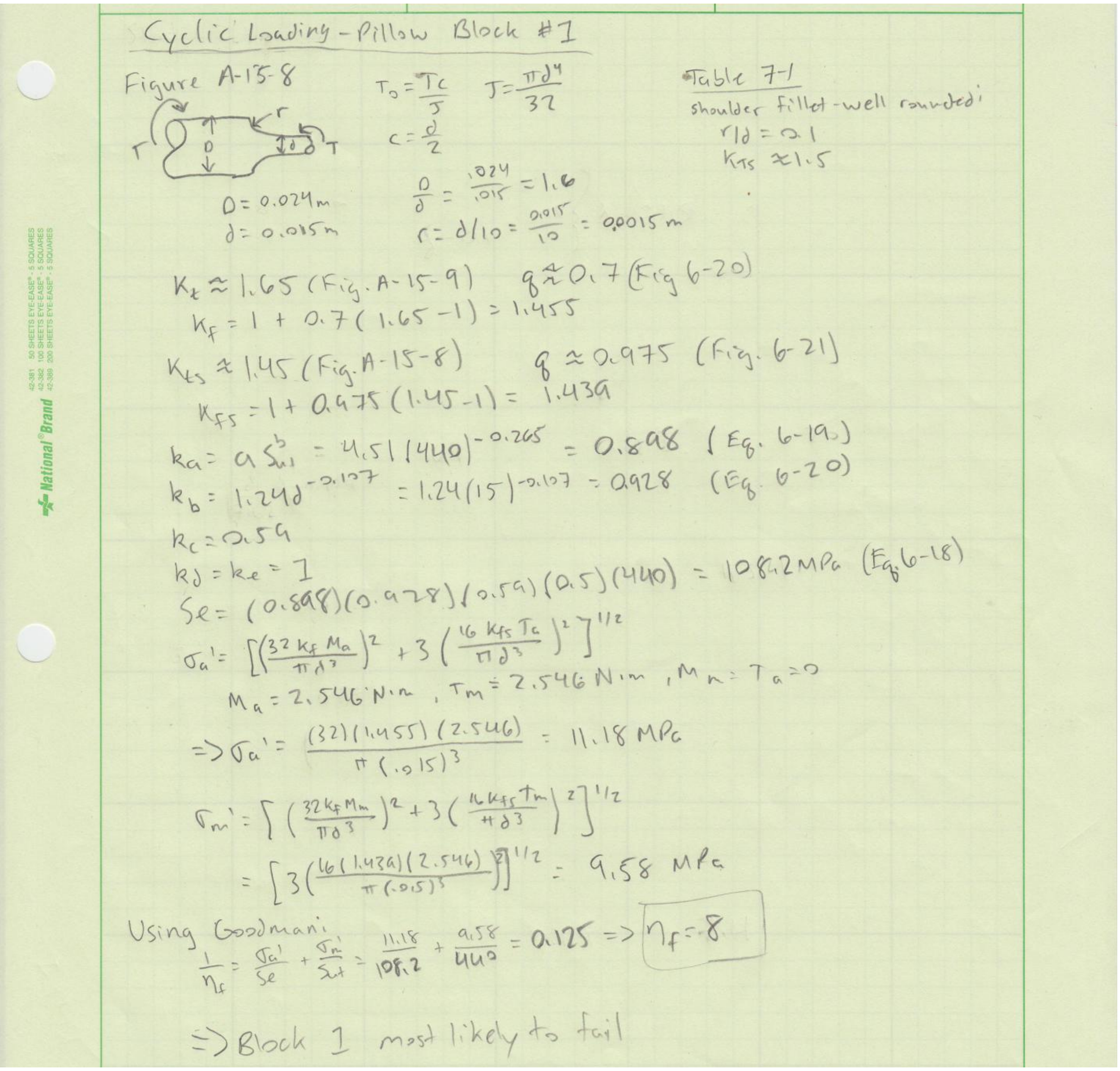


Figure11. Torque, Shear Force and Bending Moment Diagrams



Cyclic Loading - Pillow Block #1

Figure A-15-8

$\tau_0 = \frac{Tc}{J}$ $J = \frac{\pi d^4}{32}$

$c = \frac{d}{2}$

Table 7-1

shoulder fillet - well rounded:

$r/d = 0.1$

$K_{ts} \approx 1.5$

$D = 0.024\,m$

$d = 0.015\,m$

$\frac{D}{d} = \frac{.024}{.015} = 1.6$

$r = d/10 = \frac{0.015}{10} = 0.0015\,m$

$K_t \approx 1.65$ (Fig. A-15-9) $q \approx 0.7$ (Fig 6-20)

$K_f = 1 + 0.7(1.65 - 1) = 1.455$

$K_{ts} \approx 1.45$ (Fig. A-15-8) $q \approx 0.975$ (Fig. 6-21)

$K_{fs} = 1 + 0.975(1.45 - 1) = 1.439$

$k_a = a S_{ut}^{b} = 4.51(440)^{-0.265} = 0.898$ (Eq. 6-19)

$k_b = 1.24 d^{-0.107} = 1.24(15)^{-0.107} = 0.928$ (Eq. 6-20)

$k_c = 0.59$

$k_d = k_e = 1$

$S_e = (0.898)(0.928)(0.59)(0.5)(440) = 108.2\,MPa$ (Eq. 6-18)

$\sigma_a' = \left[\left(\frac{32 K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2}$

$M_a = 2.546\,N{\cdot}m$, $T_m = 2.546\,N{\cdot}m$, $M_m = T_a = 0$

$\Rightarrow \sigma_a' = \frac{(32)(1.455)(2.546)}{\pi(.015)^3} = 11.18\,MPa$

$\sigma_m' = \left[\left(\frac{32 K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$

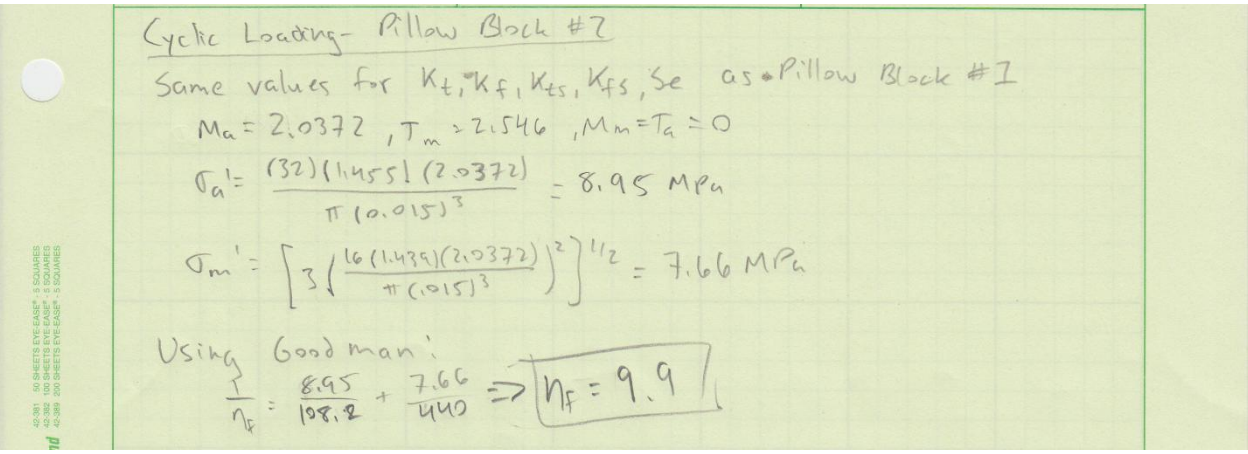
$= \left[3\left(\frac{16(1.439)(2.546)}{\pi(.015)^3}\right)^2\right]^{1/2} = 9.58\,MPa$

Using Goodman:

$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{11.18}{108.2} + \frac{9.58}{440} = 0.125 \Rightarrow \boxed{n_f = 8}$

$\Rightarrow$ Block 1 most likely to fail

Figure 12. Cyclic Loading Analysis of Pillow Block #1



Cyclic Loading - Pillow Block #2

Same values for $K_t, K_f, K_{ts}, K_{fs}, S_e$ as Pillow Block #1

$M_a = 2.0372$, $T_m = 2.546$, $M_m = T_a = 0$

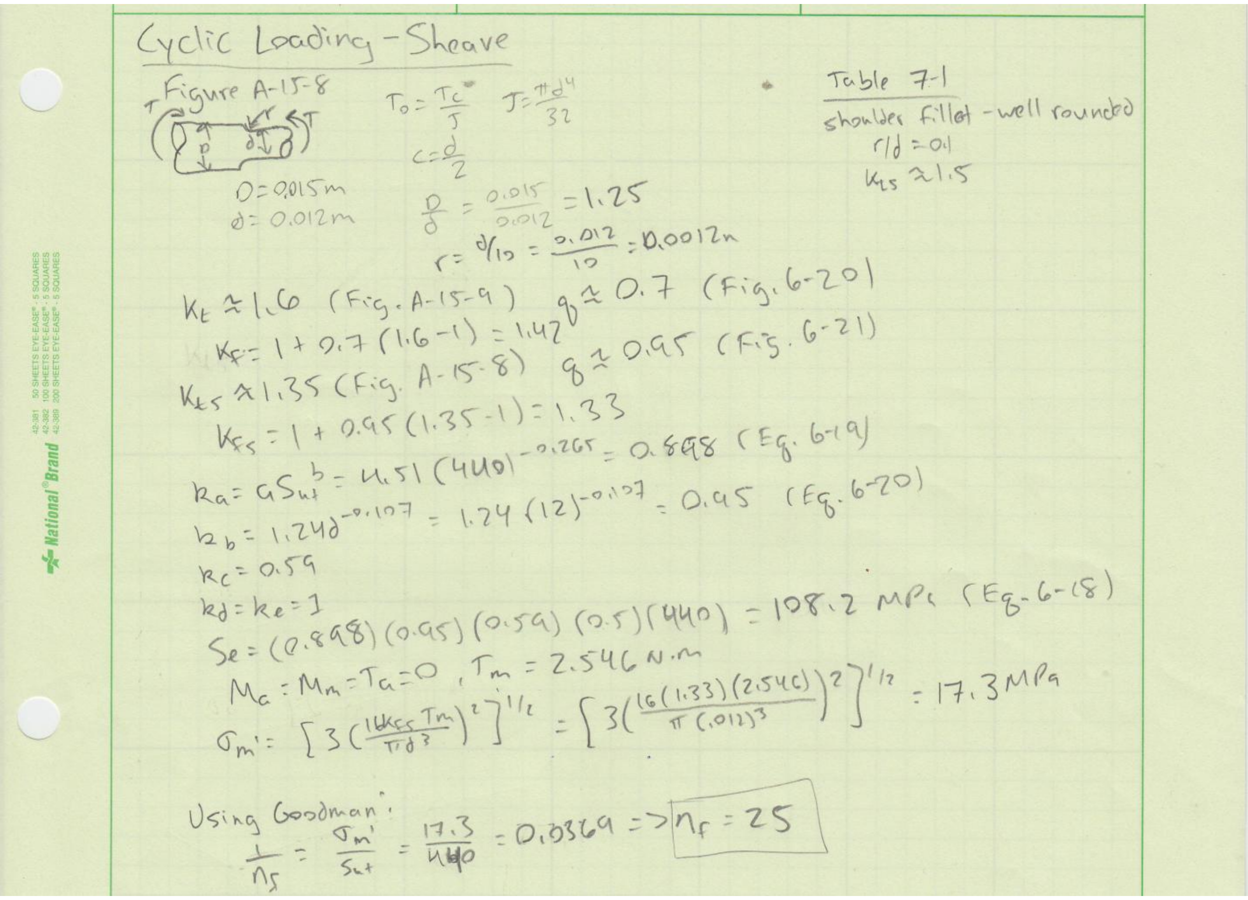
$\sigma_a' = \frac{(32)(1.455)(2.0372)}{\pi(0.015)^3} = 8.95\,MPa$

$\sigma_m' = \left[3\left(\frac{16(1.439)(2.0372)}{\pi(.015)^3}\right)^2\right]^{1/2} = 7.66\,MPa$

Using Goodman:

$\frac{1}{n_f} = \frac{8.95}{108.2} + \frac{7.66}{440} \Rightarrow \boxed{n_f = 9.9}$

Figure 13. Cyclic Loading Analysis of Pillow Block #2



Cyclic Loading - Sheave

Figure A-15-8

$D = 0.015\,m$

$d = 0.012\,m$

$T_o = \frac{Tc}{J}$ $J = \frac{\pi d^4}{32}$

$c = \frac{d}{2}$

$\frac{D}{d} = \frac{0.015}{0.012} = 1.25$

$r = d/10 = \frac{0.012}{10} = 0.0012\,m$

Table 7-1

shoulder fillet - well rounded

$r/d = 0.1$

$K_{ts} \approx 1.5$

$K_t \approx 1.6$ (Fig. A-15-9) $q \approx 0.7$ (Fig. 6-20)

$K_f = 1 + 0.7(1.6-1) = 1.42$

$K_{ts} \approx 1.35$ (Fig. A-15-8) $q \approx 0.95$ (Fig. 6-21)

$K_{fs} = 1 + 0.95(1.35-1) = 1.33$

$k_a = aS_{ut}^{\,b} = 4.51(440)^{-0.265} = 0.898$ (Eq. 6-19)

$k_b = 1.24d^{-0.107} = 1.24(12)^{-0.107} = 0.95$ (Eq. 6-20)

$k_c = 0.59$

$k_d = k_e = 1$

$S_e = (0.898)(0.95)(0.59)(0.5)(440) = 108.2\,MPa$ (Eq. 6-18)

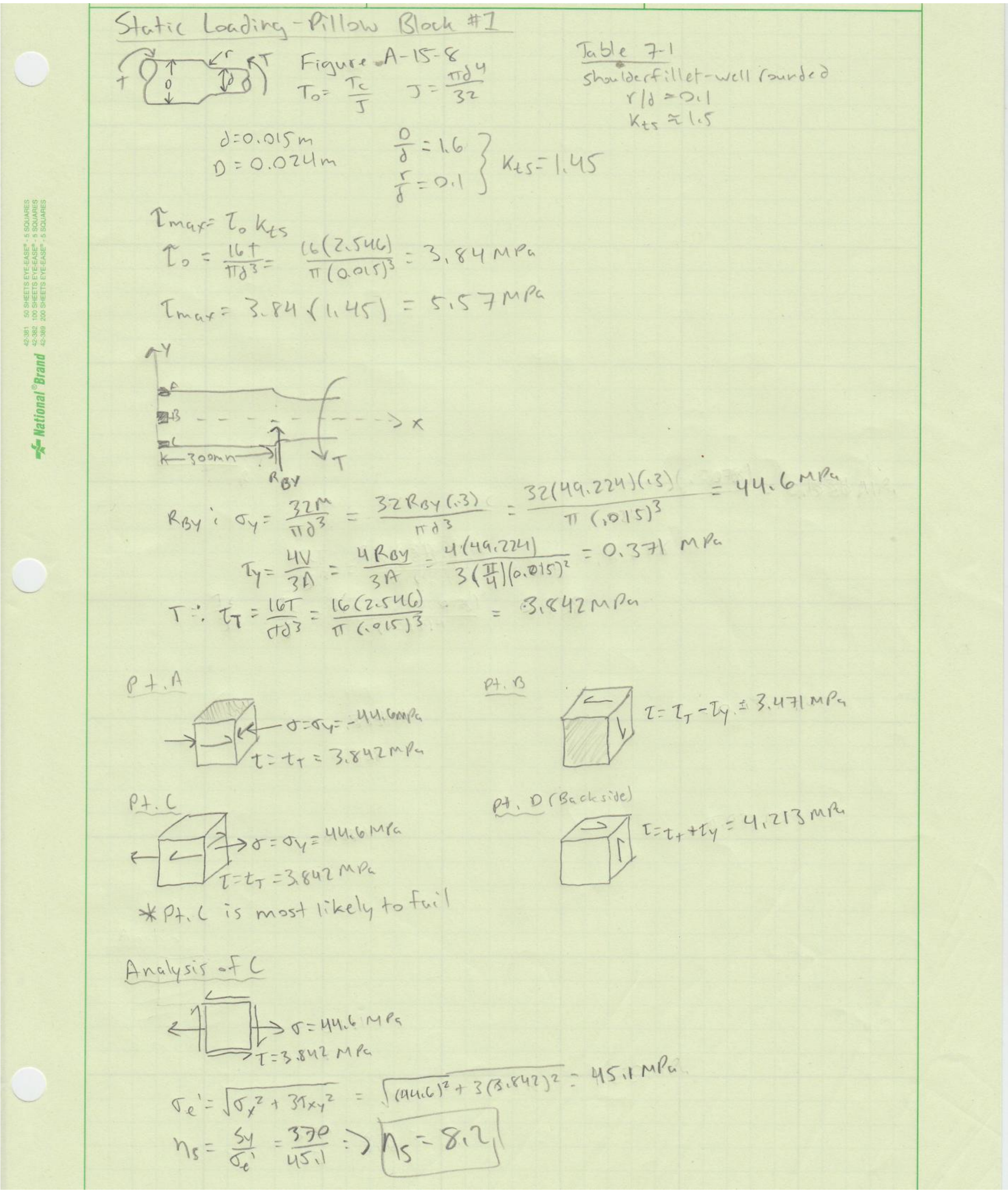
$M_a = M_m = T_a = 0$, $T_m = 2.546\,N{\cdot}m$

$\sigma_m' = \left[3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2} = \left[3\left(\frac{16(1.33)(2.546)}{\pi(.012)^3}\right)^2\right]^{1/2} = 17.3\,MPa$

Using Goodman:

$\frac{1}{n_f} = \frac{\sigma_m'}{S_{ut}} = \frac{17.3}{440} = 0.0369 \Rightarrow \boxed{n_f = 25}$

*Figure 14.*Cyclic Analysis of Sheave



Static Loading - Pillow Block #1

Figure A-15-8

$T_0 = \frac{T_c}{J}$ $\quad J = \frac{\pi d^4}{32}$

Table 7-1

shoulderfillet-well rounded

$r/d \approx 0.1$

$K_{ts} \approx 1.5$

$d = 0.015\,m$

$D = 0.024\,m$

$$\left.\begin{matrix}\frac{D}{d} = 1.6 \\ \frac{r}{d} = 0.1\end{matrix}\right\} K_{ts} = 1.45$$

$$\tau_{max} = \tau_o K_{ts}$$

$$\tau_o = \frac{16T}{\pi d^3} = \frac{16(2.546)}{\pi(0.015)^3} = 3.84\,MPa$$

$$\tau_{max} = 3.84(1.45) = 5.57\,MPa$$

$$R_{BY}: \sigma_y = \frac{32M}{\pi d^3} = \frac{32R_{BY}(.3)}{\pi d^3} = \frac{32(49.224)(.3)}{\pi(.015)^3} = 44.6\,MPa$$

$$\tau_y = \frac{4V}{3A} = \frac{4R_{BY}}{3A} = \frac{4(49.224)}{3(\frac{\pi}{4})(0.015)^2} = 0.371\,MPa$$

$$T: \tau_T = \frac{16T}{\pi d^3} = \frac{16(2.546)}{\pi(.015)^3} = 3.842\,MPa$$

Pt. A

$\sigma = \sigma_y = -44.6\,MPa$

$\tau = \tau_T = 3.842\,MPa$

Pt. B

$\tau = \tau_T - \tau_y = 3.471\,MPa$

Pt. C

$\sigma = \sigma_y = 44.6\,MPa$

$\tau = \tau_T = 3.842\,MPa$

Pt. D (Backside)

$\tau = \tau_T + \tau_y = 4.213\,MPa$

*Pt. C is most likely to fail

Analysis of C

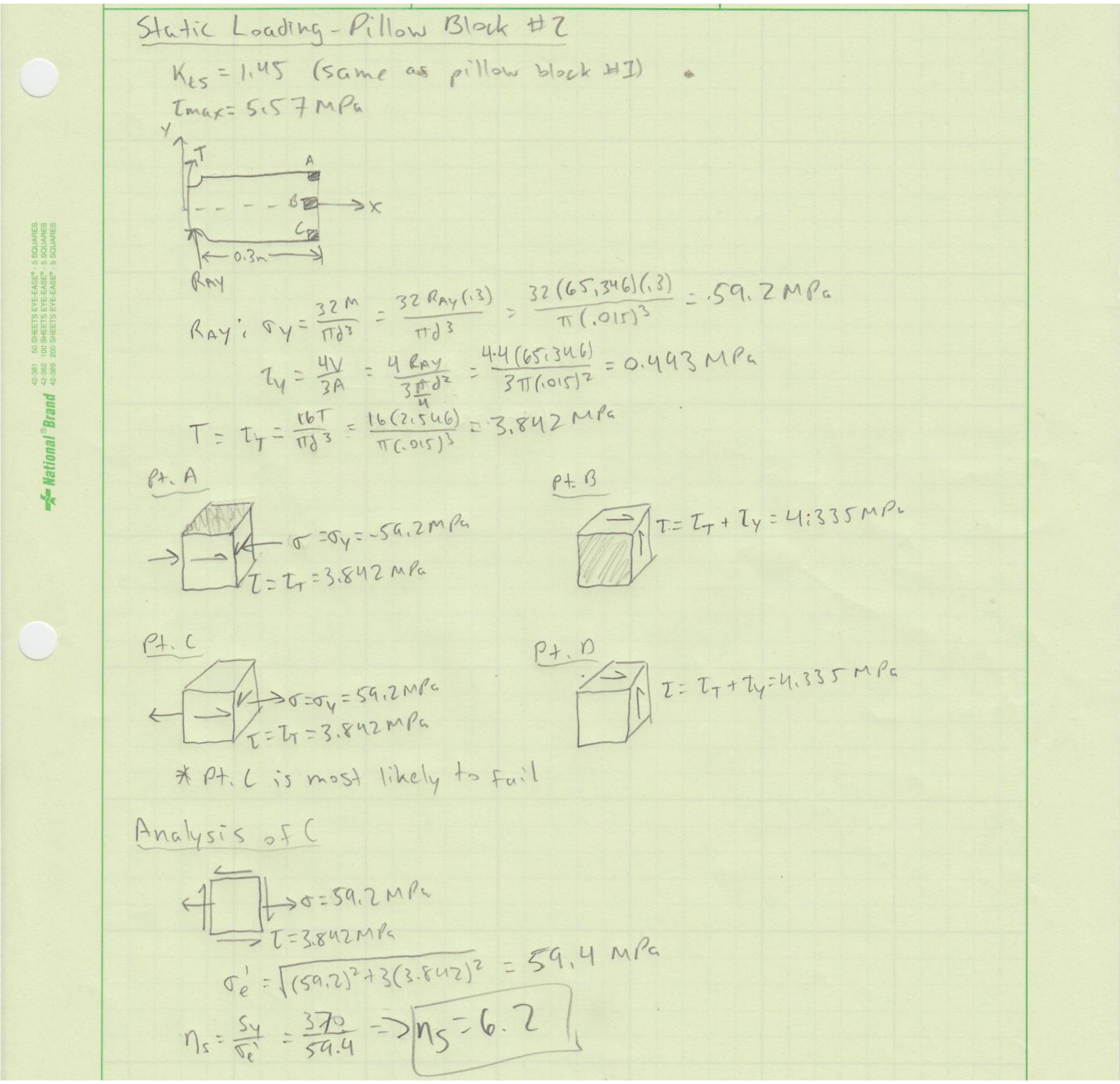
$\sigma = 44.6\,MPa$

$\tau = 3.842\,MPa$

$$\sigma_e' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} = \sqrt{(44.6)^2 + 3(3.842)^2} = 45.1\,MPa$$

$$n_s = \frac{S_y}{\sigma_e'} = \frac{370}{45.1} \Rightarrow \boxed{n_s = 8.2}$$

Figure 15. Static Analysis of Pillow Block #1



Static Loading - Pillow Block #2

$K_{ts} = 1.45$ (same as pillow block #1)

$\tau_{max} = 5.57$ MPa

$$R_{AY}:\ \sigma_y = \frac{32M}{\pi d^3} = \frac{32R_{AY}(.3)}{\pi d^3} = \frac{32(65{,}346)(.3)}{\pi(.015)^3} = 59.2\text{ MPa}$$

$$\tau_y = \frac{4V}{3A} = \frac{4R_{AY}}{3\frac{\pi}{4}d^2} = \frac{4 \cdot 4(65{,}346)}{3\pi(.015)^2} = 0.443\text{ MPa}$$

$$T:\ \tau_T = \frac{16T}{\pi d^3} = \frac{16(2.546)}{\pi(.015)^3} = 3.842\text{ MPa}$$

Pt. A

$\sigma = \sigma_y = -59.2$ MPa

$\tau = \tau_T = 3.842$ MPa

Pt. B

$\tau = \tau_T + \tau_y = 4.335$ MPa

Pt. C

$\sigma = \sigma_y = 59.2$ MPa

$\tau = \tau_T = 3.842$ MPa

Pt. D

$\tau = \tau_T + \tau_y = 4.335$ MPa

* Pt. C is most likely to fail

Analysis of C

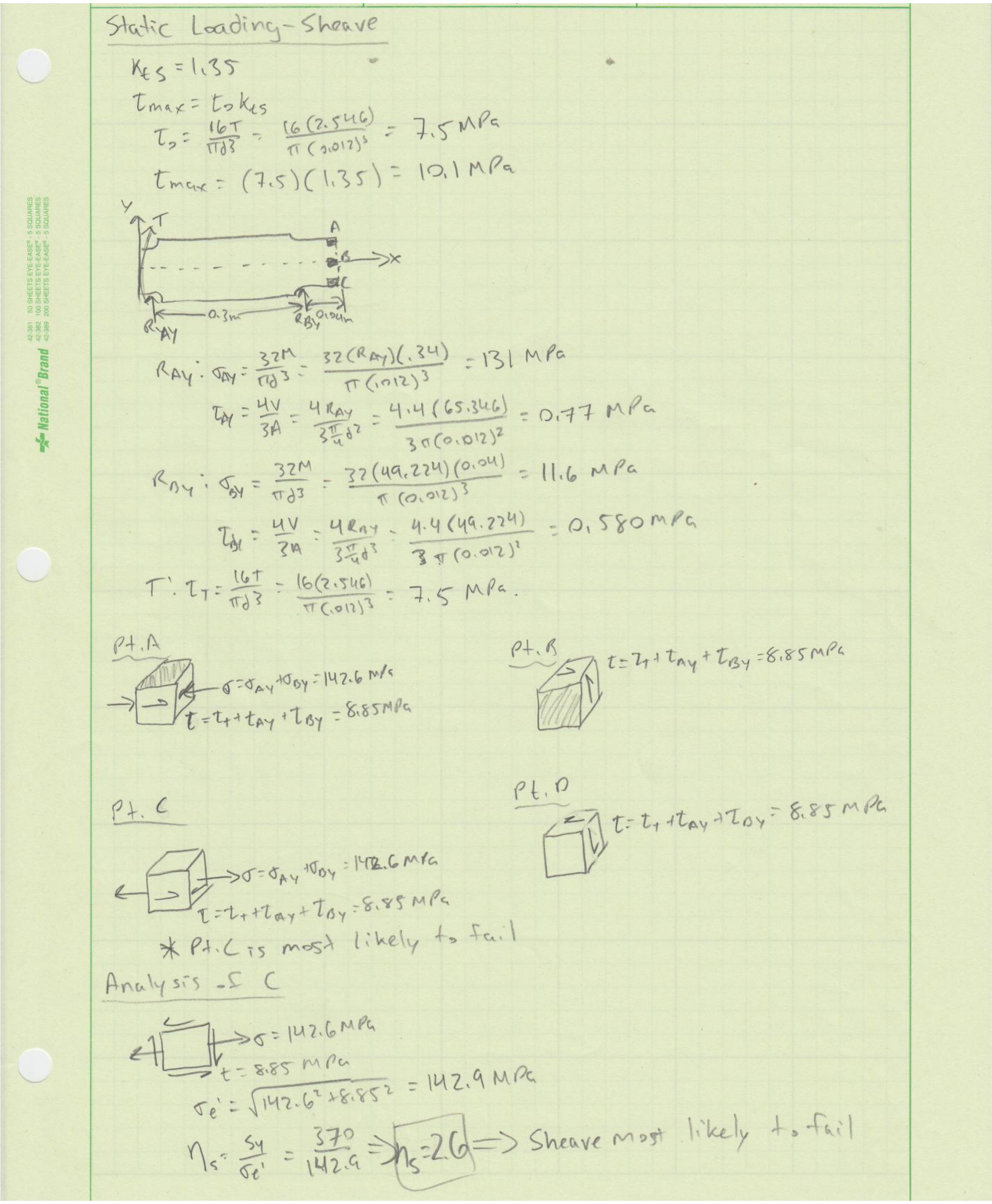
$\sigma = 59.2$ MPa

$\tau = 3.842$ MPa

$$\sigma_e' = \sqrt{(59.2)^2 + 3(3.842)^2} = 59.4\text{ MPa}$$

$$n_s = \frac{S_y}{\sigma_e'} = \frac{370}{59.4} \Rightarrow \boxed{n_s = 6.2}$$

Figure 16. Static Analysis of Pillow Block #2



Static Loading-Sheave

$K_{ts} = 1.35$

$\tau_{max} = \tau_o K_{ts}$

$\tau_o = \frac{16T}{\pi d^3} = \frac{16(2.546)}{\pi(0.012)^3} = 7.5\ MPa$

$\tau_{max} = (7.5)(1.35) = 10.1\ MPa$

$R_{AY}: \sigma_{AY} = \frac{32M}{\pi d^3} = \frac{32(R_{AY})(.34)}{\pi(.012)^3} = 131\ MPa$

$\tau_{AY} = \frac{4V}{3A} = \frac{4R_{AY}}{3\frac{\pi}{4}d^2} = \frac{4 \cdot 4(65.346)}{3\pi(0.012)^2} = 0.77\ MPa$

$R_{BY}: \sigma_{BY} = \frac{32M}{\pi d^3} = \frac{32(49.224)(0.04)}{\pi(0.012)^3} = 11.6\ MPa$

$\tau_{BY} = \frac{4V}{3A} = \frac{4R_{BY}}{3\frac{\pi}{4}d^3} = \frac{4.4(49.224)}{3\pi(0.012)^2} = 0.580\ MPa$

$T: \tau_T = \frac{16T}{\pi d^3} = \frac{16(2.546)}{\pi(.012)^3} = 7.5\ MPa.$

Pt. A

$\sigma = \sigma_{AY} + \sigma_{BY} = 142.6\ MPa$

$\tau = \tau_T + \tau_{AY} + \tau_{BY} = 8.85\ MPa$

Pt. B

$\tau = \tau_T + \tau_{AY} + \tau_{BY} = 8.85\ MPa$

Pt. C

$\sigma = \sigma_{AY} + \sigma_{BY} = 142.6\ MPa$

$\tau = \tau_T + \tau_{AY} + \tau_{BY} = 8.85\ MPa$

Pt. D

$\tau = \tau_T + \tau_{AY} + \tau_{BY} = 8.85\ MPa$

* Pt. C is most likely to fail

Analysis of C

$\sigma = 142.6\ MPa$

$\tau = 8.85\ MPa$

$\sigma_e' = \sqrt{142.6^2 + 8.85^2} = 142.9\ MPa$

$n_s = \frac{S_y}{\sigma_e'} = \frac{370}{142.9} \Rightarrow \boxed{n_s = 2.6} \Rightarrow$ Sheave most likely to fail

Figure 17. Static Analysis of Sheave

Works Cited

[1] "ANSYS Tutorials." ANSYS Inc., 2009. Web. 25 Mar. 2011. <http://www1.ansys.com/customer/content/documentation/121/ans_tut.pdf>.

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[3] *Online Metal Store | Small Quantity Metal Orders | Metal Cutting, Sales & Shipping | Buy Steel, Aluminum, Copper, Brass, Stainless | Metal Product Guides at OnlineMetals.com*. Web. 26 May 2011. <http://www.onlinemetals.com/>.

[4] Shigley, Joseph Edward., Charles R. Mischke, and Richard G. Budynas. *Mechanical Engineering Design*. New York, NY: McGraw-Hill, 2004. Print.