University of Pittsburgh
Swanson School of Engineering

Mechanical Design II
Professor Stephen Ludwick

Design Project #4

Bearing Design for a Power Transmission Shaft

June 23, 2011

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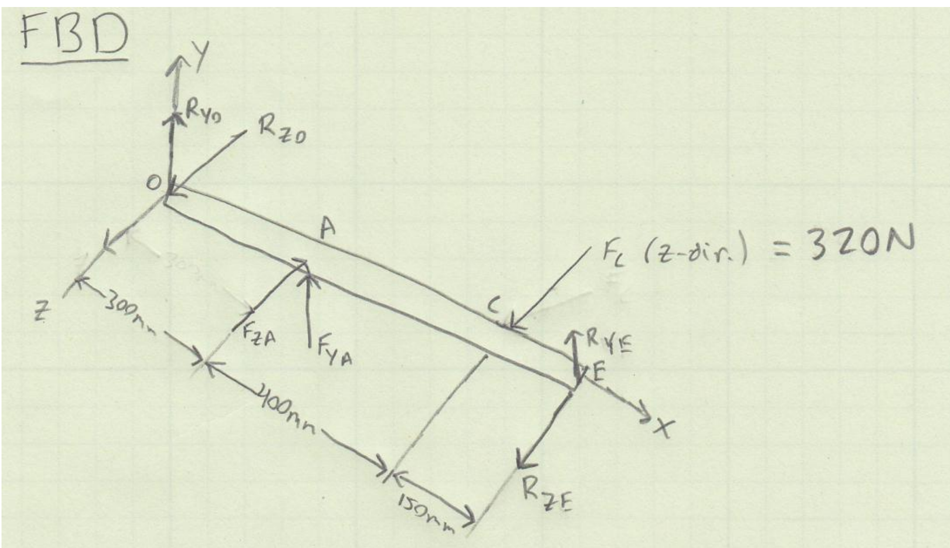
1. Project Definition

The goal of this project is to design and analyze bearings used to support a power transmission shaft. The shaft consists of two pillow blocks with bearings as well as two V-belt pulleys. Since the bearings are inside pillow blocks they can be aligned in the axial direction using a shoulder on the shaft and by bolting the pillow block to a surface. As was done in project #2, the design of a power transmission shaft, we will have to design a shaft to match the required dimensions.

2. Functional Requirements

1. Support speeds up to 1200 rpm
2. Support a radial load of 196N
3. Transmit 4.15kW of power
4. Lifetime of at least 25,000 hours
5. System reliability of 0.99 → Individual bearing reliability of 0.995
6. Must be commercially-available rolling-element bearings

3. Free Body Diagram



4. Technical Calculations

First, it was determined that a single-row deep-groove bearing would be a good choice for this application because they will take radial load as well as some thrust load. The first step in picking a bearing is to draw a free body diagram of the shaft, replacing the pulleys and bearings with forces. We can then write equations for the sum of the moments to determine the four unknowns, which are the reaction forces from the bearings (as seen below*)*.

$$\Sigma F_y = 0 = R_{Y_O} + F_{YA} + R_{YE}$$
$$\Sigma F_Z = 0 = R_{ZO} - F_{ZA} + F_C + R_{ZE}$$
$$\Sigma M_Y = 0 = F_{ZA}(.3m) - F_C(.7m) - R_{ZE}(.85m)$$
$$\Sigma M_Z = 0 = F_{YA}(.3m) + R_{YE}(.85m)$$

We must then determine the forces from the pulleys. Then we can substitute these values into the equilibrium equations and solve for the unknowns (as seen below).

Pulley Torque (T): Difference in belt tensions in the tight (t_1) and slack (t_2) sides of a pulley times the radius (r)

$$T = (t_1 - t_2)\,r$$

Pulley A Torque:

$$T_A = (P_1 - 0.15P_1)(.125m)$$

Pulley C Torque:

$$T_C = (270N - 50N)(.15m) = \underline{33N\cdot m}$$

$$\Rightarrow T_A = 66N\cdot m = (P_1 - 0.15P_1)(.125m) \Rightarrow P_1 = 310.59\ N$$
$$\Rightarrow P_2 = 0.15P_1 = 0.15(310.59N)$$
$$P_2 = 46.59\ N$$

From P_1 + P_2:

$$F_A = P_1 + P_2 = 310.59 + 46.59N = 357.18N \text{ @ } 45^\circ \text{ from } \hat{k}$$
$$F_{YA} = -F_{ZA} = 357.18 \cos 45^\circ = 252.56N$$

4 Eqns. + 4 unknowns:

$$\Sigma F_Y = 0 = R_{YO} + R_{YE} + F_{YA}$$
$$\Rightarrow R_{YO} + R_{YE} = -F_{YA}$$
$$R_{YO} + R_{YE} = -252.56N \quad (1)$$

$$\Sigma F_Z = 0 = R_{ZO} + R_{ZE} + F_C - F_{ZA}$$
$$\Rightarrow R_{ZO} + R_{ZE} = F_{ZA} - F_C$$
$$R_{ZO} + R_{ZE} = -252.56 - 320N = -572.56N \quad (2)$$

$$\Sigma M_Y = 0 = F_{ZA}(.3m) - F_C(.7m) - R_{ZE}(.85m)$$
$$\Rightarrow 0.85 R_{ZE} = -299.768 \quad (3)$$

$$\Sigma M_Z = 0 = F_{YA}(.3m) + R_{YE}(.85m)$$
$$\Rightarrow 0.85 R_{YE} = -75.768 \quad (4)$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0.85 \\ 0 & 0.85 & 0 & 0 \end{bmatrix} \begin{bmatrix} R_{YO} \\ R_{YE} \\ R_{ZO} \\ R_{ZE} \end{bmatrix} = \begin{bmatrix} -252.56 \\ -572.56 \\ -299.768 \\ -75.768 \end{bmatrix}$$

$$\Rightarrow R_{YO} = -163.4N \qquad R_{ZO} = 107\ N$$
$$R_{YE} = -89.1N \qquad R_{ZE} = -174.4N$$

$$\left.\begin{array}{l} \Rightarrow R_O = 195.3\ N \\ R_E = 195.8N \end{array}\right\} \text{Both} \approx \boxed{196N = P_r}$$

Now that we know the forces on the bearings, we can determine the approximate dynamic load rating required for this application using the lifetime equation (adjusted for our specifications).

Reliability of System $= 99\%$

$\Rightarrow$ Reliability of each bearing $= \sqrt{0.99} \approx 0.995 = R$

$$L_{0.05} = a_1 L_{10}$$

where $a_1 = 4.48 \ln\left(\frac{1}{R}\right)^{2/3}$

$$= 4.48 \ln\left(\frac{1}{0.995}\right)^{2/3} = 0.131$$

$$L_{0.05h} = 0.131\left(\frac{10^6}{n60}\right)\left(\frac{C_{10}}{P_r}\right)^3 \Rightarrow C_{10} = \sqrt[3]{\frac{L_{0.05}\cdot n \cdot 60}{10^6(0.131)}}\cdot P_r$$

$$= \sqrt[3]{\frac{(25000h)(1200rpm)(60)}{10^6(0.131)}}\cdot 196N$$

$$C_{10} = 4694.5\ N \quad \text{(approx. dyn. load rating)}$$

To get an estimate of the bore size needed, consult Table 11-2 of Shigley. From this we determine a bore size of about 10mm should be adequate. Since these bearings must actually be purchased, we go to MiSUMI USA’s website to find actual bearings. To simplify the assembly of the power transmission shaft, we look for pillow blocks that will fit our need. We then must check the selected bearing/pillow block to ensure it meets our required lifetime and reliability.

From Table 11-2 using $C_{10} \approx 4.7\ kN$

$\Rightarrow$ 10mm bore: $C_{10} = 5.07\ kN$, $C_0 = 2.24\ kN$

From MiSUMi USA:

PDR12: 12mm bore pillow block

$C_{10} = 12.8\ kN$; $C_0 = 6.6\ kN$

Check Bearing to Make Sure It Meets The Standards:

$$L_{0.05h} = 0.131\left(\frac{10^6}{n60}\right)\left(\frac{C_{10}}{P_r}\right)^3$$

$$= 0.131\left(\frac{10^6}{(1200)(60)}\right)\left(\frac{12.8}{.196}\right)^3$$

$$= 506{,}757.9 \text{ hours } \checkmark$$

$\rightarrow$ This meets our requirement of 25,000 hours

Now that we have a suitable bearing selected, we can consult Table 11-2 in Shigley to help determine some shaft dimensions. For a bore size of 12mm, we should have a filet radius of 0.6mm, and the housing shoulder diameter should be no more than 28mm to maintain concentricity and perpendicularity with the housing bore axis. In this design, the filet radius is 0.6mm and the housing shoulder diameter (the diameter of the center part of the shaft) is 25mm.

Finally, we do some simple calculations to determine some of the values for the functional requirements. This can be done at any point; it does not depend on the bearings selected.

Functional Requirements

Rotation Speed: 1200 rpm

Required Lifetime: 25,000 hours

System Reliability: 0.99

Individual Bearing Reliability: $\sqrt{0.99} \approx 0.995$

Load on Bearings: 196 N

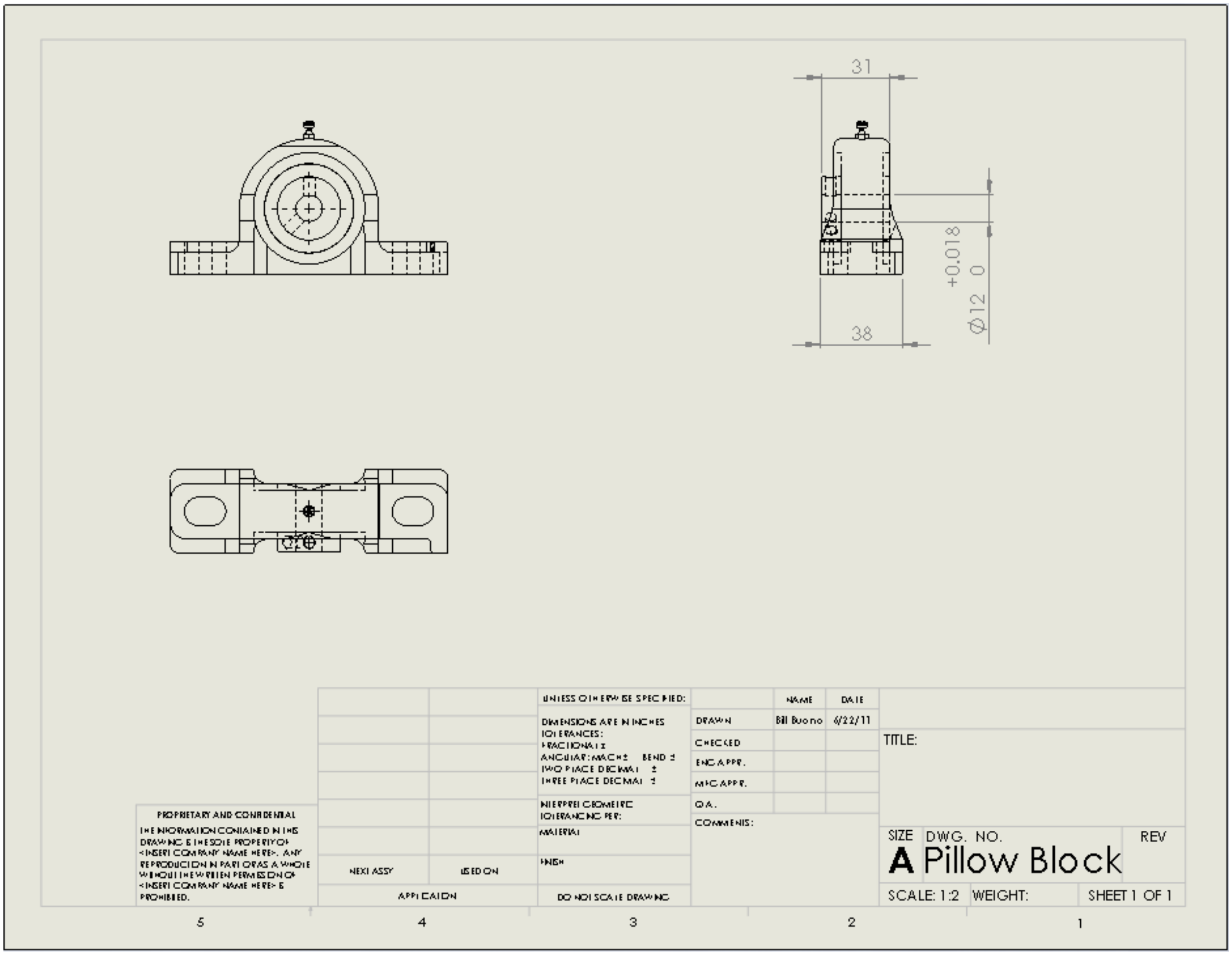
Power Transmission: $P = \frac{2\pi \omega T}{60}$

$$= \frac{2\pi (1200\,\text{rpm})(33\,\text{N}\cdot\text{m})}{60}$$

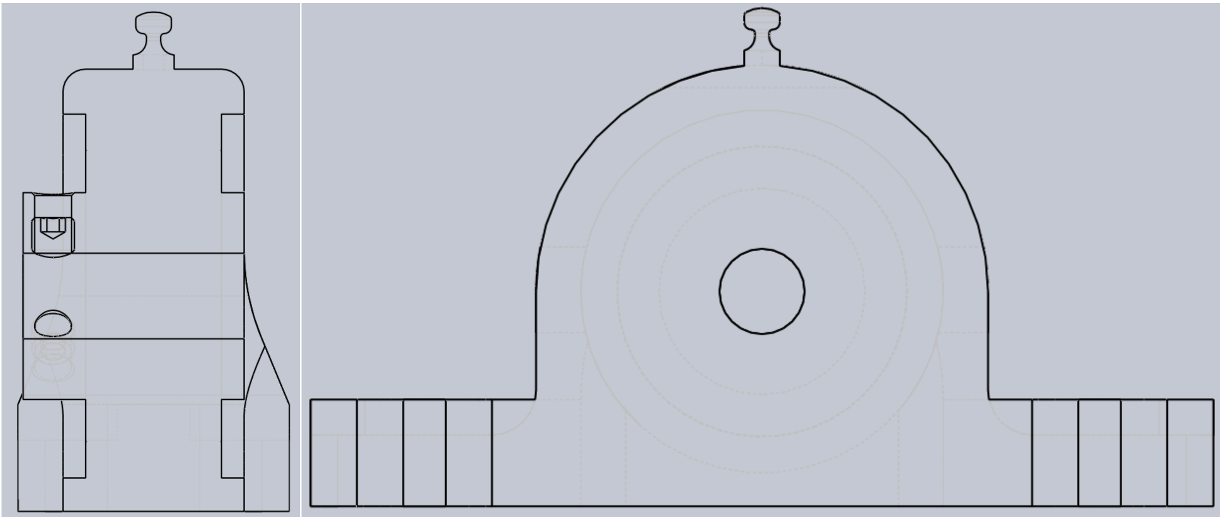
$$= 4146.9\,\text{W} = 4.15\,\text{kW}$$

5. Design Drawing

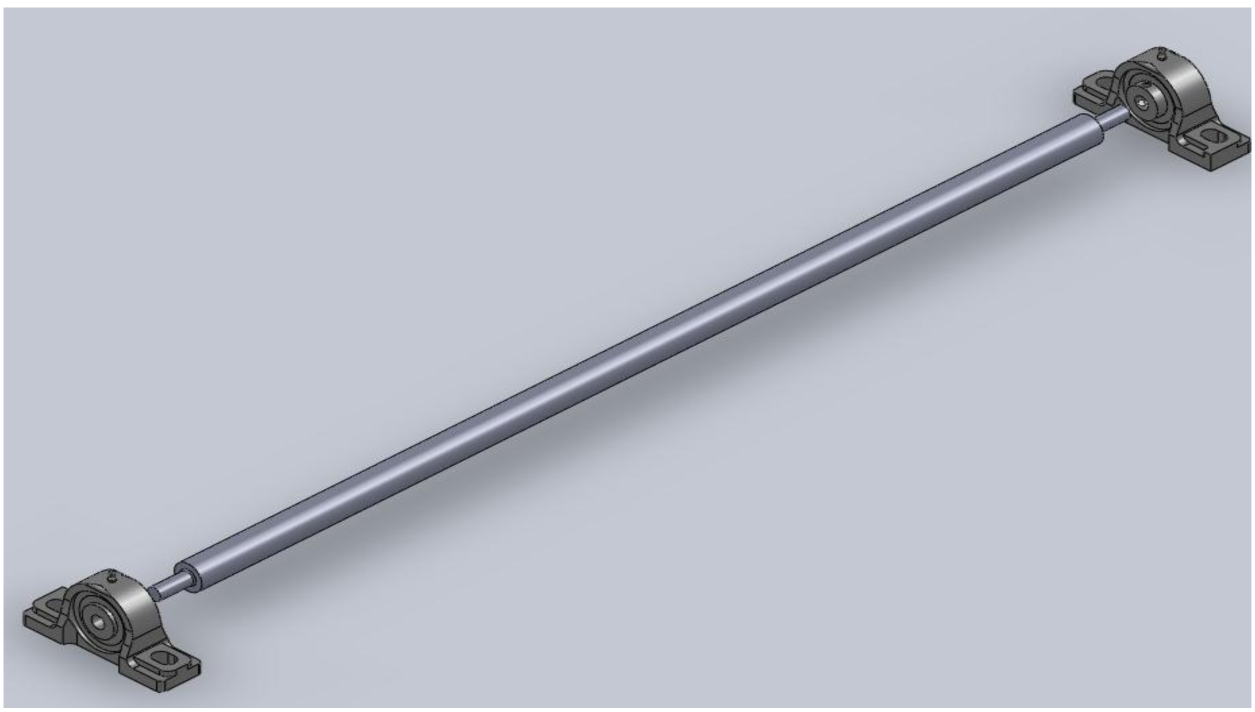
First is a drawing of the pillow block. The tolerance for the bore diameter is DH7 and was determined using Machinery’s Handbook Ed. 28.

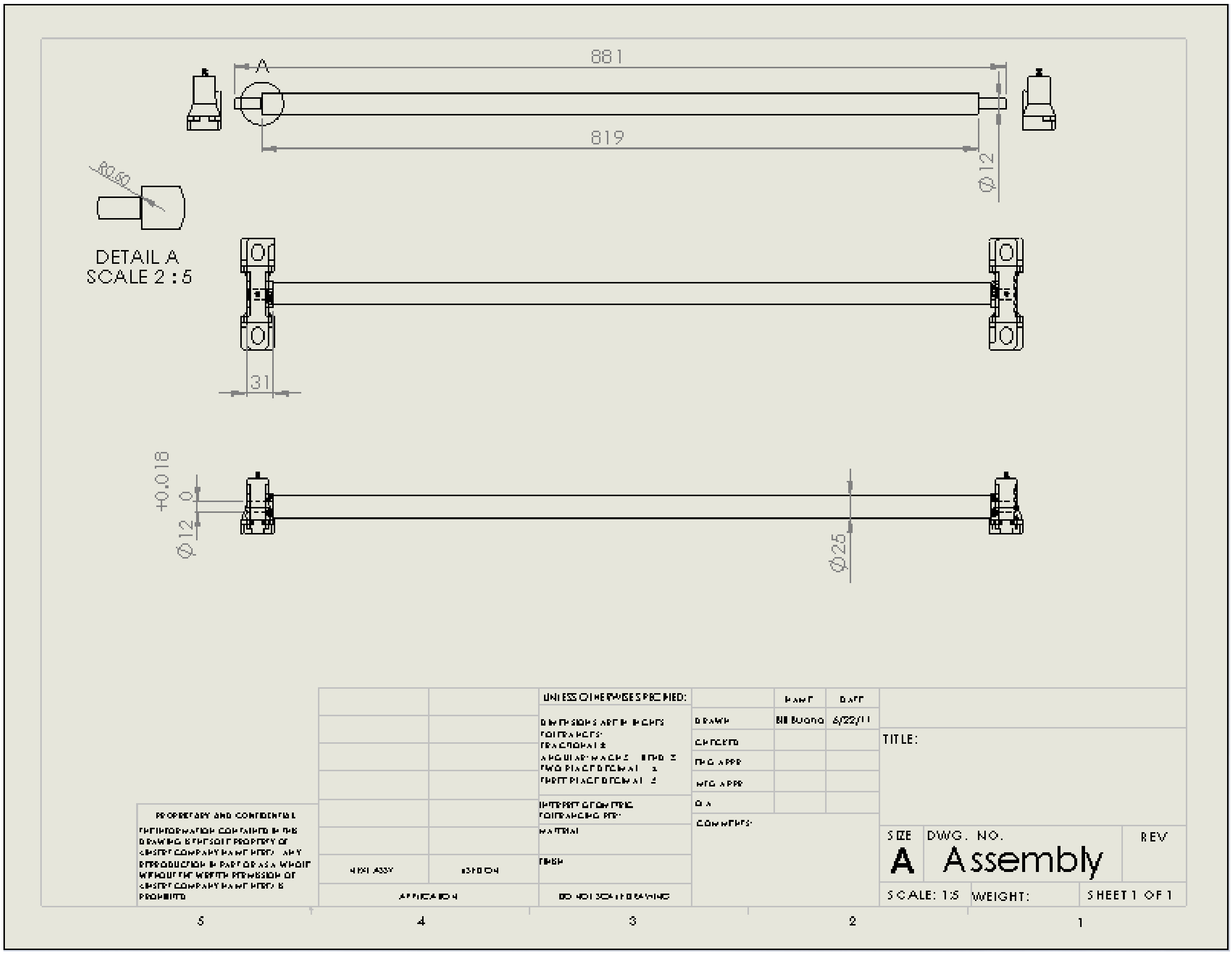


This is a cutaway of the pillow block SolidWorks model. It can be seen that there is a bore for the set screw and the bore diameter of the bearing goes all the way through.



Below is an exploded view of the shaft and pillow blocks. The pulley's are not included in this drawing or the BOM. Also, there is a drawing with the assembly on the following page. The length of the middle of the shaft is 819mm. This is different from the 850mm total length in the FBD because the FBD went from the middle of one bearing to the middle of the other.





6. BOM

Included in the bill of materials are the two pillow blocks and a cold drawn AISI1018 steel rod which can be machined to meet our design. 25mm is approximately 0.98” and 881mm is approximately 35”, so little material will be lost in machining the shaft.

Vendor	Part #	Description	Cost Per 1	Quantity	Total Cost
MiSUMi USA	PDB12	Pillow Block - Side Mount	$20.00	2	$40.00
OnlineMetals.com		Mild Steel 1019; Cold Finish Round; 1"; Cut to 36"	$17.24	1	$17.24
				Total Cost	$57.24

7. Assembly Instructions

The pillow block/bearing will come pre-assembled from the manufacturer, MiSUMi USA. The pillow block assembly is to be mounted with the set-screw side of the bearing up against the shoulder in the shaft. The set-screws can then be tightened, thus eliminating the need for a press fit of the inner ring. The proper set screws are M6x0.75 and should be torqued to 392 N-cm. Each screw should be tightened to 75% total torque, 90% total torque, 95% total torque, then 100% total torque, making sure to rotate between each screw. There were no specifications for preloading or lubrication from the manufacturer so they were deemed unnecessary for this application. Base on the general experience with rolling bearings as expressed in manufacturers' catalogs, the permissible misalignment in deep-groove ball bearings is from 0.0035 to 0.0047 rad.

8. Conclusion

It was not a difficult task selecting a suitable bore size for the deep-groove bearing needed to sustain the loads of this power transmission shaft. Using the lifetime equation, and altering it to fit the requirements of our problem, we were able to determine a baseline for the bore diameter, go to an online vendor and select the closest (without going smaller) bearing, and then rerun our calculations to ensure that the selected bearing will meet the requirements.

Works Cited

[1] "Design of Shaft." *NJIT*. Web. 22 June 2011. <http://web.njit.edu/~sengupta/met%20301/Design%20of%20Shaft.pdf>.

[2] Oberg, Erik, and Christopher J. McCauley. "4 Dimensioning, Gaging, and Measuring." *Machinery's Handbook: a Reference Book for the Mechanical Engineer, Designer, Manufacturing Engineer, Draftsman, Toolmaker, and Machinist*. New York: Industrial, 2008. Print.

[3] *Online Metal Store | Small Quantity Metal Orders | Metal Cutting, Sales & Shipping | Buy Steel, Aluminum, Copper, Brass, Stainless | Metal Product Guides at OnlineMetals.com*. Web. 26 May 2011. <http://www.onlinemetals.com/>.

[4] "Pillow Blocks." *MISUMI, the Catalog Company of Machine Components ECatalog*. Web. 23 June 2011. <http://us.misumi-ec.com/>.

[5] Shigley, Joseph Edward., Charles R. Mischke, and Richard G. Budynas. *Mechanical Engineering Design*. New York, NY: McGraw-Hill, 2004. Print.