Econ. 1100
Professor Lise Vesterlund

Problem Set 4

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Ch. 8 Slutsky Equation
Problem 1:Ms. CD has the following preferences for xylophones(x) and yams(y): $U(x,y)= x\ y$. Ms. CD has $800 to spend on the two goods, the price of a xylophone, p_x, is $1, and the price of a yam, p_y, is $1. The price of a xylophone suddenly increases to $2 ($p_x$').

a) Before the price change how many xylophones and yams does Ms. CD buy.

To solve the optimal consumption bundle for Cobb-Douglas utility, two relations are used

- The optimal consumption rule $MRS_{x,y} = -\frac{p_x}{p_y}$, where $MRS_{x,y} = -\frac{MU_x}{MU_y} = -\frac{y}{x}$ and $-\frac{p_x}{p_y}$, thus $p_x\ x = p_y\ y$.
- The budget constraint $p_x x + p_y y = m$,
- Combining the two we get the demand functions: $x = \frac{m}{2p_x}$ and $y = \frac{m}{2p_y}$
- Letting m=800, $p_x = 1$ and $p_y = 1$ we get $x^A = y^A = 400$.

b) How many xylophones and yams does she buy at the new price?

- Using the demands from a) and $(m, p'_x, p_y) = (800, 2, 1)$ we get $x^C = 200$, $y^C = 400$.

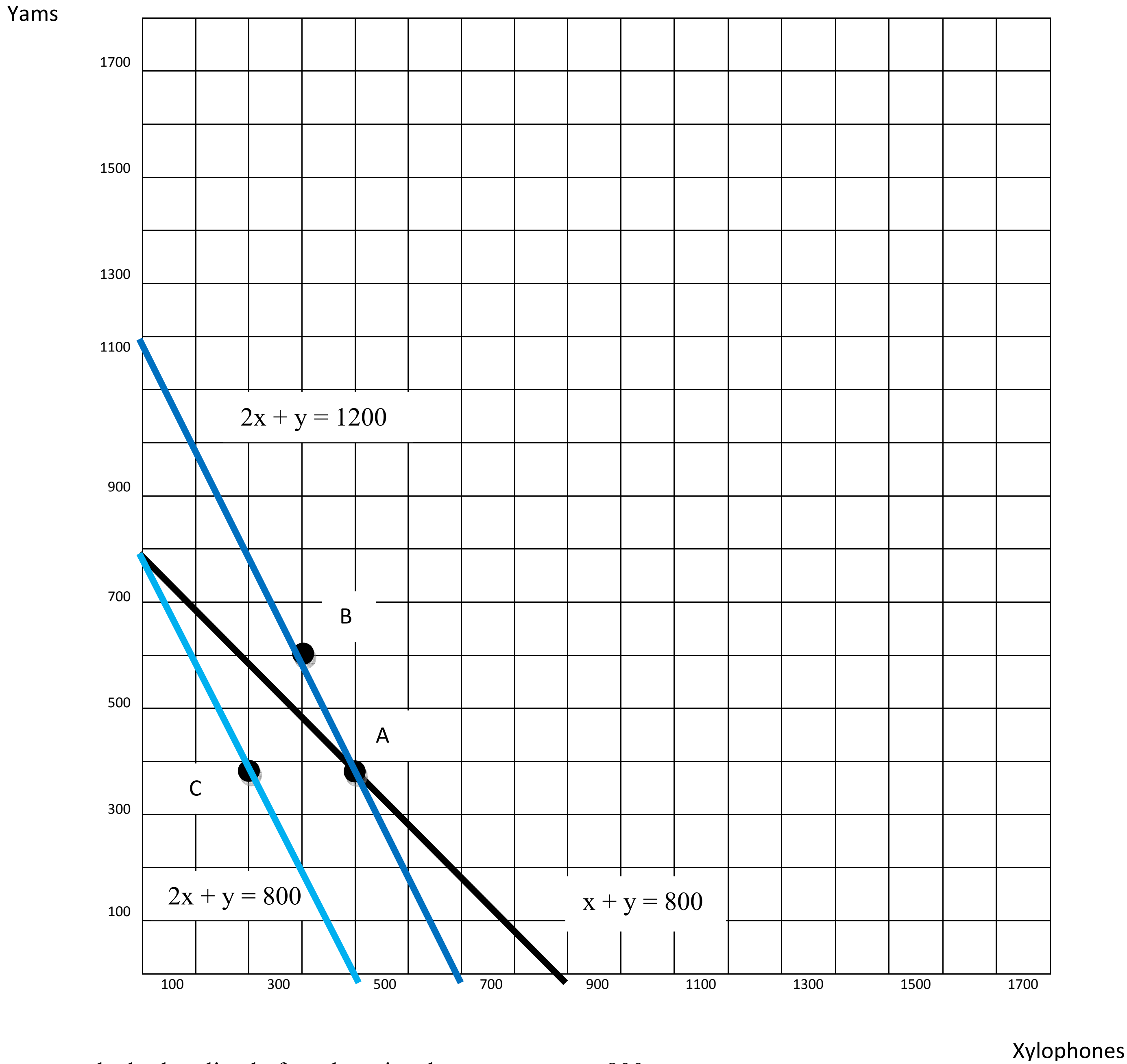
c) How large would Ms. CD's income, m', be if she after the increase in p_x exactly could afford her old consumption bundle?

At the new price system, her original bundle in a) costs 2*400 + 1*400 = 1200 (or equivalently $m'=m+\Delta p\ x^A$. Thus m'=1200.

d) How many xylophones and yams would she buy if she was faced with p_x', p_y and m'?

Using the demands in a we get $x^C = 300$, $y^C = 600$.

e) Draw three budget lines on the graph below. One illustrating the budget line before the price decrease, one after the price decrease, and finally the budget line she would face if she had income m' and the new prices. Denote her original consumption bundle by A, the final bundle by C, and the bundle she would buy at (p_x', p_y, m') by B.



the budget line before the price decrease: x + y = 800

the budget line after the price decrease: 2x + y = 800

the budget line she would face if she had income m' and the new prices: 2x + y = 1200

f) How large a decrease in the total demand for x is due to the income effect? and how large a decrease in the demand for x is due to the substitution effect?

At (p_x', p_y, m'), her preferred bundle in (p_x, p_y, m) is still on her budget line at new price system (p_x', p_y), i.e. she can just afford the original bundle. Thus the income effect is removed at (p_x', p_y, m') and any change in the demand for x from (p_x, p_y, m) to (p_x', p_y, m') is due to the substitution effect that is $\Delta x = 300 - 400 = -100$.
The rest of change in x is due to the income effect, i.e. the income effect is 200 – 300 = -100.

Problem 2: Betty's preferences for good 1 (x_1) and good 2 (x_2) can be described by $U = 2x_1 + x_2$. She has $100 to spend on the two goods, and $p_1=5$, and $p_2=1$.

a. Determine Betty's demand for x_1

Since the utility of a unit of good 1 is twice as much as the utility of a unit of good 2. Thus the price she is willing to pay for one unit of good 1 is twice as much as the price she is willing to pay for one unit of good 2.
Her demand function for x_1 is

$$x_1 = \begin{cases} \frac{m}{p_1} \; if \; p_1 < 2p_2 \\ 0 \; if \; p_1 > 2p_2 \end{cases}$$

Now since we have $-p_1/p_2 = -5/1$, the demand for x_1 is 0.

b. Determine the total change in her demand for good 1 when the price of good 1 decreases to$1.

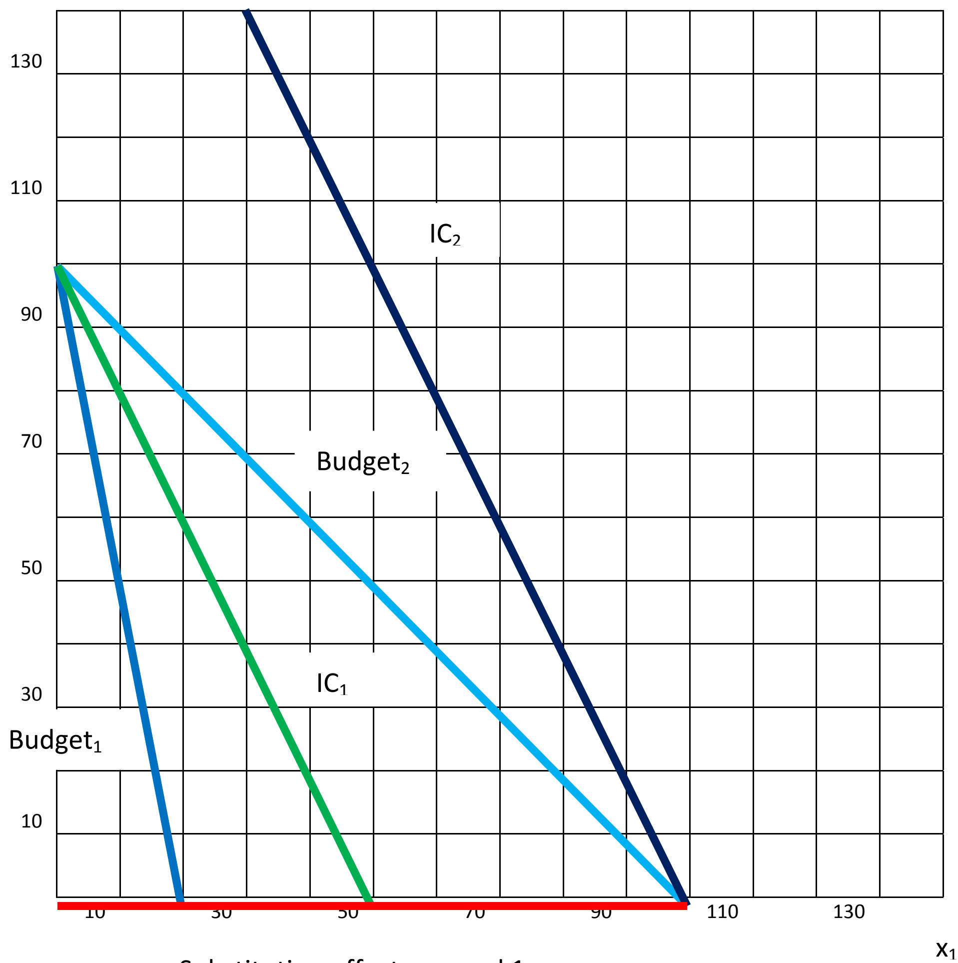
According to the demand function derived in a), when the price of good 1 decreases to$1, i.e. $-p_1/p_2 = -1$, she will spend all her budget on good 1. Thus her demand for good 1 is 100/1 = 100 and the total change in her demand for good 1 is 100 – 0 = 100.

c. What are the substitution and income effects of this price change?

After the price of good 1 decreases to $1, it is easy to see that her new budget line will also cross the original demand bundle (0, 100), i.e. she can just afford the original bundle. Thus the substitution effect on good 1 is 100 – 0 = 100 and the income effect on good 1 is 100 - 100 = 0; the substitution effect on good 2 is 0 – 100 = -100 and the income effect on good 1 is 0 - 0 = 0.

d. On the picture below draw a picture to illustrate the income and substitution effect. Be sure to illustrate the relevant budget lines and indifference curves.

x_2



Substitution effect on good 1

$Budget_1$ and $Budget_2$ are budget lines corresponding to market condition in a and b, respectively; IC_1 and IC_2 are indifference curves crossing the optimal bundles in a and b, respectively; the red line illustrates the substitution effect on good 1; since the income effect is 0, it is not shown in the graph.

Problem 3: Hank is shopping for ingredients for his margarita. His preferences for margarita mix (x_M) and tequila (x_T) can be described by U= min($2x_T$, x_M). Hank has $120 to spend on margarita, and p_T=20, and p_M=10.

a. Determine the total change in his demand for margarita mix when the price of margarita mix decreases to$5 per bottle.

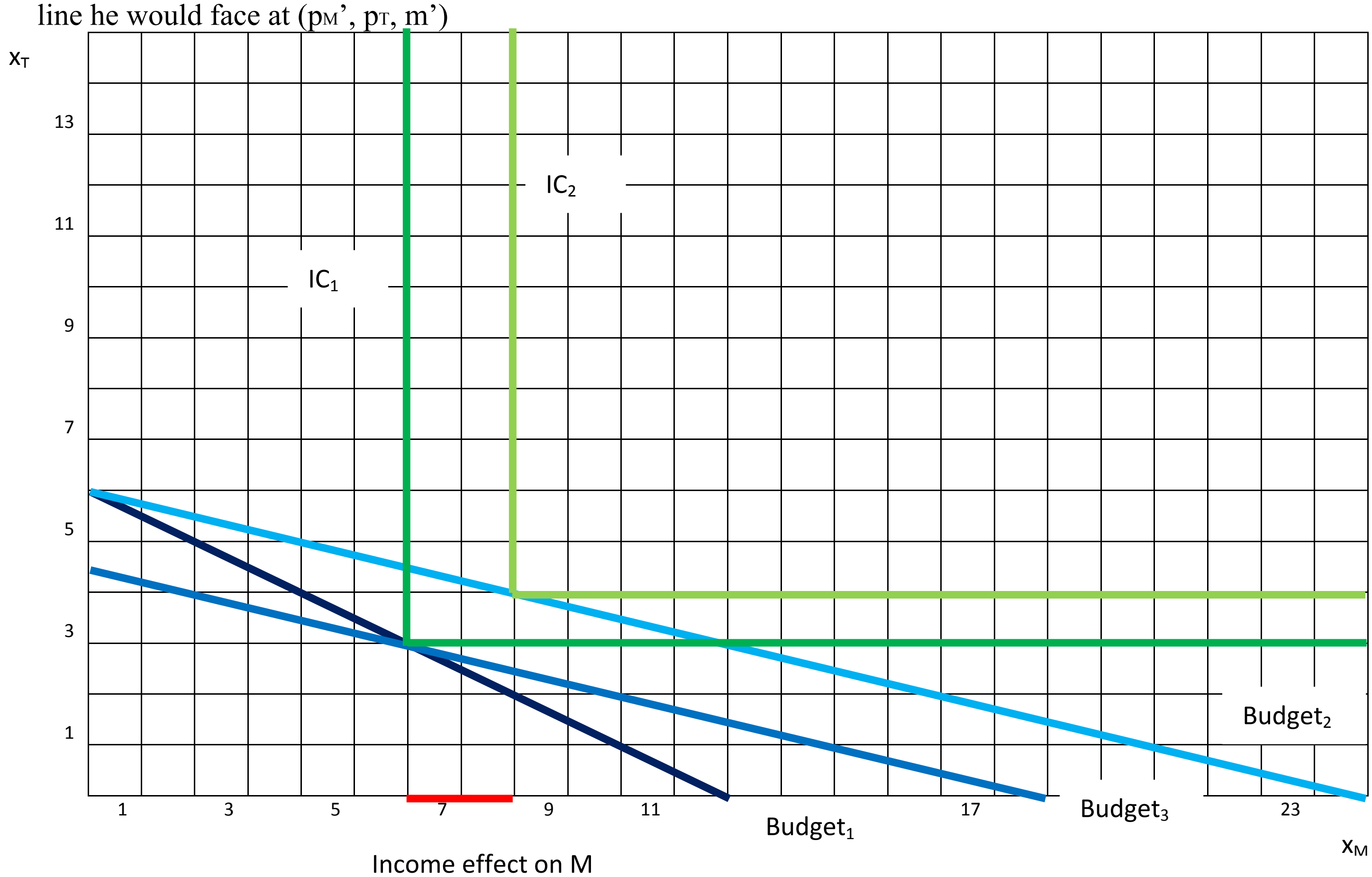
The ingredients are perfect complements. He consumes each tequila with two margarita mix. Before the price changes, a perfect pair costs 2 p_M + 1 p_T = 2*10 + 1*20 = 40. Thus she will purchase 120/40 = 3 pairs, i.e. 6 margarita mix and 3 tequila.

After the price changes, a pair costs 2 p_M' + 1 p_T = 2*5 + 1*20 = 30. Thus she will purchase 120/30 = 4 pairs, i.e. 8 margarita mix and 4 tequila.
Thus the total change in his demand for margarita mix is 8 – 6 = 2.

b. What are the substitution and income effects of this price change?

After the price changes, the current total budget is more than enough to afford the original demand bundle because 3 perfect pairs at new price levels cost only $3*(2*5 + 20) = 90 < 100$. Thus we need reduce total budget to 90 which just affords the original bundle at new price levels. Since they are perfect complements, we know that he will still consume the goods with perfect pairs. Thus he will again buy 3 perfect pairs which implies the substitute effect is 0. By subtracting the substitute effect from total change in demand we obtain the income effect that is $2 – 0 = 2$.

c. On the picture below draw a picture to illustrate the income and substitution effect. Be sure to illustrate the three budget lines (before the price change, after the price change, and the budget line he would face at (p_M', p_T, m')



Budget$_1$, Budget$_2$, Budget$_3$ are budget lines before the price change, after the price change, and the budget line he would face at (p_M', p_T, m'), respectively; IC_1 and IC_2 are indifference curves crossing the optimal bundles before the price change and after the price change, respectively; the red line illustrates the income effect on M; since the substitution effect is 0, it is not shown in the graph. Notice from Budget$_1$ to Budget$_2$, since only M's price changed (being increased), we pivot anticlockwise to account for the relative price change. Budget$_3$ should be parallel to Budget$_2$ and cross the original optimal bundle.