

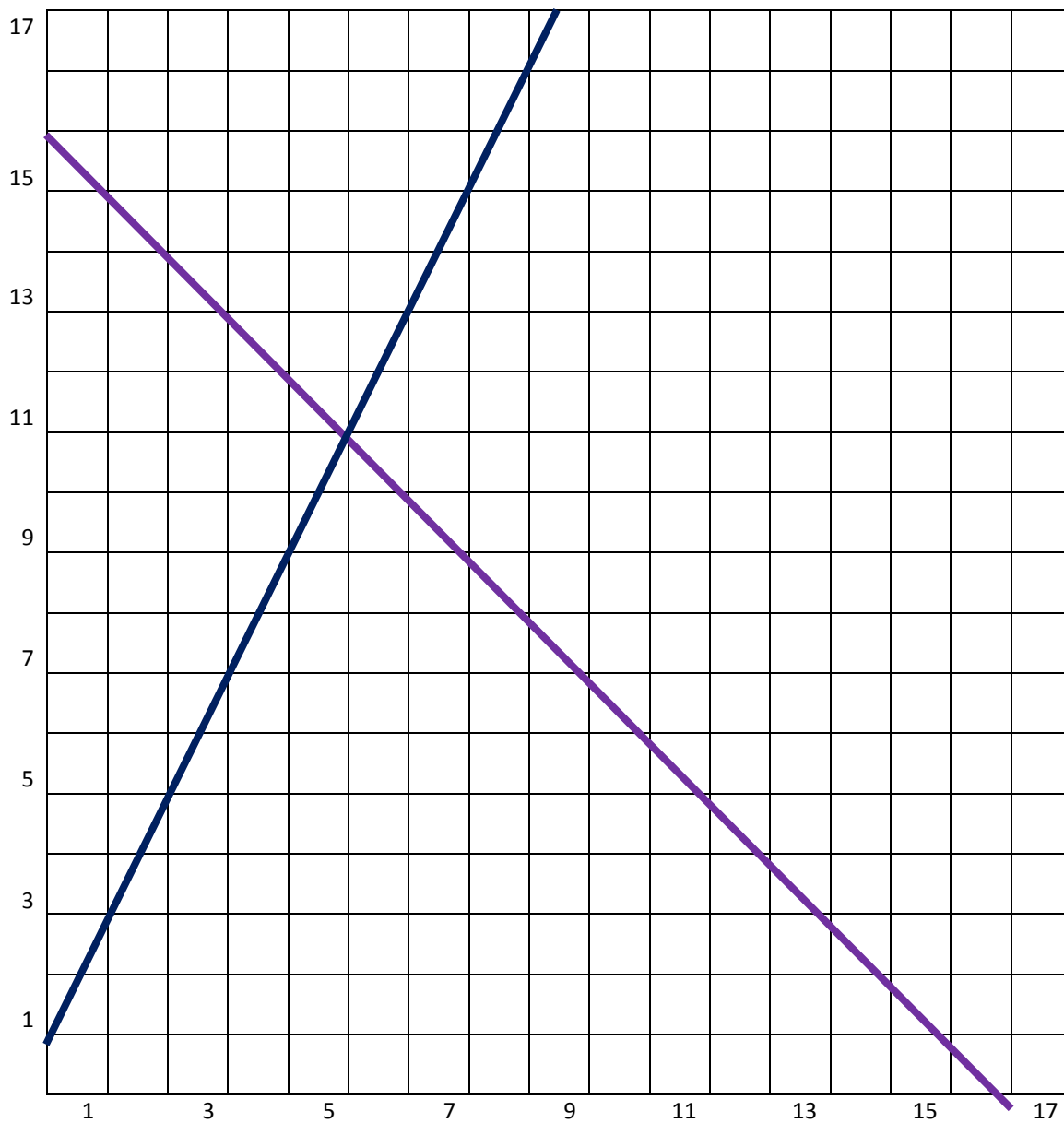
Problem Set 5

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November 04, 2009

1. Suppose the market demand is given by the equation $q_D = 16 - p_D$, and the market supply by the equation $q_S = -0.5 + 0.5 p_S$.
 - a. Draw a picture of the demand and supply curve

Price



Quantity

Notice price is on vertical axis. Thus we need to rearrange them into inverse demand and inverse supply function. They are

$$p_D = 16 - q_D \text{ and } p_S = 1 + 2q_S$$

b. Determine the equilibrium quantity and price?

In equilibrium, $p_D = p_S$ and $q_D = q_S$. We need to solve $\begin{cases} q = 16 - p \\ q = -0.5 + 0.5 p \end{cases}$. $q = 5$ and $p = 11$.

c. Calculate the consumer and producer surplus at the predicted equilibrium

The consumer surplus is the area of the triangle above $p = 11$ (i.e. equilibrium price) and below demand curve, that is $1/2 * 5 * (16 - 11) = 12\frac{1}{2}$.

The producer surplus is the area of the triangle below $p = 11$ (i.e. equilibrium price) and above supply curve, that is $1/2 * 5 * (11 - 1) = 25$.

d. Calculate the price elasticity of demand at the predicted equilibrium. Is demand elastic/unit elastic/ inelastic at the equilibrium price?

the price elasticity of demand at the predicted equilibrium = $\frac{\% \text{ change in demand}}{\% \text{ change in price}} = \frac{\Delta q_D / q_D}{\Delta p_D / p_D} = \frac{\Delta q_D}{\Delta p_D} \frac{p_D}{q_D}$
 $= -1 * \frac{11}{5} = -\frac{11}{5}$. Thus it is elastic.

e. Calculate the price elasticity of supply at the predicted equilibrium. Is supply elastic/unit elastic/ inelastic at the equilibrium price?

the price elasticity of supply at the predicted equilibrium = $\frac{\% \text{ change in supply}}{\% \text{ change in price}} = \frac{\Delta q_S / q_S}{\Delta p_S / p_S} = \frac{\Delta q_S}{\Delta p_S} \frac{p_S}{q_S}$
 $= \frac{1}{2} * \frac{11}{5} = \frac{11}{10}$. Thus it is elastic.

f. Suppose a quantity tax of \$3 was imposed on the sellers. Determine the new equilibrium quantity as well as the price the buyers will pay and the price the sellers will receive.

After imposing the quantity tax, in equilibrium $p_D = p_S + t = p_S + 3$ and $q_D = q_S$. Let $p_D = p$

Thus we solve $\begin{cases} q = 16 - p \\ q = -0.5 + 0.5 (p - 3) \end{cases}$. $q = 4$ and $p = 12$. $p_D = p = 12$, $p_S = p - 3 = 9$.

The new equilibrium quantity is 4, the price the buyers will pay is 12 and the price the sellers will receive is 9. Note that transaction take place at $p_D = 12$.

g. What is the ratio of the tax imposed on the buyers relative to that on the sellers t_D/t_S ? Use your answer to 1.d. and 1.e. to provide a verbal explanation for the size of this ratio

$$t_D/t_S = (p_D - p^*) / (p^* - p_S) = (12 - 11) / (11 - 9) = 1/2$$

Thus the sellers carry twice the tax burden of the buyers. This division of the tax burden is consistent with the elasticity of demand and supply seen in 1.d. and 1.e. above. Demand was found to be twice as elastic as supply, recall that in class we showed that $t_D/t_S = \epsilon_S / |\epsilon_D|$, thus sellers carry twice the tax burden of the buyers.

When price changes by 1 percent, the percentage change in demand is twice as much as the percentage change in supply. Thus when the same percentage change happens in demand and

supply quantity, the percentage change in equilibrium demand price is half as much as the percentage change in equilibrium supply price.

h. Calculate the tax revenue, change in consumer and producer surplus as well as deadweight loss associated with the tax.

The tax revenue equals to the equilibrium quantity multiplied by tax rate, that is $4 \cdot 3 = 12$.

After imposing the quantity tax on sellers, the buyer surplus is the area of the triangle above $p = 12$ (i.e. new price buyers pay) and below demand curve, that is $1/2 \cdot 4 \cdot (16 - 12) = 8$.

Thus the change in consumer surplus is $8 - 12 \cdot 1/2 = -4 \cdot 1/2$.

After imposing the quantity tax on sellers, the seller surplus is the area of the triangle below $p = 9$ (i.e. new price sellers receive) and above supply curve, that is $1/2 \cdot 4 \cdot (9 - 1) = 16$.

Thus the change in producer surplus is $16 - 25 = -9$.

The deadweight loss is $12 + (-4 \cdot 1/2) + (-9) = -1 \cdot 1/2$, i.e. the sum of the tax revenue, change in consumer and producer surplus.

i. Determine the equilibrium when instead of imposing the \$3 tax on the seller we impose the \$3 tax on the buyers.

After imposing the quantity tax, in equilibrium $p_D - 3 = p_S$ and $q_D = q_S$. Let $p_D = p$.

We solve $\begin{cases} q = 16 - p \\ q = -0.5 + 0.5(p - 3) \end{cases}$. $q = 4$ and $p = 12$. $p_D = p = 12$, $p_S = p - 3 = 9$.

The new equilibrium quantity is 4, the price the buyers will pay is 12 and the price the sellers will receive is 9. Note that transaction take place at $p_S = 9$.

j. Based on the predicted equilibrium - do sellers prefer that the tax is imposed on the buyers or on the sellers? Explain.

Sellers are indifferent because the amounts they sell and the prices they receive are exactly the same in two situations. It does not matter who has the legal responsibility of paying the tax.

Ch. 22 Firm Supply

Problem 2. A competitive firm has a short run cost function $c(y) = 0.5y^2 + 2y + 10$

a. What is firm's fixed cost of production?

Fixed costs are costs that must be paid independent of the level of output (y). In current case, the fix cost is 10.

b. What is the firm's variable cost?

Variable cost is the cost that varies with the level of total output. $0.5y^2 + 2y$ is the firm's variable cost.

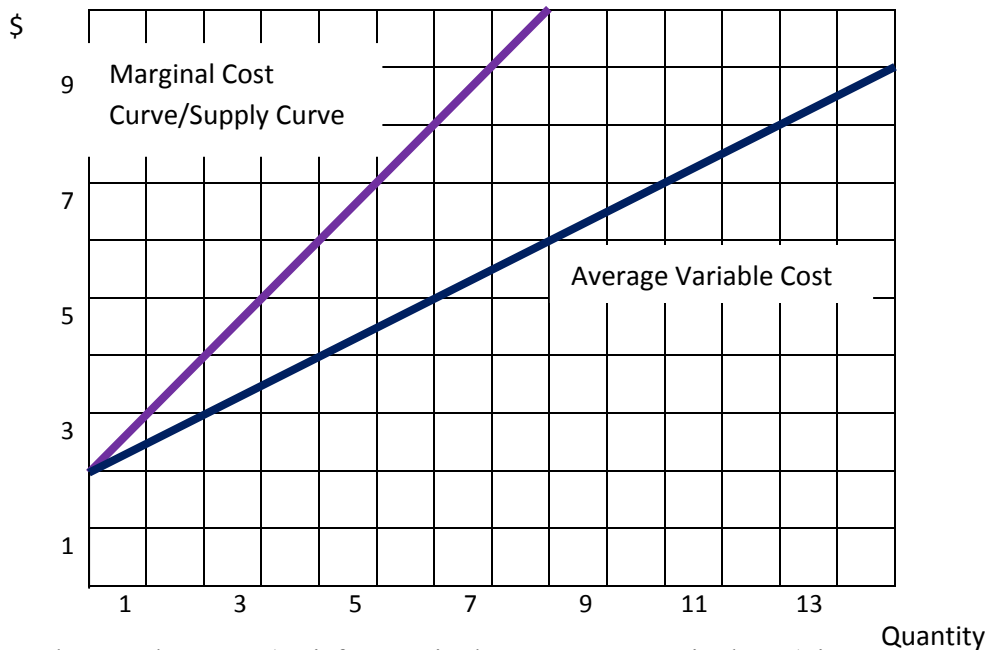
c. What is the firm's average variable cost?

The average variable cost is $(0.5y^2 + 2y)/y = 0.5y + 2$.

d. What is the firm's marginal cost?

The marginal cost measures the change in costs for a given change in output. It is $dc(y)/dy = 0.5*2y + 2 = y + 2$.

e. Illustrate the firm's marginal cost, average variable cost, and supply curves in the graph below (be sure to label the axis)



The supply curve (satisfy marginal revenue = marginal cost) is

$$y = \begin{cases} p - 2 & \text{if } p > 2 \\ 0 & \text{if } p \leq 2 \end{cases}$$

Thus it is the same as the marginal cost curve along with zero supply for prices below 2.

f. If the market price is \$8 how much does the firm choose to produce? What are the associated revenue, costs and profit of production?

At optimal output $y > 0$, it satisfies that marginal cost equals to marginal revenue. Here is marginal revenue is constant at $p = 8$. Thus $y + 2 = 8$, i.e. $y = 6$. The total revenue is $6 * 8 = 48$. The total cost is $0.5 * 6^2 + 2 * 6 + 10 = 40$. The profit is $48 - 40 = 8$.

g. If the market price is \$4 how much does the firm choose to produce? What are the associated revenue, costs and profit of production?

$MC = MR \Rightarrow y + 2 = 4 \Rightarrow y = 2$. The total revenue is $2 * 4 = 8$. The total cost is $0.5 * 2^2 + 2 * 2 + 10 = 16$. The profit is $8 - 16 = -8$ (Notice: even though the profit is negative, it is still better than not producing at all because in that case your profit is -10 due to the fixed cost).

h. If the market price is \$1 how much does the firm choose to produce? What are the associated revenue, costs and profit of production?

$MC=MR \Rightarrow y + 2 = 1 \Rightarrow y = -1$. Thus the output is 0. The total revenue is 0. The total cost is 10. The profit is -10.