

## Leontief production functions and the Heckscher-Ohlin model

A **Leontief production function** is one in which factor proportions are fixed:

for example, where one farmer always works with 1 tractor, or one carpenter requires a toolbox with 30 tools. Providing any more farmers than there are tractors for would lead not to diminishing returns, but to vanishing returns – the farmer without a tractor will produce nothing. This is of course extreme – the extra farmer could use a sickle to harvest grain, or at least be available to drive the tractor when the original farmer is ill. And since in practice the capital stock is measured by its dollar value, there might very well be the chance to equip two farmers with two cheaper tractors.

But the Leontief production function does simplify presentation of the Heckscher-Ohlin model, and is perhaps closer to the model Heckscher had in mind in his original article (Eli Heckscher, “The effect of foreign trade on the distribution of income”, 1919, reprinted in Howard S. Ellis and Lloyd Metzler, *Readings in the Theory of International Trade*, Philadelphia, Blakiston, 1949).

The Leontief production functions for goods X and Y will be of the form:

$$X = \min [ 2 L_x , 5 K_x ]$$

which means that to produce one unit of X, you need a minimum of  $\frac{1}{2}$  unit of labor and  $\frac{1}{5}$  unit of capital, and that adding any more labor or capital will not get you any more output.

The values  $\frac{1}{2}$  and  $\frac{1}{5}$  are referred to as **activity coefficients**, since they show the activity required from labor (say  $\frac{1}{2}$  of a day's work) to produce 1 unit of X, and the activity required of capital (say  $\frac{1}{5}$  of a day's work) to produce 1 unit of X.

With  $L_x = 10$  and  $K_x = 2$ , you get  $X = \min [ 20, 10 ] = 10$  units of X.

With  $L_x = 5$  and  $K_x = 2$ , you get  $X = \min [ 10, 10 ] = 10$  units of X.

The logical **capital labor ratio in the X industry** to choose will be  $K_x / L_x = 2/5 = 0.4$

Note that the same fixed proportion will be used for any amount of X you want to produce:

How much labor and capital would you use to produce 100 units of X? 300 units of X?

You should check that the K/L ratio will always be 0.4

Let the production function for the second good be:

$$Y = \min [ 10 L_y , 2 K_y ]$$

What are the activity requirements to produce 100 units of Y ?

Answer: 10 units of labor, 50 units of capital. The **capital-labor ratio in the Y industry** will be:

$$K_y / L_y = 50 / 10 = 5.0.$$

Since the K/L ratio in the Y industry is more than that in the X industry, Y is the **capital intensive industry**.

Suppose that the HOME country has a labor force of 100 and a capital stock of 300, and the FOREIGN country (ROW for “Rest of the world) has a labor force of 150 and a capital stock of 150.

The capital-labor ratio in home is  $K/L = 300 / 100 = 3$ ; the K/L ratio in the rest of the world is  $150 / 150 = 1$ .

The HOME country has a higher K/L ratio and therefore is **capital abundant**; ROW is labor **abundant**. The central Heckscher-Ohlin proposition is that the **capital abundant country has a comparative advantage in the capital intensive good**. Of course, the labor abundant country will have a comparative advantage in the labor intensive good.

**Logic:** the capital abundant country can produce relatively more of the capital intensive good; unless tastes (the indifference curve pattern) are radically different, the relative price of the capital intensive good will be lower in the capital abundant country; and traders will take advantage of the arbitrage opportunity to buy cheap and sell dear.

**Demonstration:** draw the PPFs of both countries.

First, calculate the maximum amounts of goods X and Y that can be produced in each country if ONLY one constraint were binding.

In HOME, the **labor constraint** is

$$L_{total} = L_x + L_y$$

or Labor available = labor used in the X industry + labor used in the Y industry (at full employment)

Substitute the given value for total labor to get (note the coefficients are the activity coefficients).

$$100 = 0.5 X + 0.1 Y$$

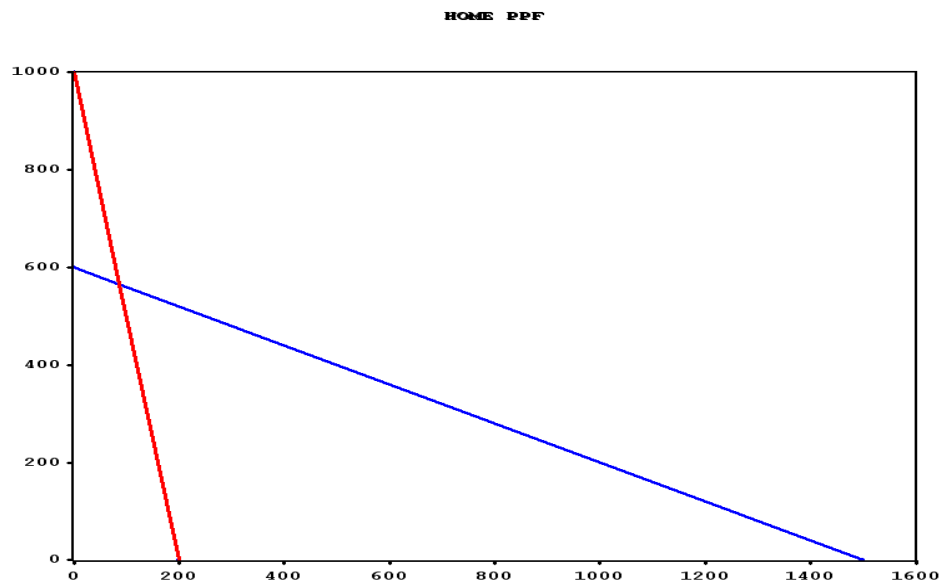
If no X is produced,  $100 / 0.1 = 1000$  units of Y can be produced;

If no Y is produced  $100 / 0.5 = 200$  units of X can be produced, according to the labor constraint.

Similarly for the capital constraint:  $K_{total} = K_x + K_y$  or  $300 = 0.2 X + 0.5 Y$

which would give a maximum value for X of 1500 and a maximum value for Y of 600.

Draw the two constraints. The area that satisfies both constraints is the PPF.



The slopes of the constraints indicate what the price ratio  $P_x / P_y$  will be if only one constraint is binding (as in the Ricardian model); if both constraints are binding (at the intersection point) the price ratio  $P_x / P_y$  will be between the two slopes. Note that the slopes are  $-0.4$  for the capital constraint, and  $-5$  for the labor constraint.

(Rewrite the labor constraint as  $Y = 10 * L_{total} - 5 X$ ; the capital constraint as  $Y = 2 * K_{total} - 0.4 X$ )

At the intersection, if  $P_y = 100$ ,  $P_x$  could be anywhere between 80 and 500, depending on demand conditions.

Repeat for the FOREIGN country:

Labor constraint:  $L_{total} = L_x + L_y$

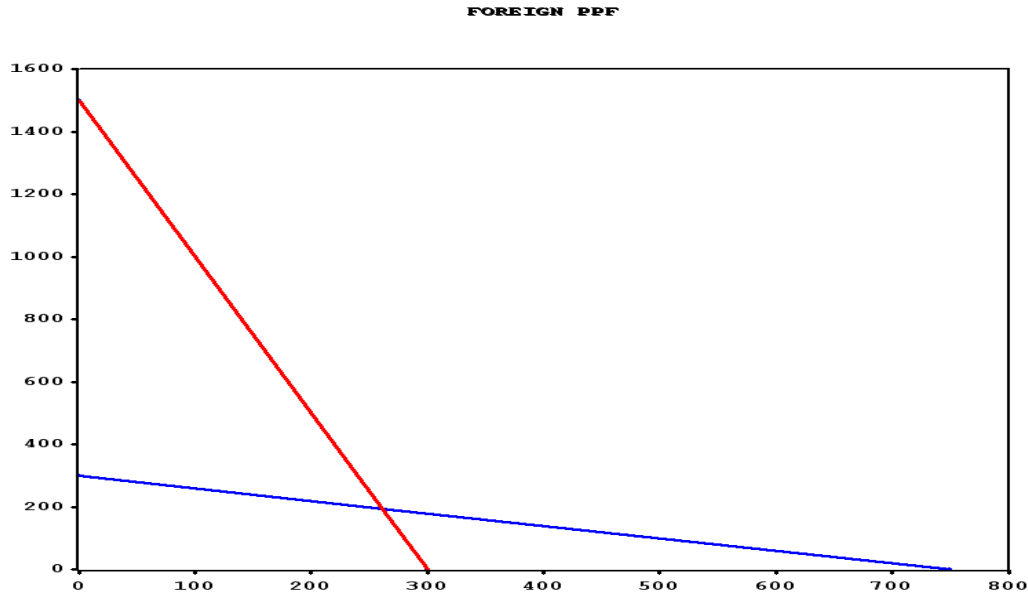
$$150 = 0.5 X + 0.1 Y$$

Maximum amount of X = 300; of Y = 1500

Capital constraint:  $K_{total} = K_x + K_y$

$$150 = 0.2 X + 0.5 Y$$

Maximum amount of X = 750; of Y = 300.



The slopes of the lines are exactly the same as for home – but the fact that the intersection of the two lines is so much farther out on the X axis makes it much more likely that:

a. the blue line will be the binding constraint

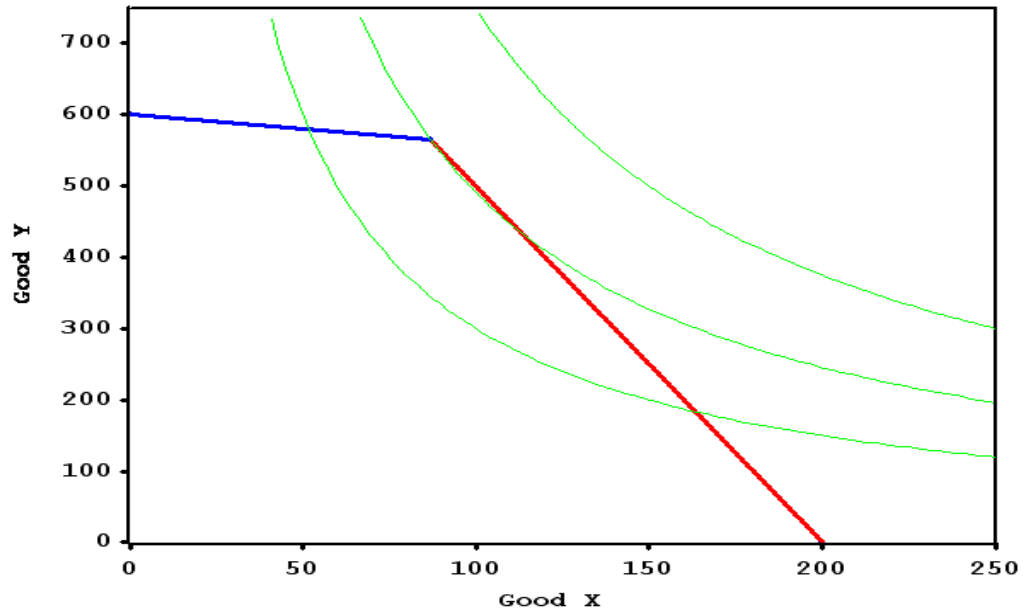
b. if the intersection is the production point, prices will again be set by the demand conditions (the slope of the indifference curve. But in foreign, the consumption bundle will include relatively more X, which means it will be less valued than the relatively scarce Y.

See the graphs on the next page, which place indifference curves through the intersection point.

Home autarky prices will be  $P_x / P_y = 5.00$ ; the indifference curve at the intersection is NOT tangent to the PPF, but cuts through it. Slightly higher utility will be attained on the labor constraint.

Foreign autarky prices will be  $P_x / P_y = 0.75$ , that is, if  $P_y = 100$ ,  $P_x = 75$

**Home PPF with indifference curves**  
Autarky Price Ratio =  $P_x / P_y = 5.00$



**Foreign PPF with indifference curve**  
Slope of IC =  $0.75 = P_x / P_y$

