The Specific Factors Model and the Marginal Value Products Graph

Dr. Tom McGahagan – to accompany Feenstra, chapter 3

Production functions are assumed to be of the form: $Qx = A\,Lx^{\alpha}$ and $Qy = B\,Ly^{\beta}$,
where α and β are both less than one (to give us diminishing returns to labor).
There are two specific factors:

- **capital, which is used in X only**
- **land, which is used in Y only**

Basic facts about the production functions:

1. The production functions show diminishing returns to labor.

Consider the production function **$Qx = 200\,Lx^{0.4}$**

If Lx = 100, Q = $200 * (100)^{0.4}$ = 200 * 6.3096 = 1261.915

If Lx = 101, Q = $200 * (101)^{0.4}$ = 200 * 6.3347 = 1266.947

The marginal product of labor as we move between 100 and 101 units of labor is
1266.9473 - 1261.9147 = 5.0326

Add another worker and you find
Lx = 102, Q = $200 * (102)^{0.4}$ = 20 * 6.33598 = 1271.9501

So the marginal product of the additional worker is 5.0028.

Not much of a decline, but if you repeat the calculations for going from 500 to 501 workers, you will find that the marginal product is only 1.9206

You should satisfy yourself that the total product with 500 workers is more than the total product with 100 workers – it is a mistake to think that diminishing marginal returns is the same as falling output.

2. The marginal productivity of labor is given by dQ / dL.

The rules of calculus allow us to find the marginal product by multiplying by the exponent and subtracting one from the exponent.

$dQx/dLx = 0.4 * 200 * Lx^{0.4-1,0} = 80 * Lx^{-0.6} = 80 / Lx^{0.6}$

If L = 100, this gives us MPL = $80 / 100^{0.6}$ = 80 / 15.8489 = 5.0477

(You may note this is a bit overstated; if you want to be more accurate, note that you are looking at the MPL between 100 and 101, and use 100.5 as Lx.
This would give MPL = 5.0326, the precise answer. But we won't have to be this precise.)

If L = 500.5, the MPL would be calculated as $80 / 500.5^{0.6}$ = 80 / 41.6526 = 1.9206.

3.The exponent on labor gives labor's share of revenue from the sale of the product.

Labor's share of revenue is w * L divided by P * Q. If Px = $ 50

$w * L = P * MPL * L = \$50 * (0.4 * 200 / Lx^{0.6}) * Lx = .4 * \$50 * (200\,Lx^{0.4})$

$w * L = 0.4 * Px * Qx$ (since the last term on the line above is the production function.

Dividing by Px * Qx, we find that

$w * Lx / Px * Qx = 0.4$, which could be expressed as **$w * Lx = 0.4 * Px * Qx$**

Hence w * L is 40 percent of revenue, which we set out to show.

You should satisfy yourself that labor's share of revenue in the Y industry will be 0.75 (or 75 percent) if the production function in the Y industry is **$Qy = 200\,Ly^{0.75}$**

Problem.. Ruritanian specific factors. The Ruritanian economy has a labor force of 600 workers
Its production functions for manufactures (X) and agricultural products (Y) are given by:

$$\mathbf{Qx = 10 * \sqrt{Lx}}$$

$$\mathbf{Qy = 20 * \sqrt{Ly}}$$

In autarky, Px = Py = $ 10

Find: a) the number of workers in the X industry and in the Y industry.
b) the output of X and Y
c) the wage rate for workers
d) the return to capital (assuming 100 units of capital)
e) the return to land (assuming 400 units of land)

Start from the profit maximizing behavior of firms, which ensures that Px * MPLx = wage = Py * MPLy

$$\mathbf{\$ 10 * 5 / \sqrt{Lx} = \$ 10 * 10 / \sqrt{Ly}}$$

$$\mathbf{\$ 50 \sqrt{Ly} = \$ 100 * \sqrt{Lx}} \quad \text{(after some multiplication and cross-multiplication)}$$

$$\mathbf{\sqrt{Ly} = 2 * \sqrt{Lx}} \quad \text{(dividing by \$ 50)}$$

$$\mathbf{Ly = 4 * Lx} \quad \text{(squaring both sides)}$$

Since Lx + Ly = 600 and Ly = 4 Lx, we have
Lx + 4 * Lx = 600
5 * Lx = 600

Lx = 120

Ly = 600 – Lx = 480

Qx = 10 * $\sqrt{120}$ = 10 * 10.9545 = 109.5445 and so **Px * Qx = $ 1095.445**

Qy = 20 * $\sqrt{480}$ = 20 * 21.9089 = 438.1780 and so **Py * Qy = $ 4381.780**

Wage rate for workers:

Px * MPLx = $ 10 * 5 / √ Lx = $ 50 / √ 120 = $ 4.5644

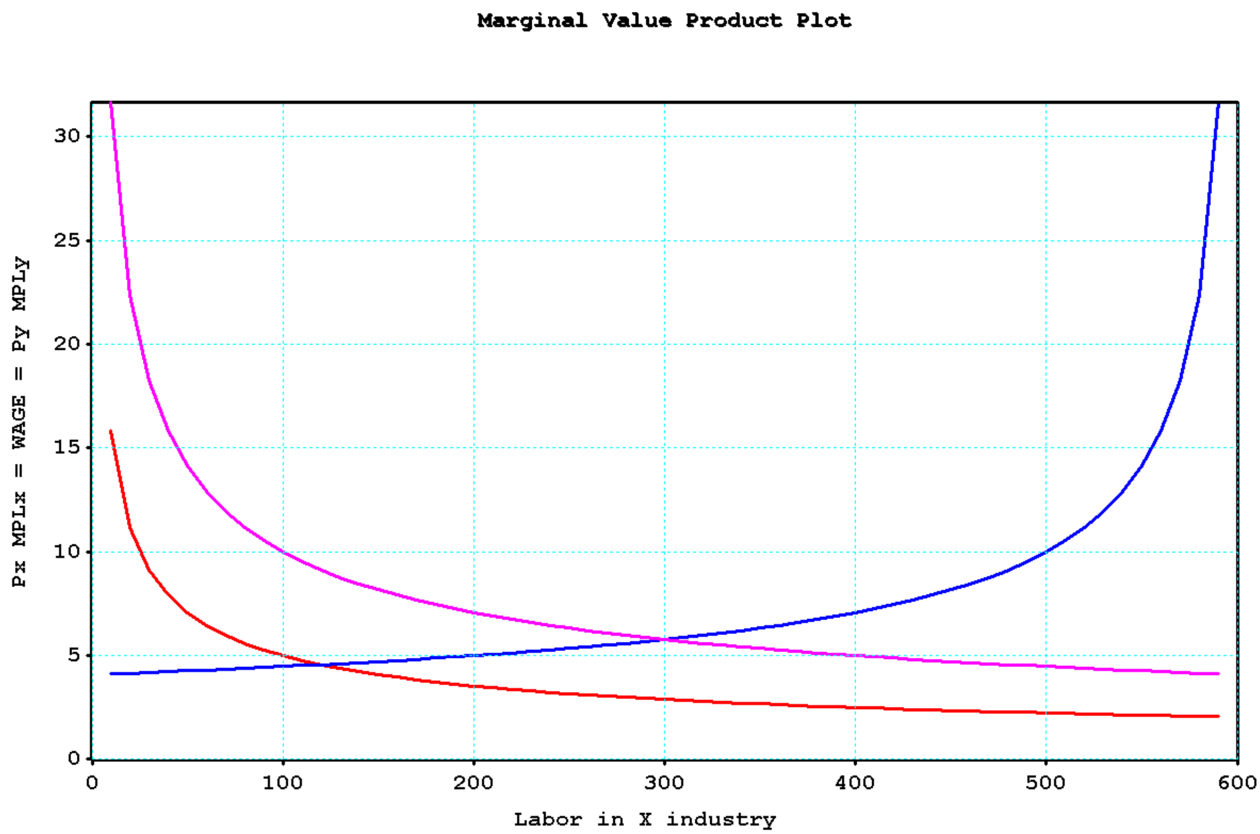
Py * MPLy = $ 10 * 10 / √ Ly = $ 100 / √ 480 = $ 4.5644

REAL WAGE: An hours' work will buy 0.4566 units of either X or Y.

The total wage bill is $ 4.5644 * 600 = $ 2738.61
The wage bill in the X industry is $ 4.5644 * 120 = $ 547.725
The wage bill in the Y industry is $ 4.5644 * 480 = $ 2190.89
Total return to capital in the X industry = Px * Qx - w * Lx = $ 1095.445 – $ 547.7225 = $ 547.7225
Rate of return to capital in the X industry = 547.7225 / 100 = $ 5.48
Total return to land in the Y industry = Py * Qy – w * Ly = $ 4381. 78 - $ 2190.89 = $ 2190.89
Rate of return to land in the Y industry = $ 2190.89 / 400 = $ 5.48

Graphic Presentation of Marginal Value Products

We graph the MVP functions (p MPL) for each industry below. Note that the length of the horizontal axis is equal to the country's workforce, and it is labeled Lx because the labor in the X industry is measured from left to right. Of course, the labor not in the X industry is employed in the Y industry, and may be computed as Ly = LF – Lx. As a result, the Py * MPLy curve seems to be a mirror image of the Px * MPLx curve.



The red line shows the marginal value product of labor in the X industry;
the blue line the marginal value product of labor in the Y industry.

The magenta line shows the impact of the price of X rising to $ 20.
Note that this will just offset the productivity disadvantage of labor in the X industry.

Problem:

If we have the same production functions as the original problem, and the price of X rises to $ 20, what will happen to:

- The workforce and output in each industry.
- The nominal and real wage (wage divided by the price of the products)
- The total return and the rate of return to capital and land.

You should find that:

Lx = Ly = 300

Qx = 10 * √ 300 = 173.2051 and Qy = 20 * √ 300 = 346.4102

Px Qx = Py Qy = 3464.1016 (up from $ 1095 in the X industry; down from $ 4382 in Y industry)

w * Lx = w Ly = 0.5 (3464.1016) = 1732.0508 (apply the fact that wage share is exponent on L)
w = (wLx + wLy) / 600 = 3464 / 600 = $ 5.77 (up from $ 4.66 before – but only by $ 1.11, not nearly enough to keep up with the $ 10 price increase in X).

REAL WAGE: in terms of X = $ 5.77 / 20 = 0.2877 – only 63 % of the previous real wage of 0.45644
in terms of Y = $ 5.77 / 10 = 0.577 , an increase of 26.5 % from the previous real wage.

Rk * K = 1732.0508 so assuming 100 units of capital, **Rk = $ 17.32 (up from $ 5.48 previously)**
Real Rk has risen in terms of manufactures, from being able to buy 0.548 units to being able to buy 17.32 / 20 = 0.866 units of manufactures.

Rt * T = 1732.0508, so assuming 400 units of capital, **Rt = $ 4.33 (down from $ 5.48 previously)**.