MATH 2090 NUMERICAL SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS

COMPUTATIONAL PROJECT 4, DUE NOVEMBER 11, 2016

1. Consider a simple pendulum problem, which is given by two coupled nonlinear equations (see [3, 2])

$$\begin{aligned} \frac{d\theta}{dt} &= \frac{v}{L}, \\ \frac{dv}{dt} &= -g\sin\theta, \end{aligned} \tag{SP}$$

where θ, v, L and g denote, respectively, angular displacement, velocity along the arc, length of the pendulum, and the acceleration due to gravity.
Set $g = 9.8$ and $L = 49$ to observe the long-time behavior of the numerical solutions.
Choose the initial condition $(\theta_0, v_0) = (0.9\pi, 0)$ at $t = 0$ and the time step $\Delta t = 0.1$, and then numerically integrate the system using LF-RA ($\nu = 0.8$), LF-RAW ($\alpha = 0.53, \nu = 0.8$), LF-hoRA ($\beta = 0.4$) and AB3 schemes over the time interval $[0, 200]$.
Plot the numerical LF-RA, LF-RAW, LF-hoRA and AB3(θ and v) solutions versus time.

2. (bonus) Consider the Lorenz system:

$$\begin{aligned} \frac{dX}{dt} &= \sigma(Y - X), \\ \frac{dY}{dt} &= -XZ + rX - Y, \\ \frac{dY}{dt} &= XY - bZ. \end{aligned} \tag{Lorenz}$$

Choose $\sigma = 12, r = 12, b = 6$, and the initial condition $(X_0, Y_0, Z_0) = (-10, -10, 25)$ at $t = 0$ (see e.g. [1]). Integrate numerically the system over the time interval $[0, 2.5]$ using LF-RA, LF-RAW, LF-hoRA and AB3 schemes, with time step $\Delta t = 0.025$ and all other filter-parameters exactly the same as in the previous test.
Plot the X numerical solutions versus t.

3. (bonus) Consider an example of reactions between atomic oxygen (O), nitrogen oxides (NO and NO2), and ozone (O3) (see [1] for more details):

$$\begin{aligned} NO_2 + h\nu &\xrightarrow{k_1} NO + O, \\ O + O_2 &\xrightarrow{k_2} O_3, \\ NO + O_3 &\xrightarrow{k_3} O_2 + NO_2, \end{aligned}$$

where $h\nu$ denotes a photon of solar radiation. Let $c = (c_1, c_2, c_3, c_4)$ represent the concentration in molecules per cubic centimeter of O, NO, NO_2 and O_3, respectively. Assuming that the background concentration of O_2 is constant, the reactions are governed by the following system:

$$\begin{aligned}
\frac{dc_1}{dt} &= k_1 c_3 - k_2 c_1,\\
\frac{dc_2}{dt} &= k_1 c_3 - k_3 c_2 c_4,\\
\frac{dc_3}{dt} &= k_3 c_2 c_4 - k_1 c_3,\\
\frac{dc_4}{dt} &= k_2 c_1 - k_3 c_2 c_4.
\end{aligned}$$

Let

$$\begin{aligned}
&k_1 = 10^{-2}\max\{0, \sin(2\pi t/t_d\} s^{-1},\\
&k_2 = 10^{-2} s^{-1}, \qquad k_3 = 10^{-16}\mathrm{cm}^3\mathrm{molecule}^{-1} s^{-1},
\end{aligned}$$

where t_d is the length of 1 day in seconds. With initial condition $c_0 = (0, 0, 5 \times 10^{11}, 8 \times 10^{11})$ molecules cm^{-3} at $t = 0$, compute the numerical solution using LF-hoRA scheme for $\beta = 0.4$ and time step $\Delta t = 40$ s.
Plot the LF-hoRA numerical solutions for O, NO, NO_2, NO_3 versus time.

REFERENCES

[1] D. R. Durran, *Numerical methods for fluid dynamics*, vol. 32 of Texts in Applied Mathematics, Springer, New York, second ed., 2010. With applications to geophysics.

[2] Y. Li and C. Trenchea, *A higher-order Robert–Asselin type time filter*, J. Comput. Phys., 259 (2014), pp. 23–32.

[3] P. D. Williams, *A proposed modification to the Robert–Asselin time filter*, Mon. Wea. Rev., 137 (2009), pp. 2538–2546.