Econ. 1100
Professor Lise Vesterlund

Problem Set 3

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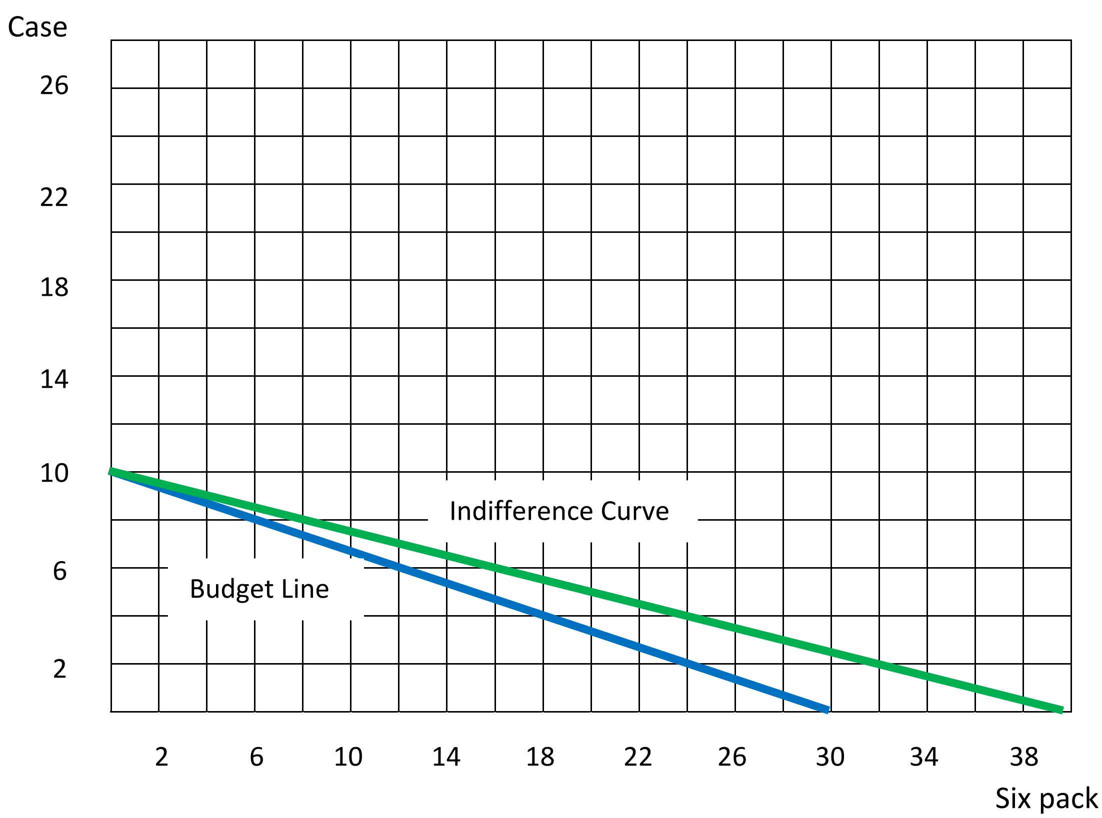
Remark: This solution is meant to be a learning tool for you. It is more comprehensive than what you need in order to get full credit. Please let me know if you find any mistakes.

Problem 1: Mary J. only cares about the amount of Pepsi that she drinks. Pepsi either comes in six pack (x_6) or in a case (x_{24}). A case has 24 cans of soda.

Since Mary only cares about the amount of Pepsi she drinks, she views the two goods as perfect substitutes. Therefore, her indifference curves are parallel straight lines.

a) Draw Mary J's budget line when her soda budget is $30 and the price of a case is (p_{24}) is $3 and the price of a six pack (p_6) is $1

The most packs and cases she can buy are 30/1=30 and 30/3=10, respectively. we obtain the budget line by connecting bundles (0, 10) and (30, 0).



b) How many six packs and cases does Mary J buy when m=30, p_{24}=3, and p_6=1?

Since Mary only cares about how much Pepsi she consumes she will purchase the good that offers the most Pepsi per dollar. For Pepsi in cases, each dollar purchases 24/3= 8 units of Pepsi; for Pepsi in packs, each dollar purchases 6/1= 6 units of Pepsi. Therefore, she will only buy Pepsi in cases. She will buy 30/3=10 cases of Pepsi and zero six-packs.

This approach is the same as comparing $MRS_{6,24}$ with exchange rate, i.e. $-p_6/p_{24} = -1/3$. $MRS_{6,24}$ is $-MU_6/MU_{24} = -6/24 = -1/4$, which implies that she is willing to give up ¼ case for a six pack. However the market requires that she gives up 1/3 of a case, thus given that the marginal rate of substitution is constant she will spend all her money on Pepsi in cases.

c) Draw a picture of her indifference curve through her utility maximizing bundle.

The slope of the indifference curve is equal to $MRS_{6,24}$, i.e. -1/4, and her utility maximizing bundle in b) is (0, 10).

d) Find Mary J's demand function for six packs (x_6)

As discussed in b), she will not buy any six packs if $MRS_{6,24} > -p_6/p_{24}$, but will spend her entire budget on six pack if $MRS_{6,24} < -p_6/p_{24}$. Thus, her demand function for six packs is

$$x_6 = \begin{cases} 0, & \text{if } p_{24} < 4p_6 \\ \frac{m}{p_6}, & \text{if } p_{24} > 4p_6 \end{cases}$$

Problem 2: Phoebe likes spending time at Central Perk. While at Central Perk she always wants two scones (x_S) for every one cup of coffee (x_C).

Since Phoebe always wants two scones (x_S) for every one cup of coffee (x_C), she views the two goods as perfect complements.

a) Draw Phoebe's budget line when her Central Perk budget is \$20, and the price of a scone is \$1 and the price of a cup of coffee is \$2

Her budget line equation is: $x_S + 2x_C = 20$, with $x_S, x_C \geq 0$. See picture below

b) How many scones and cups of coffee does she buy when m=20, p_S=1, and p_C=2?

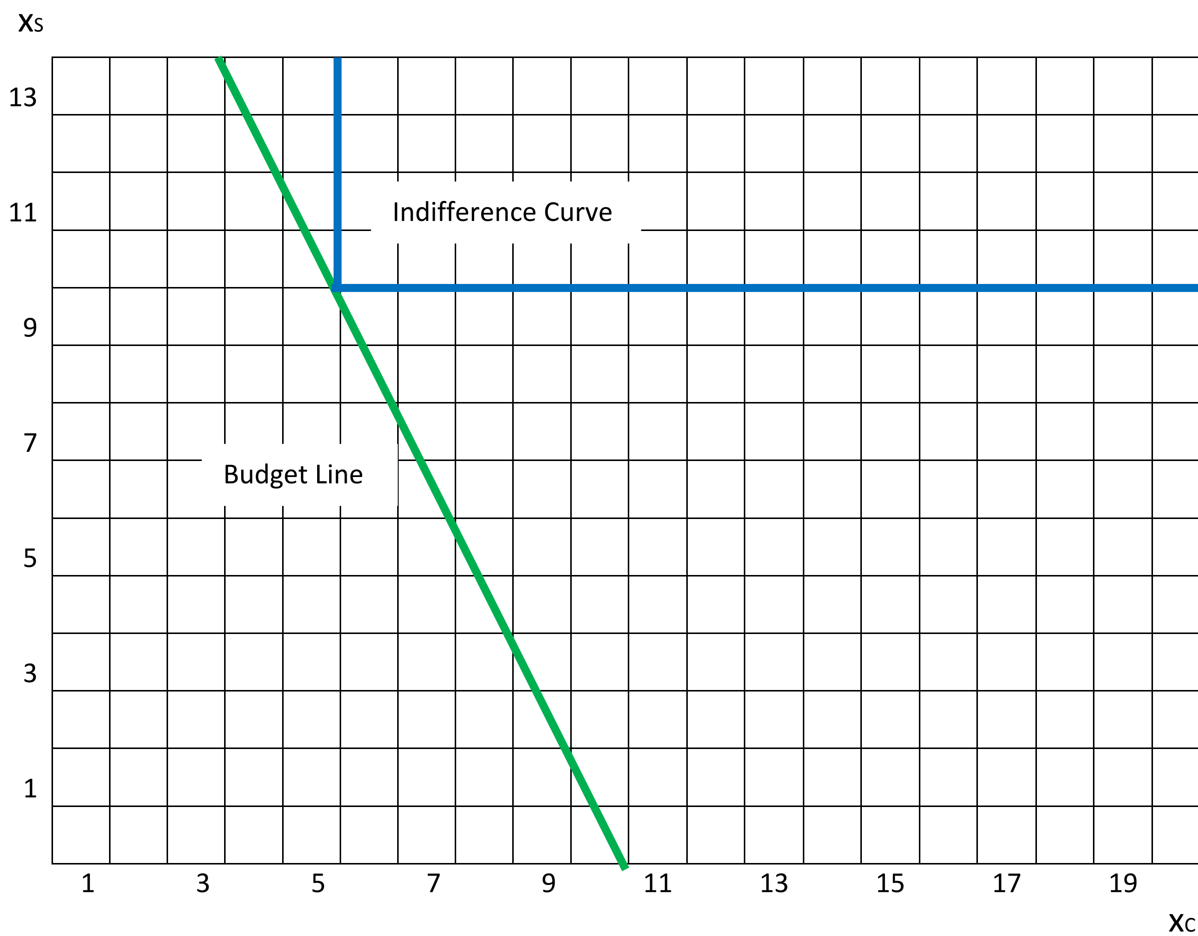
When her bundle ratio between scones and cups of coffee is 2:1, adding only one kind of consumption will not change her utility. Thus in her optimal bundle, the ration between x_S and x_C is always 2:1, i.e. $x_S = 2x_C$. In addition to the budget line equation, we have two linear equations with two unknowns

(A) $x_s = 2x_c$

(B) $x_s + 2x_c = 20$,

Solving them, we have $x_S = 10$ and $x_C = 5$.

Alternatively we know that a perfect pair has two scones and one coffee, hence the price of a perfect pair is 2×1+2=4. With an income of \$20 she can afford to buy \$20/\$4 = 5 perfect pairs. Since there are 2 scones in each pair she buys 10 scones, and since there is one coffee in each pair she buys 5 coffees.



c) Draw a picture of the indifference curve where she maximizes utility.

d) Find Phoebe's demand function for scones (x_S) (be sure to show your work)

As in b), we have (A) $x_s = 2x_c$. With the budget line equation, we have two linear equations with two unknowns

$$\text{(A)}\ x_s = 2x_c,\ \text{(B)}\ p_s x_s + p_c x_c = m,$$

$$\text{Solving them, we have } x_s = \frac{2m}{2p_S+p_C} \text{ and } x_C = \frac{m}{2p_S+p_C}.$$

Or equivalently the price of a perfect pair is $2\times p_s+p_c$. The number of perfect pairs we can afford is $\frac{m}{2p_S+p_C}$. In every pair there is one coffee hence the demand for coffee equals $\frac{m}{2p_S+p_C}$. In every pair there are two scones, hence the demand for scones equals $\frac{2m}{2p_S+p_C}$.

e) Does Phoebe view scones as a Normal or Inferior good? (Explain)

Recall the definitions: Normal goods: an increase in income increases demand. Inferior goods: an increase of income reduces demand.

Since $\frac{\partial x_S}{\partial m} = \frac{2}{2p_S+p_C} > 0$ Phoebe views scones as Normal goods.

f) Are scones a luxury or a necessity or are preferences homothetic?(Explain)

A luxury: the percentage increase in demand is greater than the percentage increase in income; A necessity: the percentage increase in demand is smaller than the percentage increase in income; Homothetic preferences: the percentage increase in demand equals the percentage increase in income

There are two ways to answer the question. First, looking at the answer to question e – I can see that the slope of my Engel curve will be a constant (i.e., it does not depend on income) therefore I know that the percentage increase in demand equals the percentage increase in income. I.e. preferences are homothetic – the good is not viewed as a luxury nor as a necessity.

Another way to answer the question is to determine the percentage change in demand relative to the percentage change in income. I.e. calculate the income elasticity of demand.

$$\frac{\text{percentage change in demand}}{\text{percentage change in income}} = \frac{\partial x_S/x_S}{\partial m/m} = \frac{\partial x_S}{\partial m}\frac{m}{x_S} = \frac{2}{2p_S + p_C}\frac{m}{\frac{2m}{2p_S + p_C}} = 1$$

Since the income elasticity equals 1 we know that preferences are homothetic.

g) Are scones ordinary or Giffen goods? (Explain)

An ordinary good: an increase in own price decreases demand.
A Giffen good: an increase in own price increase demand.

Take the partial derivative of x_s with respect to p_s

$\partial x_s/\partial p_s = -\frac{4m}{(2p_S+p_C)^2} < 0$, thus scones are ordinary goods.

Alternatively, in $x_s = \frac{2m}{2p_S+p_C}$, since p_s is in the denominator and its coefficient is positive, an increase in p_s decreases x_s.

f) Looking at your demand function. Is the cross price effect positive or negative? What does that tell you about scones and coffee? (Explain)

Again, taking the partial derivative
$\partial x_s/\partial p_c = -\frac{2m}{(2p_S+p_C)^2} < 0$, $\partial x_c/\partial p_s = -\frac{2m}{(2p_S+p_C)^2} < 0$.

Thus both cross price effects are negative and the goods are complements. $\partial x_S/\partial p_C = = -\frac{2m}{(2p_S+p_C)^2}$ < 0 means that the demand for scones goes down when the price of coffee goes up. Thus scones are complements to coffee. $\partial x_C/\partial p_S = -\frac{2m}{(2p_S+p_C)^2} < 0$ means that the demand for coffee goes down when the price of scones goes up. Thus coffee is a complement to scones.

Problem 3: Julie's preferences for pickles (x_P) and meatloaf (x_M) can be captured by the following Cobb-Douglas utility function: $U= x_P*x_M^2$

a) What is Julie's marginal utility of pickles and meatloaf?

$MU_p = \partial U/\partial x_p = x_M^2$; $MU_M = \partial U/\partial x_M = 2x_Px_M$.

b) Explain in words what the marginal utility of pickles measures

The marginal utility of pickles measures the rate of change in utility with a small change in the consumption of pickle at the bundle (x_P, x_M).

c) Determine her $MRS_{P,M}$ for an arbitrary bundle (x_P,x_M).

$MRS_{P,M} = -MU_p/MU_M = -x_M/2x_p$.

d) Draw her budget line when her budget for pickles and meatloaf is \$24 and the price of a slice of meatloaf is \$2 and the price of a pickle is \$1. (Note, Professor Vesterlund changed the budget from 32 to 24. Please check the class web-page. Thus it's OK that your answers are different.)

The budget line equation is $p_px_p+ p_Mx_M = m$. Substituting price levels and total budget, $x_P + 2x_M = 24$.

e) What is the slope of the budget line?

The slope is -1/2.

f) How many slices of meatloaf and how many pickles does she buy when m=24, p_M=2, and p_P=1? Be sure to derive the demand step by step. (Note, Professor Vesterlund changed the budget from 32 to 24. Thus it's OK that your answers are different.)

The question is equivalent to solve problem

$$\max U= x_P*x_M^2$$
$$\text{subject to } x_P + 2x_M \leq 24.$$

i.e. maximizing her utility under budget constraint. Since Julie always increases her utility by spending money on these two goods, she will spend all her budget. Thus we need to solve

$$\max U = x_P * x_M^2$$
$$\text{subject to } x_P + 2x_M = 24$$

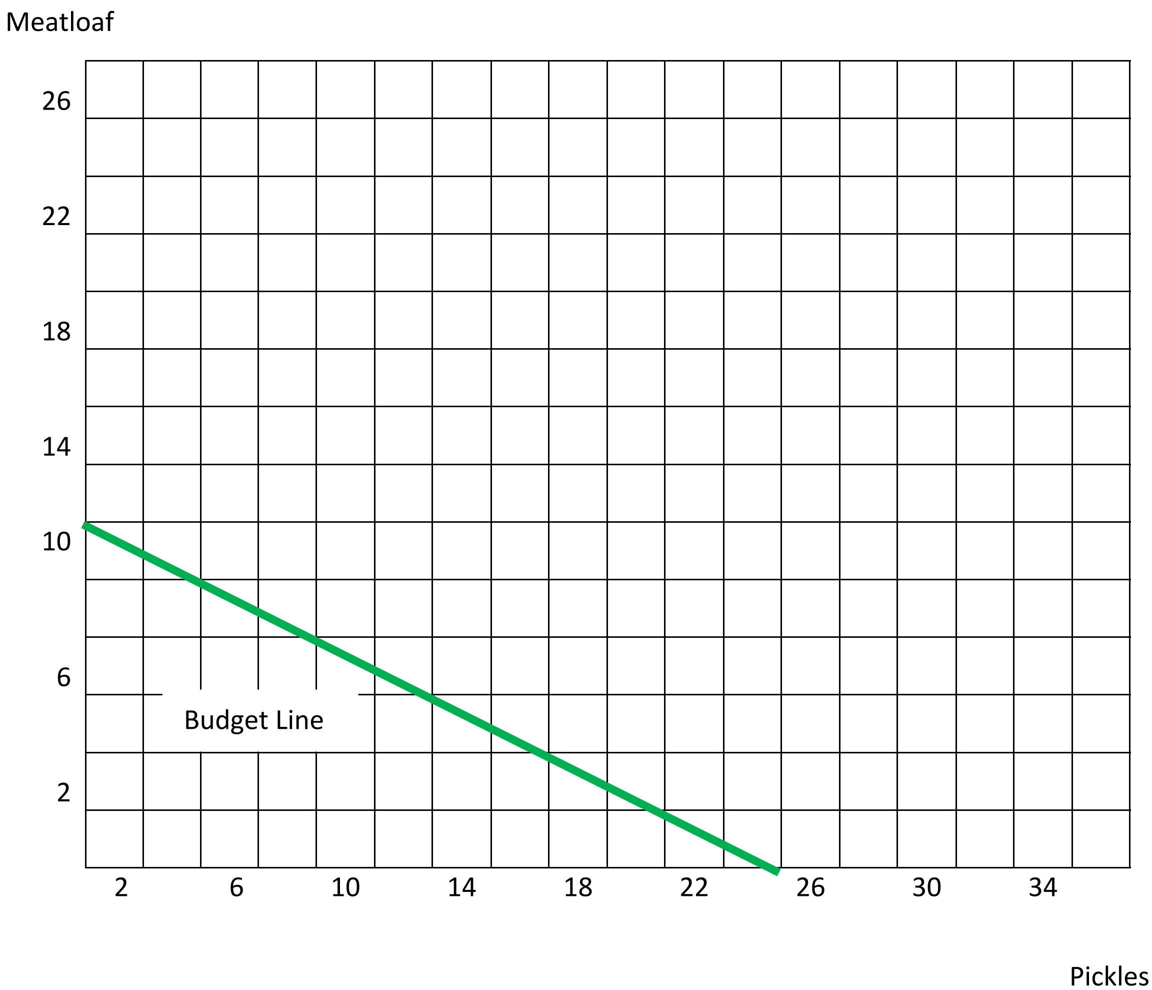
At the optimal bundle two conditions have to hold

(A) $MRS_{P,M} = -\frac{P_P}{P_M}$, that is $-\frac{X_M}{2X_P} = -\frac{1}{2}$

(B) $x_P + 2x_M = 24$

Now we have two equations (A) and (B) with 2 unknowns x_P and x_M. By solving them we obtain the optimal bundle $(x_P, x_M) = (8, 8)$. Thus she will buy 8 pickles and 8 slices of meatloaf.

(If you use budget \$32, you will get (32/3, 32/3). Thus if the units are divisible, she will buy 32/3 pickles and 32/3 slices of meatloaf.



g) Derive her demand function for arbitrary prices (p_P, p_M) and income m (be sure to do it step by step).

Similar to f), the question is equivalent to solve problem

$$\max U = x_P * x_M^2$$
$$\text{subject to } p_P x_P + p_M x_M = m$$

At the optimal bundle two conditions have to hold

(A) $MRS_{P,M} = -\frac{P_P}{P_M}$, that is $-\frac{X_M}{2X_P} = -\frac{P_P}{P_M}$

(B) $p_P x_P + p_M x_M = m$

Solving two equations with two unknowns x_P and x_M we get $x_P = \frac{m}{3P_P}$, and $x_M = \frac{2m}{3P_M}$.

h) Given your demand function, find the income elasticity of demand?

the income elasticity of demand for pickles is

$$\frac{\partial x_P}{\partial m}\frac{m}{x_p} = \frac{1}{3p_p}\frac{m}{\frac{m}{3P_P}} = 1 =$$

the income elasticity of demand for slices of meatloaf is

$$\frac{\partial x_M}{\partial m}\frac{m}{x_M} = \frac{2}{3p_M}\frac{m}{\frac{2m}{3P_M}} = 1$$

i) *Given your demand function, find the price elasticity of demand?*

The price elasticity of demand for pickles is

$$\frac{\partial x_P}{\partial p_p}\frac{p_p}{x_p} = \frac{-m}{3p_p^2}\frac{p_p}{\frac{m}{3P_P}} = -1$$

The price elasticity of demand for slices of meatloaf is

$$\frac{\partial x_M}{\partial p_M}\frac{p_M}{x_M} = \frac{-2m}{3p_M^2}\frac{p_M}{\frac{2m}{3P_M}} = -1$$

Problem 4: Suppose we observe an individual's consumption of x and y over three periods. In all periods his income is \$100, in the first period $p_x = 1$ and $p_y = 1$, in the second period $p_x = 2$ and $p_y = 1$, and in the third period $p_x = 1$ and $p_y = 2$. In the first period he consumes (x, y)=(50,50), in the second period he consumes (x, y)=(33,33), and in the third period (x, y)=(33,33). What do you think his utility function might look like?

First, the ratio between x and y are constant at one for one in all three bundles. Considering the relative price between x and y varies from 1, 2 (>1) to 1/2 (<1), it indicates the possibility that the ratio between x and y are always constant. This property is typical for perfect complements. Further, since the ratio is maintained at 1, it naturally points to U(x, y) = min{x, y}.

Problem 5: Suppose we observe an individual's consumption of x and y over three periods. In all periods his income is \$100, in the first period $p_x = 1$ and $p_y = 1$, in the second period $p_x = 2$ and $p_y = 1$, and in the third period $p_x = 1$ and $p_y = 2$. In the first period he consumes (x, y)=(100,0), in the second period he consumes (x, y)=(40,20), and in the third period (x, y)=(100,0). What do you think his utility function might look like?

First, the consumption bundles change from (100, 0), (40, 20), to (100, 0) when relative price varies from 1, 2 to 1/2. Second, bundle (100, 0) at relative price 1 and 1/2 is at the boundary. These two facts indicate extreme price sensitivity. This is part of properties for perfect substitutes. Note that the individual is willing to buy both goods when good x cost twice as much as good y, this suggest that good x is twice as good as good y. E.g. good x may be a whole gallon of milk and good y half a gallon of milk. These preferences may be captured by a utility function of the form U(x, y) = 2x + y.

Problem 6: Suppose we observe an individual's consumption of x and y over three periods. In all periods his income is \$100, in the first period $p_x = 1$ and $p_y = 1$, in the second period $p_x = 2$ and $p_y = 1$, and in the third period $p_x = 1$ and $p_y = 2$. In the first period he consumes (x, y)=(60,40), in the second period he consumes (x, y)=(30,40), and in the third period (x, y)=(60,20). What do you think his utility function might look like?

The cross price effect is zero and as seen in the table below there is constant budget share.. These are the typical properties of the Cobb-Douglas utility, i.e. $U = x^a y^b$

	$p_x = 1$ and $p_y = 1$	$p_x = 2$ and $p_y = 1$	$p_x = 1$ and $p_y = 2$
Proportion of income spent on x	$\frac{1*60}{100} = \frac{3}{5}$	$\frac{2*30}{100} = \frac{3}{5}$	$\frac{1*60}{100} = \frac{3}{5}$
Proportion of income spent on y	$\frac{1*40}{100} = \frac{2}{5}$	$\frac{1*40}{100} = \frac{2}{5}$	$\frac{2*20}{100} = \frac{2}{5}$

As seen in Varian p. 83 the demand function is then given by $(x, y) = (\frac{a}{a+b}\frac{m}{p_x}, \frac{b}{a+b}\frac{m}{p_y})$. With the proportion of total budget spent on each good is $(\frac{p_x x}{m}, \frac{p_x x}{m}) = (\frac{a}{a+b}, \frac{b}{a+b})$. Thus any Cobb-Douglas function of the form $U = x^{3k}y^{2k}$ with k > 0 can serve as her utility function.