

Games Duopolists Play
Dixit, Chapter 5
pp. 124-128

Monopoly

A review of monopoly profit maximization will help understand the math.

The monopolist is faced with a **decision problem**, not a **strategic interaction**. Although his potential customers can themselves decide whether to buy or not, none of their **individual** decisions will change his payoffs. The monopolist deals only with an abstract demand curve which sums up the collective preferences of the market.

Suppose the monopolist faces the demand curve:

$$Q = 5000 - 10 P$$

and has a constant marginal cost of \$ 200.

His per unit profit will be **P - \$ 200**.

His total profit will thus be $\pi = (P - 200) Q$

where Q is given by the demand curve, so that:

$$\pi = (P - 200) (5000 - 10 P) = 5000 P - 10 P^2 - 200 (5000) + 2000 P$$

$$\pi = 7000 P - 10 P^2 - 1,000,000$$

Note that profit will be negative 1,000,000 if $P = 0$ (our monopolist would be giving away five thousand units, each of which cost \$ 200 to make).

Profit would also be zero if our monopolist set a price of \$ 500, since the quantity demanded would be zero.

To find the price which maximizes profit, differentiate and set the derivative equal to zero.

$$d\pi / dP = 7000 - 20 P = 0 \text{ implies that } 20 P = 7000 \text{ or}$$

the profit maximizing price is **P* = \$ 350**

at that price a quantity of $Q^* = 5000 - 10 (350) = 1500$ will be sold.

Hence the monopolist's profit will be:

$$\pi = (P - 200) Q = (350 - 200) 1500 = \$ 225,000$$

Try other prices: if the monopolist prices his product at \$ 360, $Q = 5000 - 3600 = 1400$

and profit = $(360 - 200) 1400 = \$ 224,000$.

Even a price \$ 1 higher or lower would lead to a slight reduction in profit

(confirm that in either case profit would be reduced by \$ 10).

Keep in mind the monopolist's price and quantity changes for comparison with the Cournot duopoly:

$$P^* = \$ 350 \text{ and } Q^* = 1500.$$

Cournot Duopoly

The industry is made up of Mr. Xenophon and Ms. Yselt, who face demand curves which depend on their own price AND that of their competitor. They produce similar but not identical products. We assume that:

$$\begin{aligned}Q_x &= 2500 - 10 P_x + 5 P_y \\Q_y &= 2500 - 10 P_y + 5 P_x\end{aligned}$$

Note that the total demand for both products $Q = Q_x + Q_y = 5000 - 5 P_x - 5 P_y$ which would be identical to our monopolist's demand if $P_x = P_y$.

This is a game of strategy, because the price set by one will affect the profit of the other.

The problem faced by X is to maximize

$$\pi_x = (P_x - 200) Q_x = (P_x - 200) (2500 - 10 P_x + 5 P_y), \text{ or, expanding and collecting terms}$$

$$\pi_x = 4500 P_x - 10 P_x^2 + 5 P_x P_y - 500,000 + 1000 P_y$$

Differentiate and set equal to zero to find the price that maximizes profit.

Note that the derivative of a constant is zero, and that our duopolist treats his rival's price as fixed independently of his decision: they must move simultaneously. The “e” superscript for P_y is to emphasize that this is the price of Y that X **expects**.

$$d\pi_x / dP_x = 4500 - 20 P_x + 5 P_y^e = 0$$

which leaves us not with a fixed price but a **reaction function for X**:

$$20 P_x = 4500 + 5 P_y^e \quad \text{or} \quad P_x = 225 + 0.25 P_y^e \quad \text{or}$$

$$P_y^e = 4 P_x - 900$$

You should confirm that the **reaction function for Y** will be:

$$P_y = 225 + 0.25 P_x^e$$

The definition of equilibrium as a state in which expectations are realized enables us to remove the “e” and solve:

$$4 P_x - 900 = 225 + 0.25 P_x$$

$$\text{so } 3.75 P_x = 1125$$

and

$$P_x = \$ 300 . \text{ Note that this is lower than the monopolist's price.}$$

By symmetry, P_y will also be \$ 300, so that we can find that the quantity sold by X and Y will be:

$$Q_x = 2500 - 10 (300) + 5 (300) = 1000$$

$$Q_y = 2500 - 10 (300) + 5 (300) = 1000$$

Total quantity produced is 2000 – more than would be the case from a monopolized industry.

Cournot Duopoly (continued)

Calculate the profit made by either of our duopolists:

$$\begin{aligned}\pi_x &= (P_x - 200) Q_x = (300 - 200) * 1000 = 100,000 \\ \pi_y &= (P_y - 200) Q_y = (300 - 200) * 1000 = 100,000\end{aligned}$$

Total profit for both duopolists is \$ 200,000 -- \$ 25,000 less than the monopolist's profit.

Would it not be to their mutual interest to form a cartel, charge monopoly prices, and collect another \$ 12,500 each? Monopoly prices at first seem to be a natural **focal point** for both players. Agreement on prices would lead to more profit for both than the duopoly solution – but it would also be in the interest of each to cheat on the agreement:

Suppose both agree to charge \$ 350 (the monopoly price) and that Mr. X is an “honest duopolist.” Look at the reaction function of Y:

$$P_y = 225 + 0.25 P_x^e = 225 + 0.25 (350) = \$312.50$$

If X charges \$ 350 and Y charges \$ 312.50, their quantities sold will be:

$$\begin{aligned}Q_x &= 2500 - 10 (350) + 5 (312.50) = 562.50 \\ Q_y &= 2500 - 10 (312.50) + 5 (350) = 1125\end{aligned}$$

so that their profits will be:

$$\begin{aligned}\pi_x &= (P_x - 200) Q_x = (350 - 200) 562.50 = 150 * 562.50 = \$ 84,375 \\ \pi_y &= (P_y - 200) Q_y = (312.50 - 200) * 1125 = \$ 126,562.5\end{aligned}$$

The game they are playing is a Prisoner's dilemma, with “maintain the cartel price” and “cheat” the two strategies, and the profit payoffs are:

		Ms. Yseult	
		Maintain Cartel	Cheat (Charge \$ 312.50)
Mr. Xenophon	Maintain Cartel	\$ 112.5, \$ 112.5	\$ 84.4, \$126.6
	Cheat (Charge \$312.50)	\$ 126.6, \$ 84.4	\$ 105.5, \$105.5

Both sides have a dominant strategy: cheat on the cartel arrangement.

The next step in the game will of course be to consider what happens if the other player cheats optimally: you can confirm that the answer will be, if Ms. Yseult chooses \$ 312.50, for Mr. Xenophon to choose

$$P_x = 225 + 0.25 P_y^e = 225 + 0.25 (312.50) = \$ 303.125$$

and then, if Mr. Xenophon chooses \$ 303.125 for Ms. Yseult to choose

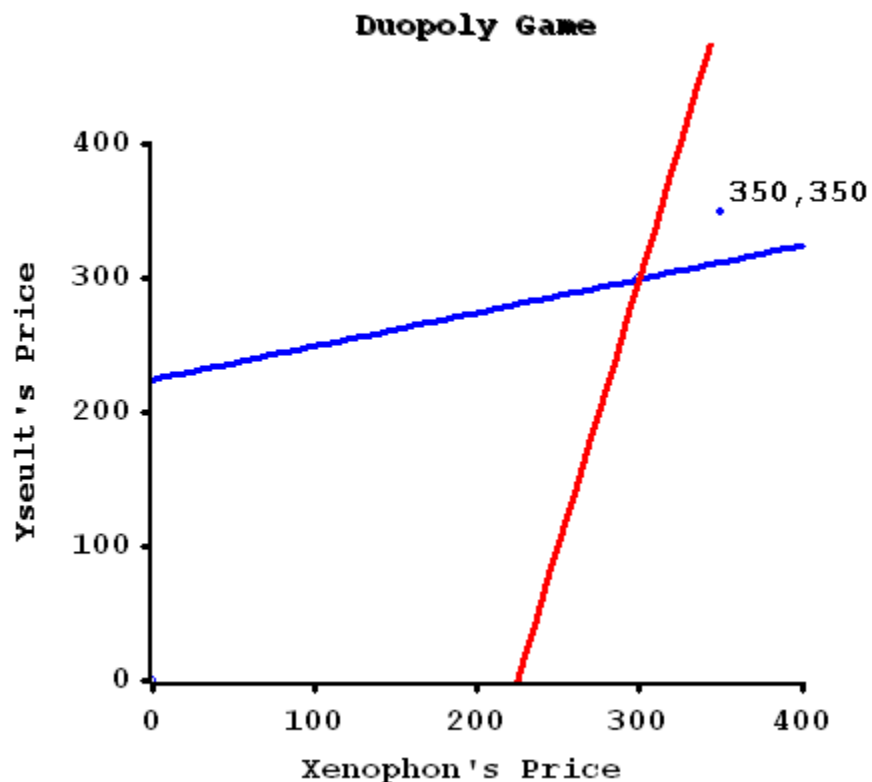
$$P_y = 225 + 0.25 P_x^e = 225 + 0.25 (303.125) = \$ 300.78$$

and we are only 78 cents away from the final duopoly price of \$ 300.

The graph below indicates the reaction curves of our duopolists;

they intersect at the Nash equilibrium (300, 300).

The cartel solution of (350, 350) is not on the reaction curves; should Yseult choose 350, Xenophon would move horizontally left to his red reaction curve; should Yseult choose 350, Yseult would move down to her blue reaction curve.



You should note that the implied dynamics is a bit of a fiction: as we will later see, if the firms are repeatedly interacting over time, it might be to their advantage to collude after all. See especially James W. Friedman, "The Legacy of Augustin Cournot", University of North Carolina at Chapel Hill Working Paper 99-05, Nov. 16, 1999, pages 15-18 for further analysis

From Duopoly to Competition

It is straightforward to generalize the Cournot model to N competitors. It is a bit more convenient here to treat quantity as the variable chosen by each of the N identical competitors; each firm will take the price chosen by all the others as given.

We keep the marginal cost constant and denote it by a lower case c .

Let the demand curve be a general function $P = D(Q)$ where Q = total quantity on the market and the quantity q_i is supplied by the i th firm, so that:

The derivative of the demand function is represented as $D'(Q)$.

$$\pi_i = (P * q_i - c * q_i) = D(Q) * q_i - c * q_i$$

In choosing quantity, each firm will look at the first order condition for profit maximization:

$$d\pi_i / dq_i = D'(Q) * q_i + D(Q) - c = 0$$

Since the firms are identical, each will choose the same quantity, so $q_i = Q / N$.

Note also that $D(Q) = P$, so we can write:

$$D'(Q) * Q / N + P - c = 0$$

We introduce the economic concept of elasticity (the absolute value of the percent change in quantity divided by percent change in price) into this equation (divide through by P and the first term becomes the negative of the reciprocal of the elasticity coefficient = $-dP / dQ * Q / P$, which we denote by ϵ) to find:

$$-\epsilon / N + 1 - c/P = 0$$

$$\text{or } c/P = 1 - \epsilon / N$$

Hence as N goes to infinity, the price charged by the firm will approach the marginal cost of production.