MATH 2090 NUMERICAL SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS

COMPUTATIONAL PROJECT 3, DUE OCTOBER 21, 2016

1. Write a code implementing the midpoint (leapfrog) method (4.21), and the Adams-Bashforth 3 method (5.9) for solving the initial value problem:

$$x'(t) = f(t, x(t)), \quad t \in (0, t_f] \tag{0.1}$$
$$x(0) = x_0. \tag{0.2}$$

   (a) Run your code for the (oscillation equation/Dahlquist test)

$$x'(t) = i\omega\, x(t), \quad t \in (0, 10], \tag{0.3}$$ {eq:osc}
$$x(0) = 1, \tag{0.4}$$

   where i is the imaginary unit, $\omega = 6$ and $h = 0.2, 0.15, 0.1, 0.08$.

   (b) Plot the LF and AB3 solutions versus the exact solution $x_{\text{exact}}(t) = e^{i\omega t} \equiv (\cos\omega t, \sin\omega t)$.

   (c) Comment on the amplitudes $|x_{\text{LF}}|, |x_{\text{AB3}}|, |x_{\text{exact}}|$. What about the frequencies?

2. (bonus) The (hoRA) time filter in conjunction with the LF method applied to the oscillation equation (0.3) is: $\forall n \geq 2$

$$v_{n+1} = u_{n-1} + 2\Delta t\, i\omega\, v_n, \tag{LF}$$
$$u_n = v_n + \frac{\beta}{2}\big(v_{n+1} - 2v_n + u_{n-1}\big) - \frac{\beta}{2}\big(v_n - 2u_{n-1} + u_{n-2}\big), \tag{hoRA}$$

   given initial conditions u_0, u_1, v_2. By eliminating the "temporary" variables v_{n+1}, v_n, (LFhoRA) can be written as the following LMM:

$$u_{n+1} - 2\beta u_n - (1 - 2\beta)u_{n-1} = i\omega\Delta t\big(2u_n - 3\beta u_{n-1} + \beta u_{n-2}\big). \tag{LFhoRA-LMM}$$

   (a) On the same graph plot the root locus curves of LFhoRA, AB3 and LF for $\beta = 0.1, 0.25, 0.4$. What are the intersections of each root locus curve with the imaginary axis?

   (b) Derive the order of consistency for the LFhoRA method.

3. (bonus) The Robert-Asselin-Williams (RAW) time filter in conjunction with the LF method applied to the oscillation equation (0.3) is

$$w_{n+1} = u_{n-1} + 2\Delta t\, i\omega\, v_n, \tag{LF}$$
$$u_n = v_n + \frac{\nu\alpha}{2}(w_{n+1} - 2v_n + u_{n-1}), \tag{RA}$$
$$v_{n+1} = w_{n+1} + \frac{\nu(\alpha - 1)}{2}(w_{n+1} - 2v_n + u_{n-1}). \tag{W}$$

(a) Eliminate the "temporary" variables w_{n+1}, v_{n+1}, v_n to write (LFRAW) as a LMM.

(b) Plot the root locus curve of the LMM with $\nu = 0.1, \alpha = 0.53$. Does it intersect the imaginary axis? What if $\nu = .2, \alpha = 0.5$?